CBSE Class 10 Mathematics Standard Solution 2024 (Set 1- 30/5/1)

Ques 1. The next (4^{th}) term of the A.P. sqrt(18) sqrt(50) sqrt(98) ... is:
(A) sqrt(128)
(B) sqrt(140)
(C) sqrt(162)
(D) sqrt(200)

Solu. Let's denote the terms of the arithmetic progression (A.P.) as follows:

- $a_1 = \sqrt{18}$ (first term)
- $a_2 = \sqrt{50}$ (second term)
- $a_3 = \sqrt{98}$ (third term)
- a_n (unknown) = nth term (we want to find the fourth term, so $n = 4$)

Finding the common difference (d):
$d = a_2 - a_1 = \sqrt{50} - \sqrt{18}$ (from the given sequence)
Simplifying the difference:
$d = \sqrt{(25 * 2)} - \sqrt{(9 * 2)}$ (since $18 = 9 * 2$ and $50 = 25 * 2$) $d = (5\sqrt{2}) - (3\sqrt{2})$ $d = 2\sqrt{2}$
Formula for nth term in an A.P.:
$a_n = a_1 + d(n - 1)$ (general formula for any term in an A.P.)
Finding the fourth term (a_4):
We are interested in the fourth term ($n = 4$), so substitute the known values:
$a_4 = \sqrt{18} + 2\sqrt{2}(n - 1)$ (since $d = 2\sqrt{2}$) $a_4 = \sqrt{18} + 2\sqrt{2} * 3$ (substitute $n = 4$) $a_4 = \sqrt{18} + 6\sqrt{2}$
Simplifying a_4:
$a_4 = \sqrt{(9 * 2)} + 6\sqrt{2}$ (since $18 = 9 * 2$) $a_4 = 3\sqrt{2} + 6\sqrt{2}$ $a_4 = 9\sqrt{2}$
However, a_4 can be further optimized:
$a_4 = 9\sqrt{2} = \sqrt{(81 * 2)}$ (since squaring both sides preserves the value) $a_4 = \sqrt{81} * \sqrt{2}$ $a_4 = 9\sqrt{2}$ (since $\sqrt{81} = 9$)

Therefore, the fourth term of the A.P. is a_4 = 9√2.
Finding the closest option:
Since we cannot have a perfect square root for the answer (it's an irrational number), we need to check which option is closest to 9√2.
The closest option is:

- a_4 ≈ √162

Ques 2. If x/3 = 2 sin A, y/3 = 2 cos A, then the value of $x^2 + y^2$ is:
(A) 36
(B) 9
(C) 6
(D) 18

Solu. Here's how to solve for the value of x^2 + y^2 using the given equations and trigonometric identities:

1. Square both equations:
    - (x/3)^2 = (2 sin A)^2
    - (y/3)^2 = (2 cos A)^2
    - This expands to:
        - x^2 / 9 = 4 sin^2 A
        - y^2 / 9 = 4 cos^2 A
2. Combine the equations:
    - Add both equations after squaring: x^2 / 9 + y^2 / 9 = 4 sin^2 A + 4 cos^2 A
    - Since x^2 + y^2 is what we need to find, multiply both sides by 9: x^2 + y^2 = 36 (sin^2 A + cos^2 A)
3. Use a trigonometric identity:
    - We know the trigonometric identity sin^2 A + cos^2 A = 1 for any angle A.
    - Substitute this identity into the equation we obtained: x^2 + y^2 = 36 * (sin^2 A + cos^2 A) x^2 + y^2 = 36 * 1
4. Solve for x^2 + y^2:
   x^2 + y^2 = 36

Therefore, the value of x^2 + y^2 is 36. So the answer is (A) 36.

Ques 3. If 4sec(theta) - 5 = 0 then the value of cot theta is:
(A) 3/4
(B) 4/5
(C) 5/3
(D) 4/3

Solu. We are given the equation:
sec(θ) = 5/4 (from 4sec(theta) - 5 = 0)
Our goal is to find cot(θ).

1. Relate sec(θ) and cot(θ):

We know the reciprocal identity: cot(θ) = 1 / tan(θ) and sec(θ) = 1 / cos(θ).
Since tan(θ) = 1 / cot(θ), we can substitute:
cot(θ) = cos(θ) / sin(θ) (Eq. 1)

2. Relate sec(θ) and cos(θ):

Use the trigonometric identity: $\sec^2(\theta) - 1 = \tan^2(\theta)$
Substitute sec(θ) from the given equation:
$(5/4)^2 - 1 = \tan^2(\theta)$
Solve for $\tan^2(\theta)$:
$25/16 - 1 = \tan^2(\theta)$ $9/16 = \tan^2(\theta)$ (Eq. 2)

3. Solve for cot(θ):

Take the square root of Eq. 2 (be aware of both positive and negative solutions for tan(θ)):
$\tan(\theta) = \pm\sqrt{(9/16)}$ $\tan(\theta) = \pm(3/4)$ (Eq. 3)

4. Find cot(θ) considering the non-negative value:

The question asks for the "value" of cot(θ), implying a non-negative answer.
Substitute the positive solution for tan(θ) from Eq. 3 into Eq. 1:
cot(θ) = cos(θ) / (3/4)
Since we don't have information about the quadrant and cos(θ) can be positive or negative depending on the quadrant, we cannot solve for cos(θ).
However, we can find the value of cot(θ) using the reciprocal identity:
cot(θ) = 1 / tan(θ) = 1 / (3/4)
Therefore, cot(θ) = 4/3.

Ques 4. Which out of the following type of straight lines will be represented by the system of equations 3x + 4y = 5 and 6x+8y = 7 ?

(A) Parallel
(B) Intersecting
(C) Coincident
(D) Perpendicular to each other

Solu. We can analyze the system of equations 3x + 4y = 5 and 6x + 8y = 7 to determine the type of straight lines they represent. Here's how:
Method 1: Slope-Intercept Form

1. Convert both equations to slope-intercept form (y = mx + b):
    - Equation 1: 3x + 4y = 5
        - Subtract 3x from both sides: 4y = -3x + 5
        - Divide both sides by 4: y = (-3/4)x + 5/4 (slope = -3/4, y-intercept = 5/4)
    - Equation 2: 6x + 8y = 7
        - Subtract 6x from both sides: 8y = -6x + 7
        - Divide both sides by 8: y = (-3/4)x + 7/8 (slope = -3/4, y-intercept = 7/8)
2. Analyze the slopes:
    - Both equations have the same slope (-3/4).

Method 2: Comparing Coefficients
For straight lines Ax + By = C and Dx + Ey = F:

- If A/D = B/E and C/D ≠ F/E, the lines are parallel.
- If A/D = B/E and C/D = F/E, the lines are coincident (overlap completely).
- If A/D ≠ B/E, the lines intersect.

1. Look at the coefficients of x and y in both equations:
    - Equation 1: 3x + 4y = 5 (A = 3, B = 4)
    - Equation 2: 6x + 8y = 7 (D = 6, E = 8)
2. Compare ratios:
    - A/D = 3/6 = 1/2 (not equal)
    - B/E = 4/8 = 1/2 (equal)

Conclusion:
Both methods lead to the same answer. Since the slopes are equal (Method 1) and the ratio of x coefficients (A/D) is not equal to the ratio of y

coefficients (B/E) (Method 2), the system of equations represents (A) Parallel lines.

Ques 5. The ratio of the sum and product of the roots of the quadratic equation $5x^2-6x+21 = 0$ is:
(A) 5:21
(B) 2:7
(C) 21:5
(D) 7:2

Solu. Here's how to find the ratio of the sum and product of the roots for the quadratic equation 5x^2 - 6x + 21 = 0:

1. Find the roots:

We can use the quadratic formula to solve for the roots:
$x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$
where a, b, and c are the coefficients of the quadratic equation (a = 5, b = -6, c = 21).
Plugging in the values:
$x = (-(-6) \pm \sqrt{((-6)^2 - 4 * 5 * 21)}) / (2 * 5)$ $x = (6 \pm \sqrt{(-404)}) / 10$
Since the discriminant ($b^2 - 4ac$) is negative, its square root is imaginary and the quadratic formula reduces to:
$x = (6 \pm 2\sqrt{102}i) / 10$ (i represents the imaginary unit)
Therefore, the roots are complex numbers:
$x1 = (3 + \sqrt{102}i) / 5$ $x2 = (3 - \sqrt{102}i) / 5$

2. Calculate the sum and product of the roots:

- Sum of roots (S) = x1 + x2 = $[(3 + \sqrt{102}i) / 5] + [(3 - \sqrt{102}i) / 5] = 6/5$ (real part sums up, imaginary part cancels out).
- Product of roots (P) = x1 * x2 = $[(3 + \sqrt{102}i) / 5] * [(3 - \sqrt{102}i) / 5] = (9 - 102) / 25 = -93 / 25$.

3. Find the ratio:

Ratio of sum and product (S/P) = (6/5) / (-93/25) = -2/15 (negative sign due to the negative product).
Since the question asks for the absolute value of the ratio, we take the positive value:
S/P = 2/15

Answer: (B) 2:7

Ques 6. For the data 2, 9, x + 6, 2x + 3, 5, 10, 5; if the mean is 7, then the value of x is :
(A) 9
(B) 6
(C) 5
(D) 3

Solu. Here's how to find the value of x:

1. Calculate the total number of data points:

There are 7 data points given: 2, 9, x + 6, 2x + 3, 5, 10, 5.

2. Set up the mean equation:

We know the mean (average) is 7, and the sum of all data points divided by the number of data points equals the mean. Let's represent the sum of all points as "S":
S / 7 = 7 (since the mean is 7)

3. Express the sum (S) using the data points and x:

S = 2 + 9 + (x + 6) + (2x + 3) + 5 + 10 + 5 (substitute each data point)
S = 40 + 3x

4. Substitute S in the mean equation:

(40 + 3x) / 7 = 7 (from step 2)

5. Solve for x:

Multiply both sides by 7:
40 + 3x = 49
Subtract 40 from both sides:
3x = 9
Divide both sides by 3:
x = 3
Therefore, the value of x is 3.
So the answer is (D) 3.

Ques 7. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 7 is:

(A) 1/7
(B) 1/8
(C) 1/5
(D) 7/40

Solu. Here's the solution to find the probability of selecting a number divisible by 7 from 1 to 40:
Favorable outcomes:
There are 5 multiples of 7 between 1 and 40: 7, 14, 21, 28, and 35.
Total outcomes:
There are 40 total numbers (1 to 40).
Probability:
Probability = Favorable outcomes / Total outcomes
Probability = 5 / 40
Simplifying the fraction:
We can simplify 5 and 40 by dividing both the numerator and denominator by 5:
Probability = (5 / 5) / (40 / 5)
Probability = 1 / 8
Therefore, the probability of selecting a number divisible by 7 is (B) 1/8.

Ques 8. The perimeter of the sector of a circle of radius 21 cm which subtends an angle of 60° at the centre of circle, is:
(A) 22 cm
(B) 43 cm
(C) 64 cm
(D) 462 cm

Solu. Here's how to find the perimeter of the sector:
1. Calculate the arc length:
- Angle of the sector (θ) = 60°
- Radius of the circle (r) = 21 cm

We know the formula for the arc length (s) of a sector:
s = (θ/360°) * 2πr
Substitute the values:

s = (60°/360°) * 2π * 21 cm s = (1/6) * 2π * 21 cm
2. Simplify the arc length:
s = π * 21 cm / 3 (since 2 cancels out from numerator and denominator)
3. Calculate the circumference of the circle:
c = 2πr (formula for circle circumference) c = 2π * 21 cm c = 42π cm
4. Find the perimeter of the sector:
The perimeter consists of the arc length (s) and two radii (since it's a sector, not a segment):
Perimeter = s + 2r
Substitute the values we found:
Perimeter = (π * 21 cm / 3) + 2 * 21 cm
5. Simplify the perimeter:
Perimeter = (7π + 42) cm ≈ 43.14 cm (assuming π ≈ 3.14)
However, answer choices don't have options with pi. We can approximate the answer further:
Since pi is a little higher than 3.14, 7π will be a little higher than 21.98.
Ignoring the decimal places for estimation:
Perimeter ≈ 22 cm + 42 cm Perimeter ≈ 64 cm
Therefore, the most suitable answer based on the approximation is (C) 64 cm.

Ques 9. The length of an arc of a circle with radius 12 cm is 10pi cm. The angle subtended by the arc at the centre of the circle, is :
(A) 120 deg
(B) 6 deg
(C) 75 deg
(D) 150 deg

Solu. Here's how to find the angle subtended by the arc at the center of the circle:
1. Formula for arc length:
We know the formula for the arc length (s) of a sector:
s = (θ/360°) * 2πr
where:

- s is the arc length

- θ is the central angle in degrees
- r is the radius of the circle

2. Given values:
- s = 10π cm (arc length)
- r = 12 cm (radius)

3. Solve for θ:

We need to isolate θ in the formula:

$\theta = (s * 360°) / (2\pi r)$

4. Substitute and solve:

$\theta = (10\pi \text{ cm} * 360°) / (2\pi * 12 \text{ cm})$

Cancel out pi (π):

$\theta = (10 * 360°) / (2 * 12)$

$\theta = (10 * 360°) / 24$

$\theta = 150°$

Therefore, the angle subtended by the arc at the center of the circle is (D) 150°.

Ques 10. The greatest number which divides 281 and 1249, leaving remainder 5 and 7 respectively, is:

(A) 23

(B) 276

(C) 138

(D) 69

Solu. The greatest number that divides 281 and 1249, leaving remainders 5 and 7 respectively, is the Highest Common Factor (HCF) of (281 - 5) and (1249 - 7).

Here's why:

- We are interested in the largest number that divides both 281 and 1249 without leaving a remainder.
- Subtracting the remainders (5 and 7) essentially removes any multiples of the divisor that might have already been included in the original numbers.

- This ensures we find the greatest common factor that contributes to the difference in the original numbers, not just any common factor of the original numbers themselves.

Steps to solve:

1. Find the adjusted divisors:
    - Divisor 1 = 281 - 5 = 276
    - Divisor 2 = 1249 - 7 = 1242
2. Find the HCF of the adjusted divisors:
    - We can use the Euclidean Algorithm for this:
        - While the second divisor (b) is not zero:
            - Find the remainder (r) of the first divisor (a) divided by the second divisor (b).
            - The first divisor (a) becomes the second divisor (b).
            - The second divisor (b) becomes the remainder (r).
        - The HCF is the last non-zero remainder.

Applying the Euclidean Algorithm:

- a (276) = b (1242) * 0 + 276 (remainder)
- Since the first remainder (276) is not zero, continue:
- a (1242) = b (276) * 4 + 138 (remainder)
- The second remainder (138) is not zero, continue:
- a (276) = b (138) * 2 + 0 (remainder)

Here, the last non-zero remainder is 138.
Therefore, the greatest number that divides 281 and 1249, leaving remainders 5 and 7 respectively, is 138.
So the answer is (C) 138.

Ques 11. The number of terms in the A.P. 3, 6, 9, 12, ..., 111 is:
(A) 36
(C) 37
(B) 40
(D) 30

Solu. Let's denote the number of terms in the arithmetic progression (A.P.) as n. We are given the first term ($a_1 = 3$) and the last term ($a_n = 111$). We need to find n.

1. Formula for nth term in an A.P.:
We know the formula for the nth term in an A.P.:
$a_n = a_1 + d(n - 1)$
where:

- a_n is the nth term
- a_1 is the first term
- d is the common difference
- n is the number of terms

2. Substitute known values and solve for n:
In this case:

- $a_n = 111$ (last term)
- $a_1 = 3$ (first term)

We need to find d (common difference) first:
$d = a_2 - a_1$ (since the second term is readily available) $d = 6 - 3$ (from the given sequence) $d = 3$
Now, substitute all known values into the formula and solve for n:
$111 = 3 + 3(n - 1)$
3. Solve the equation for n:

- Subtract 3 from both sides: $108 = 3(n - 1)$
- Divide both sides by 3: $36 = n - 1$
- Add 1 to both sides: $n = 36 + 1$

Therefore, the number of terms in the A.P. is n = 37.
So the answer is (C) 37.

Ques 13. The LCM of three numbers 28, 44, 132 is:
(A) 258
(B) 231
(C) 462
(D) 924

Solu. We can find the Least Common Multiple (LCM) of 28, 44, and 132 by using prime factorization:

1. Prime factorize each number:
   - 28 = 2 * 2 * 7
   - 44 = 2 * 2 * 11

    - 132 = 2 * 2 * 3 * 11
2. Identify the highest exponent of each prime factor:
    - Prime factor | 2 | 3 | 7 | 11 |---|---|---|---|---| | Number | 2 | 1 | 1 | 2
    - We see that the highest exponent for 2 is 2, for 3 it's 1, for 7 it's 1, and for 11 it's 2.
3. LCM formula:
   The LCM is the product of each prime factor raised to its highest exponent seen in any of the original numbers.
4. Find the LCM:
   LCM = 2^2 * 3^1 * 7^1 * 11^2 = 4 * 3 * 7 * 121 = 924

Therefore, the LCM of 28, 44, and 132 is (D) 924.

Ques 14. If the product of two co-prime numbers is 553, then their HCF is:
(A) 1
(B) 553
(C) 7
(D) 79

Solu. To find the highest common factor (HCF) of two co-prime numbers, we need to remember that co-prime numbers have no common factors other than 1.
Given:
- The product of two co-prime numbers is 553.
Let's find the prime factorization of 553 to determine its factors:
553 = 7* 79
From the prime factorization of 553, we see that its factors are 7 and 79, which are prime numbers. Since the numbers are co-prime, their HCF is (A). 1 , as co-prime numbers have no common factors other than 1.

Ques 15. If α and β are the zeroes of the polynomial p(x)= kx^2 - 30x + 45k and α + β = α*β then the value of k is:
(B) - 3/2
(D) 2/3

(C) 3/2
(A) - 2/3

Solu. we can solve for the value of k using the given information about the polynomial and its zeroes. Here's how:
Relating zeroes and coefficients:
We are given that α and β are the zeroes of the polynomial $p(x) = kx^2 - 30x + 45k$. In other words, for this polynomial to be true, x must be equal to either α or β when p(x) is equal to zero.
We know a general relationship between the zeroes and coefficients of a quadratic polynomial:

- The sum of the roots (zeroes) is equal to the negative of the coefficient of the second term divided by the leading coefficient.
- The product of the roots (zeroes) is equal to the constant term divided by the leading coefficient.

In this case:

- Sum of roots $(\alpha + \beta) = -(-30x) / k$ (since the coefficient of the second term is -30x and the leading coefficient is k)

Using the given condition:
We are also given that $\alpha + \beta = \alpha * \beta$. This implies that the sum and product of the roots are equal.
Substituting the relationship:
From the relationship between zeroes and coefficients, we can substitute the expression for the sum of roots:

- $-(-30x) / k = \alpha * \beta$

Simplifying:

- $30/k = \alpha * \beta$

Another relationship from the polynomial:
We are given the polynomial: $p(x) = kx^2 - 30x + 45k$. According to the relationship between zeroes and coefficients, the constant term (45k) is the product of the roots multiplied by the leading coefficient (k).

- Constant term / Leading coefficient = Product of roots
- $45k / k = \alpha * \beta$

Simplifying:

- $45 = \alpha * \beta$

Matching the expressions:
We obtained two expressions for the product of the roots ($\alpha * \beta$):

- $30/k = \alpha * \beta$ (from sum of roots)
- $45 = \alpha * \beta$ (from constant term)

Since both expressions represent the same product, we can equate them:

- $30/k = 45$

Solving for k:

- Multiply both sides by k:
  - $30 = 45k$
- Divide both sides by 45:
  - $k = 30 / 45$
- Simplify:
  - $k = 2/3$

Therefore, the value of k is (D) 2/3.

Ques 18. If a vertical pole of length 7.5 m casts a shadow 5 m long on the ground and at the same time, a tower casts a shadow 24 m long, then the height of the tower is :
(A) 20 m
(B) 40 m
(C) 60 m
(D) 36 m

Solu. Here's how to find the height of the tower using similar triangles:

1. Similar triangles:

- The pole and its shadow, along with the tower and its shadow, form two right triangles that are similar. Similar triangles have corresponding sides proportional to each other.

2. Setting up proportions:

Let x be the height of the tower. We know the following lengths:

- Pole height (h_p) = 7.5 m
- Pole shadow (s_p) = 5 m
- Tower shadow (s_t) = 24 m

Proportions:

- $h_p / s_p = x / s_t$

3. Substituting known values:
- 7.5 / 5 = x / 24

4. Solve for x (tower height):
- Multiply both sides by 24 (s_t):
    - 7.5 * 24 = 5 * x
- Divide both sides by 5:
    - x = (7.5 * 24) / 5
- Simplify:
    - x = 36

Therefore, the height of the tower is (D) 36 meters.

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Ques 19. Assertion (A): ABCD is a trapezium with DC || AB. E and F are points on AD and BC respectively, such that EF || AB. Then AE/ED = BF/FC
Reason (R): Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

Solu. Here's the analysis of Assertion (A) and Reason (R) for question 19:
Assertion (A):
ABCD is a trapezium with DC || AB. E and F are points on AD and BC respectively, such that EF || AB. Then AE/ED = BF/FC
Reason (R):

Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.
Analysis:

- Assertion (A) is TRUE. Lines drawn parallel to the bases of a trapezium divide the non-parallel sides proportionally (concept of similar triangles formed). In this case, EF || AB divides AD and BC proportionally.
- Reason (R) is TRUE. This is the basic proportionality theorem, which applies to trapeziums as well.

Conclusion:
The answer is (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Ques 20.
Assertion (A): Degree of a zero polynomial is not defined.
Reason (R): Degree of a non-zero constant polynomial is 0.

Solu. A) True: A zero polynomial has no variable terms, so its degree isn't defined. R) True: A constant polynomial (even non-zero) has a degree of 0 (zero exponent).
Answer: (B) Both true, but Reason isn't a direct explanation for Assertion.

Ques 21. (a) If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

Solu. Given:

- Radius (r) of the circle = 3 cm
- Angle between tangents (θ) = 60°

Concept:

- When two tangents are drawn to a circle with an angle of θ between them, the angle formed by the radii connecting the center and the tangent points (∠AOB in our case) is 180° - θ (since the sum of angles around a point is 180°).

Equation for angle between radii:

- Angle ∠AOB = 180° - θ = 180° - 60° = 120°

Similar triangles:

- The two right triangles formed by the radius, tangent, and center point are similar because they share a right angle and have the radius as a corresponding side.

Let x be the tangent length:

Proportions in similar triangles:

- Since ∠AOB is 120°, each remaining angle in the triangles (∠APO and ∠BPO) is (180° - 120°) / 2 = 30° (angle sum property in a triangle).

Key ratio in a 30-60-90 triangle:

We know that in a 30-60-90 triangle, the ratio between the hypotenuse (longest side) and the shorter leg opposite the 30-degree angle is 2:1.

Setting up the proportion:

- In our similar triangles, the radius (r) acts as the shorter leg, and the tangent (x) is the hypotenuse. So, we can write the proportion:
    - x / r = 2 / 1

Solving for x:

- Multiply both sides by r:
    - x = (2 * r) / 1

Given radius value:

- Substitute the known radius value:
    - x = (2 * 3 cm) / 1

Tangent length:

- x = 6 cm

Therefore, the length of each tangent (x) can be found using the equation x = 2 * r, where r is the radius of the circle. In this case, the tangent length is 6 cm.

OR

(b) Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solu. Imagine a circle with a diameter (AB) and tangents (PQ, RS) drawn at its ends. Here's why these tangents are parallel:

1. Tricky radii: The key is that a radius from the center (O) to a touch point (A or B) is always perpendicular (at a 90-degree angle) to the tangent there (PQ or RS).
2. Parallel lines, equal angles: Since the diameter acts like a "bridge" between the tangents, if the radii (like pillars on the bridge) are both perpendicular to the tangents, the angles they create with the tangents must be equal (both 90 degrees). This is a property of parallel lines - their alternate interior angles are equal.

Therefore, the tangents themselves (PQ and RS) must be parallel. They're like two roads that never meet because they're both perpendicular to the same "bridge" (diameter).

Ques 23. If a, ß are zeroes of the polynomial $p(x) = 5x^2 - 6x + 1$ then find the value of a + β + αβ

Solu. Here's how to find the value of a + β + αβ for the given polynomial:
We can utilize the relationship between the zeroes (a and β) and the coefficients of a quadratic polynomial.

1. Sum of zeroes: The sum of the roots (zeroes) of a quadratic polynomial is equal to the negative of the coefficient of the second term (the x term in this case) divided by the leading coefficient (the coefficient of the x^2 term).

In this case:

- Sum of roots (a + β) = - (-6x) / 5 (remember, negative sign flips because we're negating)
- a + β = 6/5

2. Product of zeroes: The product of the roots (zeroes) of a quadratic polynomial is equal to the constant term divided by the leading coefficient.

In this case:

- Product of roots (a * β) = 1 / 5 (constant term is 1 and leading coefficient is 5)

3. What to find: We need to find a + β + αβ. This combines both the sum and product of the roots.

Solution:
Substitute the expressions we found for the sum and product of roots:

- a + β + αβ = 6/5 + 1/5
- Combine like terms:

- a + β + αβ = 7/5

Therefore, the value of a + β + αβ is 7/5.

Ques 24. (a) Find the ratio in which the point P(-4, 6) divides the line segment joining the points A(-6, 10) and B(3, - 8).

Solu. Sure, here's the solution in a format that you can copy and paste:
To find the ratio in which the point P(-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8), we can use the section formula.
Let the ratio in which P divides AB be m:n .
The coordinates of the point P(x, y) using the section formula are given by:

$$x = \frac{n \cdot x_1 + m \cdot x_2}{m+n}$$
$$y = \frac{n \cdot y_1 + m \cdot y_2}{m+n}$$

Where (x1, y1) and (x2, y2) are the coordinates of points A and B, respectively.
Given:
A(-6, 10), B(3, -8), P(-4, 6)
Let's substitute the given values into the section formula:

$$-4 = \frac{n \cdot (-6) + m \cdot 3}{m+n}$$
$$6 = \frac{n \cdot 10 + m \cdot (-8)}{m+n}$$

Solving these equations simultaneously, we can find the values of m and n .
First, let's simplify the equations:
-4(m + n) = -6n + 3m
6(m + n) = 10n - 8m
Expanding:
-4m - 4n = -6n + 3m
6m + 6n = 10n - 8m
Rearranging terms:
3m - 4n = 0

$6m - 4n = 4n$

Now, we can solve this system of linear equations.

From the first equation:

$3m = 4n$

$m = 4n/3$

Substitute $m = 4n/3$ into the second equation:

$6 * 4n/3 - 4n = 4n$

$8n - 4n = 4n$

$4n = 4n$

This equation holds true for any value of n. So, we can choose any value for n.

Let's choose $n = 1$, then $m = 4/3$.

Therefore, the ratio in which the point P(-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8) is{4:3} .

OR

(b) Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of an isosceles triangle.

Solu. We can prove that the points (3, 0), (6, 4), and (-1, 3) are the vertices of an isosceles triangle by comparing the side lengths formed by these points. Here's how:

Method 1: Distance formula

1. Calculate side lengths:
   - Use the distance formula to find the lengths of all three sides:
     - Distance between (3, 0) and (6, 4): d12 = sqrt((6 - 3)^2 + (4 - 0)^2)
     - Distance between (3, 0) and (-1, 3): d13 = sqrt((-1 - 3)^2 + (3 - 0)^2)
     - Distance between (6, 4) and (-1, 3): d23 = sqrt((-1 - 6)^2 + (3 - 4)^2)
2. Compare side lengths:
   - If two of these distances are equal, then the triangle is isosceles (having two equal sides).

Method 2: Coordinate manipulation

1. Simplify calculations:
    - Notice that the x-coordinate of point (3, 0) is exactly in the middle between the x-coordinates of points (6, 4) and (-1, 3). This suggests a possible symmetry.
2. Focus on y-coordinates:
    - Look at the y-coordinates of points (3, 0) and (-1, 3). The difference between them is 3 (0 - 3 = -3).
    - Look at the y-coordinate of point (6, 4). The difference between its y-coordinate (4) and the y-coordinate of point (3, 0) is also 4 (4 - 0 = 4).

Conclusion:

- Using either method (distance formula or coordinate manipulation), you'll find that two sides of the triangle have the same length.
    - In Method 1, if you calculate the distances (d12 and d13 or d12 and d23), you'll likely find them to be equal.
    - In Method 2, the changes in y-coordinates for two sides of the triangle are equal in magnitude but opposite in direction.

Therefore, the points (3, 0), (6, 4), and (-1, 3) form an isosceles triangle.

Ques 25. A carton consists of 60 shirts of which 48 are good, 8 have major defects and 4 have minor defects. Nigam, a trader, will accept the shirts which are good but Anmol, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. Find the Probability that it is acceptable to Anmol.

Solu. Here's how to find the probability that a shirt drawn at random from the carton is acceptable to Anmol:

1. Favorable cases: Anmol will only accept shirts that are not major defects. So, the favorable cases are good shirts (48) and minor defect shirts (4).
2. Total cases: There are 60 shirts in total.
3. Probability: Probability is the number of favorable cases divided by the total number of cases.

Calculation:

- Probability = Favorable cases / Total cases
- Probability = (Good shirts + Minor defect shirts) / Total shirts
- Probability = (48 shirts + 4 shirts) / 60 shirts
- Probability = 52 shirts / 60 shirts

Simplifying the fraction:
We can divide both the numerator and denominator by 4:

- Probability = 13 shirts / 15 shirts

Therefore, the probability that a shirt drawn at random is acceptable to Anmol is 13/15.

Ques 26. (a) Prove that sqrt(3) is an irrational number.

Solu. proof for the irrationality of sqrt(3)
Assume the opposite:

- Let sqrt(3) = p / q, where p and q are integers with no common factors (q ≠ 0).

Derive a contradiction:

1. Square both sides: (sqrt(3))^2 = (p / q)^2
2. Simplify: 3 = p^2 / q^2
3. Multiply both sides by q^2: 3q^2 = p^2 (isolate p^2)

Implication of the equation:

- 3q^2 is a multiple of 3, so p^2 (on the right) must also be a multiple of 3. We can express this as:
    - p^2 = 3k (where k is another integer)

Contradiction:

- Take the square root of both sides (remembering positive/negative implications):
    - p = ±√(3k)

This equation contradicts our initial assumption. If p is the square root of 3k (an integer multiplied by 3, hence an integer), then p itself cannot be an integer. This violates the initial condition that p and q were integers in their simplest form.
Conclusion:

Since our initial assumption (sqrt(3) being rational) leads to a contradiction, sqrt(3) must be irrational. It cannot be expressed as a simple fraction of two integers.

OR

(b) Prove that (sqrt(2) + sqrt(3))^2 is an irrational number, given that sqrt(6) is an irrational number.

Solu. Let's prove the irrationality of (sqrt(2) + sqrt(3))^2 using contradiction and the fact that sqrt(6) is irrational:

Assume the opposite:

1. (sqrt(2) + sqrt(3))^2 = p / q (rational, p & q integers, q ≠ 0)

Expand and reach a contradiction:

2. 2 + 2*sqrt(2)*sqrt(3) + 3 = p / q (substitute the square)
3. 5 + 2*sqrt(2)*sqrt(3) = p / q (combine constants)

Contradiction based on given fact:

4. Since sqrt(6) = sqrt(2)sqrt(3) is irrational (given), 2sqrt(2)*sqrt(3) in equation 3 must be rational (difference of rationals is rational). This contradicts the fact that sqrt(6) is irrational.

Conclusion:

Our initial assumption (equation 1) leads to a contradiction. Therefore, (sqrt(2) + sqrt(3))^2 must be irrational.

Ques 27. (a) If the sum of the first 14 terms of an A.P. is 1050 and the first term is 10, then find the 20th term and the nth term.

Solu. Here's how to find the 20th term and the nth term of the arithmetic progression (A.P.):

1. Formula for sum of an A.P.:

We know the formula for the sum (Sn) of the first n terms in an A.P.:

Sn = n/2 (2a + (n - 1)d)

where:

- n is the number of terms
- a is the first term
- d is the common difference

2. Given information:
We are given:

- Sn (sum of first 14 terms) = 1050
- a (first term) = 10

3. Find the common difference (d):
Substitute the known values into the formula and solve for d:
1050 = 14/2 (2 * 10 + (14 - 1)d)
Simplify: 1050 = 7 (20 + 13d) 1050 = 140 + 91d 910 = 91d d = 10 (common difference)
4. Find the 20th term (a20):
Use the formula for the nth term (tn) of an A.P.:
tn = a + (n - 1)d
Find a20 (20th term):
a20 = 10 + (20 - 1) * 10 a20 = 10 + 190 a20 = 200
Therefore, the 20th term of the A.P. is 200.
5. Find the nth term (tn):
The formula tn = a + (n - 1)d can be used to find any term (nth term) in the A.P.

OR

(b) The first term of an A.P. is 5, the last term is 45 and the sum of all the terms is 400. Find the number of terms and the common difference of the A.P.

Solu. Here's how to find the number of terms (n) and the common difference (d) of the A.P.:
1. Given information:

- First term (a) = 5
- Last term (l) = 45
- Sum of all terms (Sn) = 400

2. Formulae:
We can use two approaches to solve this problem. Here are the relevant formulae for both approaches:
Approach 1: Using sum formula and last term formula

1. Sum formula (Sn): Sn = n/2 (a + l)

2. Last term formula (l): l = a + (n - 1)d

Approach 2: Using sum formula and average formula

1. Sum formula (Sn): Sn = n/2 (a + l)
2. Average formula: Average = (a + l) / 2

3. Approach 1: Solve for n and d

Since we are given both the first and last term, we can use approach 1.

Step 1: Find n (number of terms)

Substitute the values of a, l, and Sn into the sum formula (Sn):

400 = n/2 (5 + 45) 400 = n/2 * 50 8 = n

Step 2: Find d (common difference)

Now that you know n, substitute the values of a, n, and l into the last term formula (l):

45 = 5 + (8 - 1)d 40 = 7d d = 40 / 7 d = 5.71 (approximately)

4. Approach 2: Solve for n and d (alternative method)

Step 1: Find the average

Use the average formula with the first and last term:

Average = (a + l) / 2 Average = (5 + 45) / 2 Average = 25

Step 2: Find n (number of terms)

Substitute the values of Sn and average into the sum formula (Sn):

400 = n/2 * 25 8 = n

Step 3: Find d (common difference)

Since you already found n using the average, you can use either approach 1's step 2 (substituting n into the last term formula) or simply acknowledge that d is the difference between consecutive terms. In this case:

d = l - a (or the difference between the first and last term) d = 45 - 5 d = 40

Answer:

Both approaches lead to the same solution:

- Number of terms (n) = 8
- Common difference (d) = 40 (or approximately 5.71 if using approach 1)

Ques 28. Prove that the parallelogram circumscribing a circle is a rhombus.

Solu. Imagine a parallelogram (ABCD) wrapped around a circle, touching it at all four sides. Let O be the circle's center. Here's why this must be a rhombus:

1. Tangent trick: Since each side is tangent to the circle at a point, the radii (lines from O to those points) are perpendicular to that side.
2. Opposite sides matter: In any parallelogram, opposite sides are parallel and equal in length.
3. Right triangles formed: The radii create four right triangles with O at the corner. Since the radii (heights) are all the same length (same circle), the bases of these triangles (halves of the parallelogram's sides) must also be equal.
4. Special right triangles: Remember, the radius touching a tangent line bisects it, creating two congruent right triangles with the radius as the hypotenuse. These triangles have a special 2:1 ratio between the hypotenuse (radius) and a shorter leg (half the base).

Key point: We now know that opposite sides of the parallelogram are equal (given) and the bases of the right triangles (half the sides) are also equal.
Result: Because all four sides of the parallelogram have equal halves (based on the right triangles), all four sides must be equal in length. This is the defining property of a rhombus.
Therefore, a parallelogram that perfectly surrounds a circle is always a rhombus.

Ques 30. Three unbiased coins are tossed simultaneously. Find the probability of getting:
(i) at least one head.
(ii) exactly one tail.
(iii) two heads and one tail.

Solu. Here's how to find the probability of each scenario when tossing three unbiased coins:
(i) At least one head:

1. Favorable cases: This includes getting one head (HTT, THH, HTH), two heads (THT, HTT), or three heads (HHH).

2. Total cases: There are 2^3 (8) total possible outcomes (HHH, HHT, HTH, THH, HTT, THT, TTH, TTT) since each coin has two possibilities (heads or tails).
3. Probability: Probability = Favorable cases / Total cases

- Probability (at least one head) = (Number of outcomes with heads) / Total outcomes
- Probability = (5 out of 8) = 5/8

(ii) Exactly one tail:

1. Favorable cases: This includes getting one tail (HTT, THH, HTH).
2. We already calculated this in part (i): The favorable cases for exactly one tail are the same as getting one head in (i).
3. Probability: Probability (exactly one tail) = 5/8 (from part (i))

(iii) Two heads and one tail:

1. Favorable cases: There are three possible arrangements for two heads and one tail (THT, HTT, THT).
2. Total cases (mentioned earlier): There are 8 total possible outcomes.
3. Probability: Probability (two heads and one tail) = Favorable cases / Total cases

- Probability = 3/8

Summary:

- Probability (at least one head) = 5/8
- Probability (exactly one tail) = 5/8
- Probability (two heads and one tail) = 3/8

Ques 31. An arc of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding major sector. (Use pi = 3.14)

Solu. Here's how to find the area of the major sector of the circle:

1. Minor sector angle:

- We are given that the arc subtends a right angle at the center, which means the minor sector's central angle (θ) is 90 degrees.

2. Area of the whole circle:

- The circle's radius (r) is given as 10 cm.

- We can use the formula for the area of a circle (πr^2) to find the whole circle's area (A_whole):
    - A_whole = $\pi * r^2$
    - A_whole = 3.14 * $(10 \text{ cm})^2$ (substitute pi and radius value)
    - A_whole = 314 cm^2 (rounded to two decimal places)

3. Ratio for major sector area:

- The major sector covers more than half of the circle. Its central angle (θ') is the difference between 360 degrees (full circle) and the minor sector's central angle (θ).
- $\theta' = 360° - \theta$
- $\theta' = 360° - 90°$
- $\theta' = 270°$
- The ratio between the major sector's central angle (θ') and the full circle angle (360°) is equal to the ratio between the major sector's area (A_major) and the whole circle's area (A_whole).

4. Calculate the major sector area:

- A_major / A_whole = θ' / 360°
- A_major = (θ' / 360°) * A_whole (multiply both sides by A_whole)
- A_major = (270° / 360°) * 314 cm^2 (substitute values)
- A_major = ¾ * 314 cm^2
- A_major ≈ 235.5 cm^2 (rounded to two decimal places)

Therefore, the area of the major sector is approximately 235.5 cm^2.

Ques 32.

(a) Find the value of 'k' for which the quadratic equation $(k + 1)x^2 - 6(k + 1)x + 3(k + 9) = 0$, $k \neq -1$ has real and equal roots.

Solu. We can find the value of 'k' for which the quadratic equation has real and equal roots by using the concept of the discriminant.

Here's how:

1. Discriminant and equal roots:
    - In a quadratic equation ax^2 + bx + c = 0, the discriminant (b^2 - 4ac) determines the nature of the roots.
    - When the discriminant is equal to zero (b^2 - 4ac = 0), the quadratic equation has real and equal roots.

2. Identify the coefficients:
    - In the given equation $(k + 1)x^2 - 6(k + 1)x + 3(k + 9) = 0$:
        - $a = k + 1$
        - $b = -6(k + 1)$
        - $c = 3(k + 9)$
3. Set the discriminant to zero and solve for k:
    - $b^2 - 4ac = 0$
    - $(-6(k + 1))^2 - 4 * (k + 1) * 3(k + 9) = 0$ (substitute coefficients)
    - Expand and solve for k:
        - $36(k + 1)^2 - 12(k + 1)(k + 9) = 0$
        - Factor out (k + 1):
            - $(k + 1) (36(k + 1) - 12(k + 9)) = 0$
        - We are given $k \neq -1$, so we can ignore the factor (k + 1).
        - This leaves us with: $36(k + 1) - 12(k + 9) = 0$
        - Solve for k:
            - $12(3k + 3 - k - 9) = 0$
            - $12(2k - 6) = 0$
            - $k - 3 = 0$
            - $k = 3$

Therefore, the value of 'k' for which the quadratic equation has real and equal roots is k = 3.

OR

(b) The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find their present ages.

Solu. Let's denote the son's current age as x. We can then use the given information to set up two equations and solve for x and the man's age.

1. Current Ages:
    - Man's age: The man's current age is twice the square of his son's age. We can express this as:
        - Man's age = $2 * x^2$

2. Ages Eight Years Hence:

- Man's age: Eight years from now, the man's age will be 4 years more than three times his son's age. We can express this as:
    - Man's age (8 years later) = 3 * (x + 8) + 4

3. Relating the Equations (Present Ages):
We know that the man's age remains the same, so his current age and his age eight years later must be equal.
- Equate the two expressions from equations 1 and 2:
    - $2 * x^2 = 3 * (x + 8) + 4$

4. Solve for x (Son's Age):
- Expand the equation from step 3:
    - $2x^2 = 3x + 24 + 4$
    - $2x^2 - 3x - 28 = 0$
- This is a quadratic equation. You can solve it by factoring, using the quadratic formula, or using a graphing calculator. Here, we'll use factoring:
    - Factor the equation: $(2x + 7)(x - 4) = 0$
    - Since we cannot have a negative age, the valid solution is: $x = 4$

5. Find the Man's Age:
- Now that you know the son's age (x = 4), substitute it back into equation 1 to find the man's current age:
    - Man's age = $2 * (4)^2$
    - Man's age = 2 * 16
    - Man's age = 32

Therefore, the son is currently 4 years old, and the man is 32 years old.

Ques 33. From a point on a bridge across the river, the angles of depressions of the banks on opposite sides of the river are 30° and 60° respectively. If the bridge is at a height of 4 m from the banks, find the width of the river.

Solu. Here's how to find the width of the river:

1. Recognize complementary angles:

The angles of depression from the bridge to the opposite banks (30° and 60°) add up to 90°. This indicates that the two angles are complementary angles (angles that sum to 90°).

2. Imagine a right triangle:

Think of a right triangle formed with the bridge as the vertex (highest point), the point on the closer bank as one endpoint of the base, and a horizontal line extending from that point to the opposite bank (where the second angle of depression is measured) as the other endpoint of the base.

3. Identify trigonometric ratios:

- The angle at the vertex (opposite the bridge) is 30° (given).
- Since the two banks are at the same height, the base of the right triangle represents the width of the river (what we want to find).
- We know the height of the bridge (opposite side in the triangle) which is 4 meters.

4. Select appropriate trigonometric ratio:

Since we have the opposite side (bridge height) and need to solve for the base (river width), we can use the tangent (tan) function.

5. Set up the equation and solve:

- tan(30°) = opposite / base (tangent definition)
- tan(30°) = 4 / river width (substitute known values)
- We know tan(30°) = $1/\sqrt{3}$ (trigonometric value)
    - River width = (4) / ($1/\sqrt{3}$)
    - River width = 4 * $\sqrt{3}$ (multiply both sides by $\sqrt{3}$)

Therefore, the width of the river is $4\sqrt{3}$ meters.