

# MATHEMATICS

1. A variable line  $\frac{x}{a} + \frac{y}{b} = 1$  is such that  $a + b = 4$ . The locus of the midpoint of the portion of the line intercepted between the axes is
- 1)  $x + y = 4$
  - 2)  $x + y = 8$
  - 3)  $x + y = 1$
  - 4)  $x + y = 2$
2. The point  $(5, -7)$  lies outside the circle
- 1)  $x^2 + y^2 - 8x = 0$
  - 2)  $x^2 + y^2 - 5x + 7y = 0$
  - 3)  $x^2 + y^2 - 5x + 7y - 1 = 0$
  - 4)  $x^2 + y^2 - 8x + 7y - 2 = 0$
3. If the circles  $x^2 + y^2 = 9$  and  $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$  touch each other internally, then  $\alpha =$
- 1)  $\pm \frac{4}{3}$
  - 2) 1
  - 3)  $\frac{4}{3}$
  - 4)  $-\frac{4}{3}$
4. The locus of the midpoints of the line joining the focus and any point on the parabola  $y^2 = 4ax$  is a parabola with the equation of directrix as
- 1)  $x + a = 0$
  - 2)  $2x + a = 0$
  - 3)  $x = 0$
  - 4)  $x = \frac{a}{2}$
5. The tangents drawn at the extremities of a focal chord of the parabola  $y^2 = 16x$
- 1) intersect on  $x = 0$
  - 2) intersect on the line  $x + 4 = 0$
  - 3) intersect at an angle of  $60^\circ$
  - 4) intersect at an angle of  $45^\circ$

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(Space for Rough Work)

6. On the set  $Z$ , of all integers  $*$  is defined by  $a * b = a + b - 5$ . If  $2 * (x * 3) = 5$  then  $x =$
- 1) 0
  - 2) 3
  - 3) 5
  - 4) 10
7. Which of the following is false ?
- 1) Addition is commutative in  $N$ .
  - 2) Multiplication is associative in  $N$ .
  - 3) If  $a * b = a^b$  for all  $a, b \in N$  then  $*$  is commutative in  $N$ .
  - 4) Addition is associative in  $N$ .
8. If  $\vec{a} \cdot \hat{i} = \vec{a} \cdot (\hat{i} + \hat{j}) = \vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$  then  $\vec{a} =$
- 1)  $\hat{i} + \hat{j}$
  - 2)  $\hat{i} - \hat{k}$
  - 3)  $\hat{i}$
  - 4)  $\hat{i} + \hat{j} - \hat{k}$
9. If  $\vec{a}$  and  $\vec{b}$  are unit vectors and  $|\vec{a} + \vec{b}| = 1$  then  $|\vec{a} - \vec{b}|$  is equal to
- 1)  $\sqrt{2}$
  - 2) 1
  - 3)  $\sqrt{5}$
  - 4)  $\sqrt{3}$
10. The projection of  $\vec{a} = 3\hat{i} - \hat{j} + 5\hat{k}$  on  $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$  is
- 1)  $\frac{8}{\sqrt{35}}$
  - 2)  $\frac{8}{\sqrt{39}}$
  - 3)  $\frac{8}{\sqrt{14}}$
  - 4)  $\sqrt{14}$

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(Space for Rough Work)



16. The equation  $\frac{x^2}{2-\lambda} - \frac{y^2}{\lambda-5} - 1 = 0$  represents an ellipse if

1)  $\lambda > 5$

2)  $\lambda < 2$

3)  $2 < \lambda < 5$

4)  $2 > \lambda > 5$

17. The equation to the normal to the hyperbola  $\frac{x^2}{16} - \frac{y^2}{9} = 1$  at  $(-4, 0)$  is

1)  $2x - 3y = 1$

2)  $x = 0$

3)  $x = 1$

4)  $y = 0$

18. The converse of the contrapositive of the conditional  $p \rightarrow \sim q$  is

1)  $p \rightarrow q$

2)  $\sim p \rightarrow \sim q$

3)  $\sim q \rightarrow p$

4)  $\sim p \rightarrow q$

19. The perimeter of a certain sector of a circle is equal to the length of the arc of the semicircle. Then the angle at the centre of the sector in radians is

1)  $\pi - 2$

2)  $\pi + 2$

3)  $\frac{\pi}{3}$

4)  $\frac{2\pi}{3}$

20. The value of  $\tan 67\frac{1}{2}^\circ + \cot 67\frac{1}{2}^\circ$  is

1)  $\sqrt{2}$

2)  $3\sqrt{2}$

3)  $2\sqrt{2}$

4)  $2 - \sqrt{2}$

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(Space for Rough Work)

21. If  $e_1$  and  $e_2$  are the eccentricities of a hyperbola  $3x^2 - 3y^2 = 25$  and its conjugate, then

1)  $e_1^2 + e_2^2 = 2$

2)  $e_1^2 + e_2^2 = 4$

3)  $e_1 + e_2 = 4$

4)  $e_1 + e_2 = \sqrt{2}$

22. If  $p$  and  $q$  are prime numbers satisfying the condition  $p^2 - 2q^2 = 1$ , then the value of  $p^2 + 2q^2$  is

1) 5

2) 15

3) 16

4) 17

23. If  $A(\text{adj } A) = 5I$  where  $I$  is the identity matrix of order 3, then  $|\text{adj } A|$  is equal to

1) 125

2) 25

3) 5

4) 10

24. The number of solutions for the equation  $\sin 2x + \cos 4x = 2$  is

1) 0

2) 1

3) 2

4) Infinite

25.  $\int e^x \cdot x^5 dx$  is

1)  $e^x [x^5 + 5x^4 + 20x^3 + 60x^2 + 120x + 120] + C$

2)  $e^x [x^5 - 5x^4 - 20x^3 - 60x^2 - 120x - 120] + C$

3)  $e^x [x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120] + C$

4)  $e^x [x^5 + 5x^4 + 20x^3 - 60x^2 - 120x + 120] + C$

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(Space for Rough Work)

26. If  $f(x)$  is an even function and  $f'(x)$  exists, then  $f'(e) + f'(-e)$  is

1)  $> 0$

2)  $0$

3)  $\geq 0$

4)  $< 0$

27. If  $\alpha$  is a complex number satisfying the equation  $\alpha^2 + \alpha + 1 = 0$  then  $\alpha^{31}$  is equal to

1)  $\alpha$

2)  $\alpha^2$

3)  $1$

4)  $i$

28. The derivative of  $\sin(x^3)$  w.r.t.  $\cos(x^3)$  is

1)  $-\tan(x^3)$

2)  $\tan(x^3)$

3)  $-\cot(x^3)$

4)  $\cot(x^3)$

29. A unit vector perpendicular to both the vectors  $\hat{i} + \hat{j}$  and  $\hat{j} + \hat{k}$  is

1)  $\frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

2)  $\frac{\hat{i} + \hat{j} - \hat{k}}{3}$

3)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

4)  $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

30. If  $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$  and  $B = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$  then

1)  $A = -B$

2)  $A = B$

3)  $B = 0$

4)  $B = A^2$

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(Space for Rough Work)

31. The locus of a point which moves such that the sum of its distances from two fixed points is a constant is
- 1) a circle
  - 2) a parabola
  - 3) an ellipse
  - 4) a hyperbola
32. The centroid of the triangle  $ABC$  where  $A \equiv (2, 3)$ ,  $B \equiv (8, 10)$  and  $C \equiv (5, 5)$  is
- 1)  $(5, 6)$
  - 2)  $(6, 5)$
  - 3)  $(6, 6)$
  - 4)  $(15, 18)$
33. If  $3x^2 + xy - y^2 - 3x + 6y + K = 0$  represents a pair of lines, then  $K =$
- 1) 0
  - 2) 9
  - 3) 1
  - 4) -9
34. The equation of the smallest circle passing through the points  $(2, 2)$  and  $(3, 3)$  is
- 1)  $x^2 + y^2 + 5x + 5y + 12 = 0$
  - 2)  $x^2 + y^2 - 5x - 5y + 12 = 0$
  - 3)  $x^2 + y^2 + 5x - 5y + 12 = 0$
  - 4)  $x^2 + y^2 - 5x + 5y - 12 = 0$
35. The characteristic roots of the matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$  are
- 1) 1, 3, 6
  - 2) 1, 2, 4
  - 3) 4, 5, 6
  - 4) 2, 4, 6

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(Space for Rough Work)

36. If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ , then  $A^{-1} =$

1)  $\frac{-1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

2)  $\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

3)  $\begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$

4)  $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

37. The set  $\{-1, 0, 1\}$  is not a multiplicative group because of the failure of

1) Closure law

2) Associative law

3) Identity law

4) Inverse law

38. The angle of elevation of the top of a TV tower from three points  $A, B$  and  $C$  in a straight line through the foot of the tower are  $\alpha, 2\alpha$  and  $3\alpha$  respectively. If  $AB = a$ , the height of the tower is

1)  $a \tan \alpha$

2)  $a \sin \alpha$

3)  $a \sin 2\alpha$

4)  $a \sin 3\alpha$

39. The angles  $A, B$  and  $C$  of a triangle  $ABC$  are in A.P. If  $b : c = \sqrt{3} : \sqrt{2}$ , then the angle  $A$  is

1)  $30^\circ$

2)  $15^\circ$

3)  $75^\circ$

4)  $45^\circ$

40.  $\sin \left( 2 \sin^{-1} \sqrt{\frac{63}{65}} \right) =$

1)  $\frac{2\sqrt{126}}{65}$

2)  $\frac{4\sqrt{65}}{65}$

3)  $\frac{8\sqrt{63}}{65}$

4)  $\frac{\sqrt{63}}{65}$

(Space for Rough Work)



41. The general solution of  $|\sin x| = \cos x$  is (when  $n \in \mathbb{Z}$ ) given by

1)  $n\pi + \frac{\pi}{4}$

2)  $2n\pi \pm \frac{\pi}{4}$

3)  $n\pi \pm \frac{\pi}{4}$

4)  $n\pi - \frac{\pi}{4}$

42. The real root of the equation  $x^3 - 6x + 9 = 0$  is

1) -6

2) -9

3) 6

4) -3

43. The digit in the unit's place of  $5^{834}$  is

1) 0

2) 1

3) 3

4) 5

44. The remainder when  $3^{100} \times 2^{50}$  is divided by 5 is

1) 1

2) 2

3) 3

4) 4

45.  $\int \frac{\sin x \cos x}{\sqrt{1 - \sin^4 x}} dx =$

1)  $\frac{1}{2} \sin^{-1}(\sin^2 x) + C$

2)  $\frac{1}{2} \cos^{-1}(\sin^2 x) + C$

3)  $\tan^{-1}(\sin^2 x) + C$

4)  $\tan^{-1}(2 \sin x) + C$

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(Space for Rough Work)

46. The value of  $\int_{-2}^2 (ax^3 + bx + c) dx$  depends on the

- |                 |                          |
|-----------------|--------------------------|
| 1) value of $b$ | 2) value of $c$          |
| 3) value of $a$ | 4) values of $a$ and $b$ |

47. The area of the region bounded by  $y = 2x - x^2$  and the  $x$ -axis is

- |                            |                            |
|----------------------------|----------------------------|
| 1) $\frac{8}{3}$ sq. units | 2) $\frac{4}{3}$ sq. units |
| 3) $\frac{7}{3}$ sq. units | 4) $\frac{2}{3}$ sq. units |

48. The differential equation  $y \frac{dy}{dx} + x = c$  represents

- 1) a family of hyperbolas
- 2) a family of circles whose centres are on the  $y$ -axis
- 3) a family of parabolas
- 4) a family of circles whose centres are on the  $x$ -axis

49. If  $f(x^5) = 5x^3$ , then  $f'(x) =$

- |                            |                          |
|----------------------------|--------------------------|
| 1) $\frac{3}{5\sqrt{x^2}}$ | 2) $\frac{3}{5\sqrt{x}}$ |
| 3) $\frac{3}{x}$           | 4) $5\sqrt{x}$           |

50.  $f(x) = 2a - x$  in  $-a < x < a$   
 $= 3x - 2a$  in  $a \leq x$ .

Then which of the following is true?

- |   |  |
|---|--|
| 1) $f(x)$ is discontinuous at $x = a$         | 2) $f(x)$ is not differentiable at $x = a$ |
| 3) $f(x)$ is differentiable at all $x \geq a$ | 4) $f(x)$ is continuous at all $x < a$     |

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(Space for Rough Work)

51. The maximum area of a rectangle that can be inscribed in a circle of radius 2 units is (in square units)

1) 4

2)  $8\pi$

3) 8

4) 5

52. If  $Z$  is a complex number such that  $Z = -\bar{Z}$ , then

1)  $Z$  is purely real

2)  $Z$  is purely imaginary

3)  $Z$  is any complex number

4) Real part of  $Z$  is the same as its imaginary part

53. The value of  $\sum_{K=1}^6 \left[ \sin \frac{2K\pi}{7} - i \cos \frac{2K\pi}{7} \right]$  is

1)  $i$

2) 0

3)  $-i$

4)  $-1$

54.  $\lim_{x \rightarrow \infty} x \sin \left( \frac{2}{x} \right)$  is equal to

1)  $\infty$

2) 0

3) 2

4)  $\frac{1}{2}$

55. A stone is thrown vertically upwards and the height  $x$  ft. reached by the stone in  $t$  seconds is given by  $x = 80t - 16t^2$ . The stone reaches the maximum height in

1) 2 seconds

2) 2.5 seconds

3) 3 seconds

4) 1.5 seconds

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(Space for Rough Work)

56. The maximum value of  $\frac{\text{Log } x}{x}$  in  $(2, \infty)$  is

1) 1

2)  $\frac{2}{e}$

3)  $e$

4)  $\frac{1}{e}$

57. If  $f(x) = be^{ax} + ae^{bx}$ , then  $f''(0) =$

1) 0

2)  $2ab$

3)  $ab(a+b)$

4)  $ab$

58. If  $\sqrt{\frac{1+\text{Cos } A}{1-\text{Cos } A}} = \frac{x}{y}$ , then the value of  $\text{Tan } A =$

1)  $\frac{x^2 + y^2}{x^2 - y^2}$

2)  $\frac{2xy}{x^2 + y^2}$

3)  $\frac{2xy}{x^2 - y^2}$

4)  $\frac{2xy}{y^2 - x^2}$

59.  $\int \frac{\text{Sec } x}{\text{Sec } x + \text{Tan } x} dx =$

1)  $\text{Tan } x - \text{Sec } x + C$

2)  $\text{Log } (1 + \text{Sin } x) + C$

3)  $\text{Sec } x + \text{Tan } x + C$

4)  $\text{Log } \text{Sin } x + \text{Log } \text{Cos } x + C$

60. If  $\int f(x) dx = g(x)$ , then  $\int f(x) g(x) dx =$

1)  $\frac{1}{2} f^2(x)$

2)  $\frac{1}{2} g^2(x)$

3)  $\frac{1}{2} [g'(x)]^2$

4)  $f'(x) g(x)$

(Space for Rough Work)