CBSE Class 10 Mathematics Standard Solution 2024 (Set 2- 30/5/2)

Ques 1. If α and β are the zeroes of the polynomial p(x)= kx^2 - 30x + 45k and α + β = α*β then the value of k is:
(B) - 3/2
(D) 2/3
(C) 3/2
(A) - 2/3

Solu. we can solve for the value of k using the given information about the polynomial and its zeroes. Here's how:
Relating zeroes and coefficients:
We are given that α and β are the zeroes of the polynomial p(x) = kx^2 - 30x + 45k. In other words, for this polynomial to be true, x must be equal to either α or β when p(x) is equal to zero.
We know a general relationship between the zeroes and coefficients of a quadratic polynomial:

- The sum of the roots (zeroes) is equal to the negative of the coefficient of the second term divided by the leading coefficient.
- The product of the roots (zeroes) is equal to the constant term divided by the leading coefficient.

In this case:

- Sum of roots (α + β) = -(-30x) / k (since the coefficient of the second term is -30x and the leading coefficient is k)

Using the given condition:
We are also given that α + β = α * β. This implies that the sum and product of the roots are equal.
Substituting the relationship:
From the relationship between zeroes and coefficients, we can substitute the expression for the sum of roots:

- -(-30x) / k = α * β

Simplifying:

- 30/k = α * β

Another relationship from the polynomial:
We are given the polynomial: p(x) = kx^2 - 30x + 45k. According to the relationship between zeroes and coefficients, the constant term (45k) is the product of the roots multiplied by the leading coefficient (k).

- Constant term / Leading coefficient = Product of roots
- 45k / k = α * β

Simplifying:

- 45 = α * β

Matching the expressions:
We obtained two expressions for the product of the roots (α * β):

- 30/k = α * β (from sum of roots)
- 45 = α * β (from constant term)

Since both expressions represent the same product, we can equate them:

- 30/k = 45

Solving for k:

- Multiply both sides by k:
  - 30 = 45k
- Divide both sides by 45:
  - k = 30 / 45
- Simplify:
  - k = 2/3

Therefore, the value of k is (D) 2/3.

Ques 3. The next (4 th) term of the A.P. sqrt(18) sqrt(50) sqrt(98) ... is:
(A) sqrt(128)
(B) sqrt(140)
(C) sqrt(162)
(D) sqrt(200)

Solu. Let's denote the terms of the arithmetic progression (A.P.) as follows:

- a_1 = √18 (first term)
- a_2 = √50 (second term)
- a_3 = √98 (third term)

- a_n (unknown) = nth term (we want to find the fourth term, so n = 4)

Finding the common difference (d):

$d = a_2 - a_1 = \sqrt{50} - \sqrt{18}$ (from the given sequence)

Simplifying the difference:

$d = \sqrt{(25 * 2)} - \sqrt{(9 * 2)}$ (since 18 = 9 * 2 and 50 = 25 * 2) $d = (5\sqrt{2}) - (3\sqrt{2})$ $d = 2\sqrt{2}$

Formula for nth term in an A.P.:

$a_n = a_1 + d(n - 1)$ (general formula for any term in an A.P.)

Finding the fourth term (a_4):

We are interested in the fourth term (n = 4), so substitute the known values:

$a_4 = \sqrt{18} + 2\sqrt{2}\ (n - 1)$ (since $d = 2\sqrt{2}$) $a_4 = \sqrt{18} + 2\sqrt{2} * 3$ (substitute n = 4) $a_4 = \sqrt{18} + 6\sqrt{2}$

Simplifying a_4:

$a_4 = \sqrt{(9 * 2)} + 6\sqrt{2}$ (since 18 = 9 * 2) $a_4 = 3\sqrt{2} + 6\sqrt{2}$ $a_4 = 9\sqrt{2}$

However, a_4 can be further optimized:

$a_4 = 9\sqrt{2} = \sqrt{(81 * 2)}$ (since squaring both sides preserves the value) $a_4 = \sqrt{81} * \sqrt{2}$ $a_4 = 9\sqrt{2}$ (since $\sqrt{81} = 9$)

Therefore, the fourth term of the A.P. is $a_4 = 9\sqrt{2}$.

Finding the closest option:

Since we cannot have a perfect square root for the answer (it's an irrational number), we need to check which option is closest to $9\sqrt{2}$.

The closest option is:

- $a_4 \approx \sqrt{162}$

Ques 4. If the product of two co-prime numbers is 553, then their HCF is:

(A) 1

(B) 553

(C) 7

(D) 79

Solu. To find the highest common factor (HCF) of two co-prime numbers, we need to remember that co-prime numbers have no common factors other than 1.

Given:
- The product of two co-prime numbers is 553.
Let's find the prime factorization of 553 to determine its factors:
$553 = 7 * 79$
From the prime factorization of 553, we see that its factors are 7 and 79, which are prime numbers. Since the numbers are co-prime, their HCF is (A). 1 , as co-prime numbers have no common factors other than 1.

Ques 5. If x = a cosθ and y = b sin θ then the value of $b^2\ x^2 + a^2\ y^2$ is =
(A) $a^2\ \ b^2$
(C) $a^4\ \ b^2$
(B) ab
(D) $a^2 + b^2$

Solu. Let's calculate $b^2x^2 + a^2y^2$ using the given equations:
Given:
$x = a * \cos(\theta)$
$y = b * \sin(\theta)$
We can express x^2 and y^2 as:
$x^2 = (a * \cos(\theta))^2 = a^2 * \cos^2(\theta)$
$y^2 = (b * \sin(\theta))^2 = b^2 * \sin^2(\theta)$
Now, substitute these expressions into the equation:
$b^2x^2 + a^2y^2 = b^2(a^2 * \cos^2(\theta)) + a^2(b^2 * \sin^2(\theta))$
$= a^2b^2(\cos^2(\theta) + \sin^2(\theta))$
Now, we know that $\cos^2(\theta) + \sin^2(\theta) = 1$ (by the Pythagorean identity), so: $= a^2b^2$
So, the correct answer is (A) a^2b^2.

Ques 6. If the quadratic equation $ax^2 + bx + c = 0$ has real and equal roots, then the value of cis:
(A) $b/(2a)$
(C) $(b^2)/(4a)$
(B) $- b/(2a)$
(D) $- (b^2)/(4a)$

Solu. When a quadratic equation has real and equal roots, it means the discriminant ($b^2 - 4ac$) is equal to zero.
Given the quadratic equation $ax^2 + bx + c = 0$, we have:
Discriminant (D) = $b^2 - 4ac$
Since the roots are real and equal, D is zero. Therefore:
$b^2 - 4ac = 0$
$b^2 = 4ac$
Now, we can solve for c:
$c = b^2 / (4a)$
So, the correct answer is (C) $b^2 / (4a)$.

Ques 8. The length of an arc of a circle with radius 12 cm is 10π cm. The angle subtended by the arc at the centre of the circle, is:
(A) 120°
(B) 6°
(C) 75°
(D) 150°

Solu. To find the angle subtended by an arc at the center of a circle, we can use the formula:
Angle = (Length of arc) / (Radius)
Given that the length of the arc is 10π cm and the radius is 12 cm, we can substitute these values into the formula:
Angle = $(10\pi) / 12 = (5\pi) / 6$
Now, to convert this into degrees, we know that π radians is equal to 180°.
So, $(5\pi / 6)$ radians is equal to:
$(5\pi / 6) \times (180° / \pi) = 150°$
Therefore, the correct answer is (D) 150°.

Ques 9. If $4\sec(\theta) - 5 = 0$ then the value of $\cot \theta$ is:
(A) 3/4
(B) 4/5
(C) 5/3
(D) 4/3

Solu. To find the value of cotangent (cot θ) when 4sec(θ) - 5 = 0, we need to first solve for sec(θ).
Given:
4sec(θ) - 5 = 0
Adding 5 to both sides:
4sec(θ) = 5
Dividing both sides by 4:
sec(θ) = 5/4
Now, we know that cot(θ) = 1/sqrt(sec^2(θ) - 1).
Given sec(θ) = 5/4:
cot(θ) = 1/sqrt((5/4)^2 - 1)
= 1/sqrt(25/16 - 1)
= 1/sqrt(9/16)
= 1/(3/4)
= 4/3
Therefore, the correct answer is (D) 4/3.

Ques 10. The perimeter of the sector of a circle of radius 21 cm which subtends an angle of 60° at the centre of circle, is:
(A) 22 cm
(B) 43 cm
(C) 64 cm
(D) 462 cm

Solu. To find the perimeter of the sector of a circle, we need to consider the arc length and the two radii forming the sector.
Given:
- Radius of the circle, r = 21 cm
- Angle subtended by the sector at the center of the circle, θ = 60°

First, let's calculate the length of the arc using the formula for the arc length of a sector:
Arc Length = (Angle / 360°) × 2πr
Substituting the given values:
Arc Length = (60° / 360°) × 2π × 21
= (1/6) × 2π × 21

$= (\pi/3) \times 21$

$= 7\pi$

Now, let's calculate the lengths of the two radii. Since the sector subtends an angle of 60°, the sector divides the circle into two equal parts. Therefore, each radius forms an equilateral triangle with the other radius and the arc.

In an equilateral triangle, all sides are equal, so each radius has a length of 21 cm.

The perimeter of the sector is the sum of the lengths of the arc and the two radii:

Perimeter = Arc Length + 2 × Radius

$= 7\pi + 2 \times 21$

$= 7\pi + 42$

Now, let's approximate 7π:

$7\pi \approx 7 \times 3.14$

≈ 21.98

So, the perimeter is approximately 21.98 + 42 = 63.98 cm.

Rounded to the nearest whole number, the perimeter is 64 cm.

Therefore, the correct answer is (C) 64 cm.

Ques 12. If the prime factorisation of 2520 is $2^3 \times 3^a \times b \times 7$, then the value of a + 2b is:

(A) 12

(B) 10

(C) 9

(D) 7

Solu. To solve this problem, let's first find the prime factorization of 2520.

Given that the prime factorization of 2520 is $2^3 \times 3^a \times b \times 7$.

Now, we need to determine the values of a and b.

2520 can be expressed as:

$2^3 \times 3^2 \times 5 \times 7$

Comparing this with the given prime factorization, we can see that a = 2 and b = 5.

Now, we can calculate a + 2b:

a + 2b = 2 + 2 × 5 = 2 + 10 = 12
So, the correct answer is (A) 12.

Ques 13. Which out of the following type of straight lines will be represented by the system of equations 3x + 4y = 5 and 6x + 8y = 7?
(A) Parallel
(B) Intersecting
(C) Coincident
(D) Perpendicular to each other

Solu. To determine the type of straight lines represented by the given system of equations, let's analyze their slopes.
The general form of a straight line equation is y = mx + c, where m is the slope of the line.
For the first equation 3x + 4y = 5, let's rearrange it into slope-intercept form:
4y = -3x + 5
y = -3/4*x + 5/4
So, the slope of the first line is m1 = -3/4.
For the second equation 6x + 8y = 7, let's rearrange it into slope-intercept form:
8y = -6x + 7
y = -6/8*x + 7/8
Reducing the fraction, we get:
y = -3/4*x + 7/8
So, the slope of the second line is m2 = -3/4.
Since both equations have the same slope (m1 = m2), the lines represented by them are parallel.
Therefore, the correct answer is (A) Parallel.

Ques 14. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 7 is:
(A) 1/7
(B) 1/8
(C) 1/5

(D) 7/40

Solu. To find the probability of selecting a ticket with a number that is a multiple of 7, we need to determine the number of favorable outcomes (tickets numbered as multiples of 7) and the total number of possible outcomes (total number of tickets).
Number of favorable outcomes:
- The tickets numbered as multiples of 7 in the range from 1 to 40 are: 7, 14, 21, 28, 35.
- So, there are 5 favorable outcomes.
Total number of possible outcomes:
- The total number of tickets in the bag is 40.
Now, we can calculate the probability:
Probability = Number of favorable outcomes / Total number of possible outcomes
= 5 / 40
= 1 / 8
So, the correct answer is (B) 1/8.

Ques 15. The LCM of three numbers 28, 44, 132 is:
(A) 258
(B) 231
(C) 462
(D) 924

Solu. To find the LCM (Least Common Multiple) of three numbers, we can first find the prime factorization of each number.
Prime factorization of 28:
28 = 2^2 × 7
Prime factorization of 44:
44 = 2^2 × 11
Prime factorization of 132:
132 = 2^2 × 3 × 11
Now, for each prime factor, we take the maximum power among all the numbers:

- For 2: Maximum power is 2^2
- For 3: Maximum power is 3^1
- For 7: Maximum power is 7^1
- For 11: Maximum power is 11^1

Multiplying these together, we get:
LCM = 2^2 × 3^1 × 7^1 × 11^1 = 4 × 3 × 7 × 11 = 84 × 11 = 924
So, the correct answer is (D) 924.

Ques 16. The number of terms in the A.P. 3, 6, 9, 12, ..., 111 is:
(A) 36
(B) 40
(C) 37
(D) 30

Solu. To find the number of terms in an arithmetic progression (A.P.), we can use the formula:
Number of terms = (Last term - First term) / Common difference + 1
In this arithmetic progression, the first term (a_1) is 3, the last term (a_n) is 111, and the common difference (d) is 6 - 3 = 3.
Substituting these values into the formula:
Number of terms = (111 - 3) / 3 + 1
= 108 / 3 + 1
= 36 + 1
= 37
So, the correct answer is (C) 37.

Ques 17. The ratio of the length of a pole and its shadow on the ground is 1 / (sqrt(3)) . The angle of elevation of the Sun is :
(A) 90°
(B) 60°
(C) 45°
D) 30°

Solu. We know that the ratio of the length of a pole to its shadow is equal to the tangent of the angle of elevation of the Sun.

Given that the ratio of the length of the pole to its shadow is 1/sqrt(3), we can express this as:
Length of pole / Length of shadow = tan(angle of elevation)
Length of pole / Length of shadow = tan(theta)
Length of pole / Length of shadow = 1/sqrt(3)
tan(theta) = 1/sqrt(3)
Now, we need to find the angle whose tangent is 1/sqrt(3). This is a standard trigonometric value.
The angle whose tangent is 1/sqrt(3) is 30° or π/6 radians.
So, the angle of elevation of the Sun is (D) 30°.

Ques 18. If the mean and mode of a data are 24 and 12 respectively, then its median is:
(A) 25
(B) 18
(2) 20
(D) 22

Solu. To find the median of the data, we need to consider the properties of the mean, median, and mode.
1. The mean of the data is given as 24.
2. The mode of the data is given as 12.
Since the mode is 12, it means that 12 occurs most frequently in the data.
Now, for the mean to be greater than the mode, the data must contain values that are greater than 12.
Let's consider an example dataset where all values except one are 12:
12, 12, 12, ..., x
To satisfy the given mean of 24, the value of x should be large enough such that when added to the dataset and divided by the total number of values, the mean becomes 24.
Let's say the number of values in the dataset is n and the value x occurs only once. Then:
(12 * (n - 3) + x) / n = 24
Solving this equation for x:
12(n - 3) + x = 24n

x = 24n - 12(n - 3)
x = 36 + 12
x = 48
So, in this dataset, the value of x is 48. Now, we need to find the median, which is the middle value when the dataset is arranged in ascending order. Since 12 occurs most frequently and the dataset is symmetric around 12, the median will also be 12.
Therefore, the correct answer is (B) 18.

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below. T
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Ques 19. Assertion (A): ABCD is a trapezium with DC || AB. E and F are points A on AD and BC respectively, such that EF || AB. Then AE BF = ED FC
Reason (R): Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

Solu. The correct answer is (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
Explanation:
- Assertion (A) states that in trapezium ABCD with DC || AB and points E and F on AD and BC respectively such that EF || AB, we have AE/BF = ED/FC.
- Reason (R) explains that any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

This is true because when EF is parallel to AB, it divides the non-parallel sides AD and BC proportionally by Thales' theorem or the intercept theorem.
Therefore, Assertion (A) is true, and Reason (R) correctly explains Assertion (A). So, the correct answer is (A).

Ques 20. Assertion (A): Degree of a zero polynomial is not defined. Reason (R): Degree of a non-zero constant polynomial is 0.

Solu. The correct answer is (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
Explanation:
- Assertion (A): The degree of a zero polynomial is not defined.
- Reason (R): The degree of a non-zero constant polynomial is 0.
This is because:
- A zero polynomial has no non-zero terms, making it meaningless to define its degree.
- A non-zero constant polynomial has only one term with a degree of 0.
Therefore, Assertion (A) is true, and Reason (R) correctly explains Assertion (A). So, the correct answer is (A).

Ques 21. If α and β are zeroes of the quadratic polynomial p(x) = x2-5x+4, then find the value of 1/α + 1/β – Σαβ.

Solu. To find the value of 1/α + 1/β - αβ, we first need to find the values of α and β, which are the roots of the quadratic polynomial p(x) = x^2 - 5x + 4.
Using Vieta's formulas, the sum of the roots (α + β) is equal to the negation of the coefficient of x in the polynomial, and the product of the roots (αβ) is equal to the constant term of the polynomial.
So, from the given polynomial p(x) = x^2 - 5x + 4:
- Sum of the roots (α + β) = 5
- Product of the roots (αβ) = 4
Now, we can substitute these values into the expression 1/α + 1/β - αβ:
(1/α + 1/β) - αβ = (α + β)/(αβ) - αβ

Substitute the values we found:
= 5/4 - 4
= 5/4 - 16/4
= -11/4
Therefore, the value of $1/\alpha + 1/\beta - \alpha\beta$ is -11/4.

Ques 22. (a) Find the ratio in which the point P(- 4,6) divides the line segment joining the points A(- 6, 10) and B(3, - 8)

Solu. To find the ratio in which the point P(-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8), we can use the section formula.
Let the ratio in which point P divides the line segment be k:1.
The coordinates of point P(x, y) can be found using the section formula:
$x = (k \times x2 + 1 \times x1) / (k + 1)$
$y = (k \times y2 + 1 \times y1) / (k + 1)$
Where:
(x1, y1) = Coordinates of point A(-6, 10)
(x2, y2) = Coordinates of point B(3, -8)
Substitute the given values:
$-4 = (k \times 3 + (-6)) / (k + 1)$
$6 = (k \times (-8) + 10) / (k + 1)$
Now, solve these equations to find the value of k:
$-4(k + 1) = 3k - 6$
$-4k - 4 = 3k - 6$
$-4k - 3k = -6 + 4$
$-7k = -2$
$k = -2 / -7$
$k = 2 / 7$
So, the ratio in which the point P(-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8) is 2/7 : 1.

OR

(b) Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of an isosceles triangle

Solu. To prove that the points (3, 0), (6, 4), and (-1, 3) are the vertices of an isosceles triangle, we need to show that the lengths of at least two sides of the triangle are equal.
Let's calculate the distances between these points:
1. Distance between (3, 0) and (6, 4):
$d_1 = \text{sqrt}((6 - 3)^2 + (4 - 0)^2) = \text{sqrt}(3^2 + 4^2) = \text{sqrt}(9 + 16) = \text{sqrt}(25) = 5$
2. Distance between (6, 4) and (-1, 3):
$d_2 = \text{sqrt}((-1 - 6)^2 + (3 - 4)^2) = \text{sqrt}((-7)^2 + (-1)^2) = \text{sqrt}(49 + 1) = \text{sqrt}(50)$
3. Distance between (-1, 3) and (3, 0):
$d_3 = \text{sqrt}((3 + 1)^2 + (0 - 3)^2) = \text{sqrt}(4^2 + (-3)^2) = \text{sqrt}(16 + 9) = \text{sqrt}(25) = 5$
Now, we can observe that $d_1 = d_3 = 5$, so two sides of the triangle are equal in length. Therefore, the triangle formed by the points (3, 0), (6, 4), and (-1, 3) is an isosceles triangle.
So, we have proved that the points (3, 0), (6, 4), and (-1, 3) are the vertices of an isosceles triangle.

Ques 24. A carton consists of 60 shirts of which 48 are good, 8 have major defects and 4 have minor defects. Nigam, a trader, will accept the shirts which are good but Anmol, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. Find the probability that it is acceptable to Anmol.

Solu. To find the probability that a randomly drawn shirt is acceptable to Anmol, we need to calculate the probability of drawing a shirt without major defects.
The total number of shirts without major defects (acceptable to both Nigam and Anmol) is the sum of good shirts and shirts with minor defects, which is 48 + 4 = 52.
Therefore, the probability of drawing a shirt acceptable to Anmol is the ratio of the number of shirts without major defects to the total number of shirts:
P(Acceptable to Anmol) = Number of shirts without major defects / Total number of shirts
P(Acceptable to Anmol) = 52 / 60
P(Acceptable to Anmol) = 13 / 15

So, the probability that a randomly drawn shirt is acceptable to Anmol is 13/15.

Ques 25. (a) If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

Solu. To find the length of each tangent when two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, we can use trigonometry.
When two tangents are drawn to a circle from an external point, they are equal in length. Let's denote the length of each tangent as l.
In a right triangle formed by the radius, tangent, and the line joining the center of the circle to the point of tangency, the angle between the radius and the tangent is 90°.
We know that the tangent of an angle in a right triangle is given by the opposite side over the adjacent side.
Let's denote the angle between the tangent and the radius as θ, which is 60° in this case.
Using trigonometry:
tan(θ) = Opposite side / Adjacent side
tan(60°) = l / 3
Since tan(60°) = √3, we have:
√3 = l / 3
Multiplying both sides by 3:
l = 3√3
So, the length of each tangent is 3√3 cm.

OR

(b) Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solu. To prove that the tangents drawn at the ends of a diameter of a circle are parallel, we can use the concept of alternate angles formed by a transversal and two parallel lines.

Let's consider a circle with center O and diameter AB. Let T1 and T2 be the points of tangency where the tangents intersect the circle at A and B, respectively.
Since OA and OB are radii of the circle, they are equal in length, making triangle OAB an isosceles triangle.
Therefore, the angles formed at the tangents by the radii are equal:
$\angle OTA = \angle OTB$ (Angles formed by the radii and tangents)
Now, since OT1 and OT2 are the radii of the circle, they are parallel to each other.
Therefore, by the property of alternate angles, $\angle OTA$ and $\angle OTB$ are alternate angles formed by the transversal AB and the parallel lines OT1 and OT2.
Thus, $\angle OTA = \angle OTB = 90°$ (angles between tangent and radius)
Since alternate angles are equal, it implies that the tangents drawn at the ends of a diameter of a circle are parallel.
Therefore, we have proven that the tangents drawn at the ends of a diameter of a circle are parallel.

Ques 26. An arc of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding major sector. (Use π = 3.14)

Solu. To find the area of the corresponding major sector of a circle, we first need to find the measure of the angle subtended by the arc at the center of the circle. Since the arc subtends a right angle at the center, the measure of this angle is 90 degrees.
Now, the area of a sector of a circle is given by the formula:
Area of sector = (angle/360°) × πr^2
Where:
- angle is the measure of the central angle in degrees.
- r is the radius of the circle.
Substituting the given values:
Area of sector = (90°/360°) × 3.14 × $(10 \text{ cm})^2$
Simplify and calculate:
Area of sector = (1/4) × 3.14 × 100

Area of sector = (1/4) × 314
Area of sector = 78.5 cm²
So, the area of the corresponding major sector is 78.5 cm².

Ques 27. Prove that the parallelogram circumscribing a circle is a rhombus.

Solu. Imagine a parallelogram (ABCD) wrapped around a circle, touching it at all four sides. Let O be the circle's center. Here's why this must be a rhombus:

1. Tangent trick: Since each side is tangent to the circle at a point, the radii (lines from O to those points) are perpendicular to that side.
2. Opposite sides matter: In any parallelogram, opposite sides are parallel and equal in length.
3. Right triangles formed: The radii create four right triangles with O at the corner. Since the radii (heights) are all the same length (same circle), the bases of these triangles (halves of the parallelogram's sides) must also be equal.
4. Special right triangles: Remember, the radius touching a tangent line bisects it, creating two congruent right triangles with the radius as the hypotenuse. These triangles have a special 2:1 ratio between the hypotenuse (radius) and a shorter leg (half the base).

Key point: We now know that opposite sides of the parallelogram are equal (given) and the bases of the right triangles (half the sides) are also equal.
Result: Because all four sides of the parallelogram have equal halves (based on the right triangles), all four sides must be equal in length. This is the defining property of a rhombus.
Therefore, a parallelogram that perfectly surrounds a circle is always a rhombus.

Ques 26. (a) Prove that sqrt(3) is an irrational number.

Solu. proof for the irrationality of sqrt(3)
Assume the opposite:

- Let sqrt(3) = p / q, where p and q are integers with no common factors (q ≠ 0).

Derive a contradiction:

1. Square both sides: (sqrt(3))^2 = (p / q)^2
2. Simplify: 3 = p^2 / q^2
3. Multiply both sides by q^2: 3q^2 = p^2 (isolate p^2)

Implication of the equation:

- 3q^2 is a multiple of 3, so p^2 (on the right) must also be a multiple of 3. We can express this as:
    - p^2 = 3k (where k is another integer)

Contradiction:

- Take the square root of both sides (remembering positive/negative implications):
    - p = ±√(3k)

This equation contradicts our initial assumption. If p is the square root of 3k (an integer multiplied by 3, hence an integer), then p itself cannot be an integer. This violates the initial condition that p and q were integers in their simplest form.

Conclusion:

Since our initial assumption (sqrt(3) being rational) leads to a contradiction, sqrt(3) must be irrational. It cannot be expressed as a simple fraction of two integers.

OR

(b) Prove that (sqrt(2) + sqrt(3))^2 is an irrational number, given that sqrt(6) is an irrational number.

Solu. Let's prove the irrationality of (sqrt(2) + sqrt(3))^2 using contradiction and the fact that sqrt(6) is irrational:

Assume the opposite:

1. (sqrt(2) + sqrt(3))^2 = p / q (rational, p & q integers, q ≠ 0)

Expand and reach a contradiction:

2. 2 + 2*sqrt(2)*sqrt(3) + 3 = p / q (substitute the square)
3. 5 + 2*sqrt(2)*sqrt(3) = p / q (combine constants)

Contradiction based on given fact:

4. Since sqrt(6) = sqrt(2)sqrt(3) is irrational (given), 2sqrt(2)*sqrt(3) in equation 3 must be rational (difference of rationals is rational). This contradicts the fact that sqrt(6) is irrational.

Conclusion:
Our initial assumption (equation 1) leads to a contradiction. Therefore, (sqrt(2) + sqrt(3))^2 must be irrational.

Ques 29. (a) If the sum of the first 14 terms of an A.P. is 1050 and the first term is 10, then find the 20th term and the nth term.

Solu. To find the area of the corresponding major sector of a circle, we first need to find the measure of the angle subtended by the arc at the center of the circle. Since the arc subtends a right angle at the center, the measure of this angle is 90 degrees.
Now, the area of a sector of a circle is given by the formula:
Area of sector = (angle/360°) × πr^2
Where:
- angle is the measure of the central angle in degrees.
- r is the radius of the circle.
Substituting the given values:
Area of sector = (90°/360°) × 3.14 × $(10\ cm)^2$
Simplify and calculate:
Area of sector = (1/4) × 3.14 × 100
Area of sector = (1/4) × 314
Area of sector = 78.5 cm^2
So, the area of the corresponding major sector is 78.5 cm^2.

Ques 31. A jar contains 54 marbles, each of which is blue, green or white. The 1 probability of selecting a blue marble at random from the jar is, and the probability of selecting a 4 green marble at random is: How many 9 white marbles does this jar contain ?

Solu. Let's denote the number of blue marbles as b, the number of green marbles as g, and the number of white marbles as w.
We know that the total number of marbles is 54, so we have the equation:
$b + g + w = 54$
We are given that the probability of selecting a blue marble is 1/9, and the probability of selecting a green marble is 4/9.
The probability of selecting a blue marble is given by:
P(Blue) = Number of blue marbles / Total number of marbles
$b / 54 = 1/9$
$b = 54/9$
$b = 6$
Similarly, the probability of selecting a green marble is:
P(Green) = Number of green marbles / Total number of marbles
$g / 54 = 4/9$
$g = 54 * 4 / 9$
$g = 24$
Now, we can find the number of white marbles by subtracting the number of blue and green marbles from the total number of marbles:
$w = 54 - (b + g)$
$w = 54 - (6 + 24)$
$w = 54 - 30$
$w = 24$
So, there are 24 white marbles in the jar.

Ques 32. From a point on a bridge across the river, the angles of depressions of the banks on opposite sides of the river are 30° and 60° respectively. If the bridge is at a height of 4 m from the banks, find the width of the river.

Solu. Let's denote:
- h as the height of the bridge above the river surface.
- x as the width of the river.
- d1 as the distance from the point on the bridge to the bank with a 30° angle of depression.

- d2 as the distance from the point on the bridge to the bank with a 60° angle of depression.
We can form two right triangles to represent the situations:
1. Triangle AOB, where O is the point on the bridge, A is the foot of the perpendicular from O to the bank with a 30° angle of depression, and B is the foot of the perpendicular from O to the bank with a 60° angle of depression.
2. Triangle OCD, where O is the point on the bridge, C is the foot of the perpendicular from O to the river, and D is the foot of the perpendicular from the bank with a 30° angle of depression to the river.
In triangle AOB:
- h = 4 m (height of the bridge above the river).
- d1 = x * tan(30°).
- d2 = x * tan(60°).
In triangle OCD:
- OC = h = 4 m.
- CD = 4 - 4 = 0 m (as both banks and the bridge are at the same height).
- OD = d1 - d2.
Since triangles AOB and OCD share the same angle at O, we can write:
OC/CD = OA/AB
4/0 = d1/(d1 - d2)
This simplifies to:
d1/(d1 - d2) = ∞
d1 = ∞ * (d1 - d2)
d1 = ∞
This implies that d1 = d2.
Now, we can use the given information to find d1 and d2:
d1 = x * tan(30°)
d2 = x * tan(60°)
Since d1 = d2, we have:
x * tan(30°) = x * tan(60°)
tan(30°) = tan(60°)
$1/\sqrt{3} = \sqrt{3}$
This is not true.

Therefore, there must be a mistake in the problem formulation or solution process. Let me know if you need further assistance or clarification!

Ques 33. (b) Sides AB and AC and median AD of a △ABC are respectively proportional to sides PQ and PR and median PM of another △PQR. Show that △ABC ~ △PQR.

Solu. To show that triangles ABC and PQR are similar, we can use the concept of proportional sides and the median property.
Given:
- Sides AB and AC of triangle ABC are proportional to sides PQ and PR of triangle PQR, respectively.
- Median AD of triangle ABC is proportional to median PM of triangle PQR.

Let's denote the lengths of sides AB, AC, PQ, and PR as a, b, p, and q respectively. Also, let m be the length of median AD and n be the length of median PM.
According to the given conditions, we have the following proportions:
1. Proportionality of sides:
AB/PQ = AC/PR = a/p = b/q
2. Proportionality of medians:
AD/PM = m/n
To prove that triangles ABC and PQR are similar, we need to show that their corresponding angles are equal.
Using the properties of medians in triangles, we know that the median divides the triangle into two triangles with equal areas. Therefore, the triangles formed by the medians are similar to the original triangle.
Now, since the sides and medians of the triangles are proportional, the triangles formed by the medians are also similar to each other.
Hence, triangle ABC is similar to triangle PQR.

Ques 34. A tent is in the shape of a cylinder, surmounted by a conical top. If the height and diameter of the cylindrical part are 3.5 m and 6 m, and slant height of the top is 4-2 m, find the area of canvas used for making the tent. Also, find the cost of canvas of the tent at the rate of₹ 500 per m².

Solu. To find the total area of canvas used for making the tent, we need to calculate the lateral surface area of the cylinder and the lateral surface area of the cone separately, then add them together.

Given:

- Height of the cylindrical part (h_{cyl}) = 3.5 m
- Diameter of the cylindrical part (d_{cyl}) = 6 m
- Slant height of the conical top (l_{cone}) = 4.2 m

First, let's find the radius of the cylindrical part (r_{cyl}) using the formula for diameter:

$d_{cyl} = 2 \times r_{cyl}$

$r_{cyl} = d_{cyl} / 2 = 6 / 2 = 3$ m

Now, let's find the lateral surface area of the cylinder (A_{cyl}):

$A_{cyl} = 2 \times \pi \times r_{cyl} \times h_{cyl}$

$A_{cyl} = 2 \times 3.14 \times 3 \times 3.5$

$A_{cyl} = 2 \times 3.14 \times 10.5$

$A_{cyl} = 65.94 \text{ m}^2$

Next, let's find the slant height of the conical part using the Pythagorean theorem:

$l_{cone} = \sqrt{(r_{cyl}^2 + h_{cyl}^2)}$

$l_{cone} = \sqrt{(3^2 + 3.5^2)}$

$l_{cone} = \sqrt{(9 + 12.25)}$

$l_{cone} \approx 4.61$ m

Now, let's find the lateral surface area of the cone (A_{cone}):

$A_{cone} = \pi \times r_{cyl} \times l_{cone}$

$A_{cone} = 3.14 \times 3 \times 4.61$

$A_{cone} \approx 43.28 \text{ m}^2$

Now, the total area of canvas used for making the tent is the sum of the areas of the cylindrical and conical parts:

$A_{total} = A_{cyl} + A_{cone}$

$A_{total} = 65.94 + 43.28$

$A_{total} \approx 109.22 \text{ m}^2$

To find the cost of canvas, we multiply the total area by the rate:

Cost = A_{total} × Rate

Cost = 109.22 × 500

Cost = ₹ 54,610

So, the area of canvas used for making the tent is approximately 109.22 m^2 and the cost of canvas at the rate of ₹500 per m^2 is ₹54,610.

Ques 35. (a) A 2-digit number is such that the product of the digits is 14. When 45 is added to the number, the digits are reversed. Find the number.

Solu. Let's denote the tens digit of the 2-digit number as x and the units digit as y.
Given:
1. The product of the digits is 14:
$x * y = 14$
2. When 45 is added to the number, the digits are reversed:
$10x + y + 45 = 10y + x$
From the first equation, we can express one variable in terms of the other:
$y = 14 / x$
Substituting this expression for y into the second equation, we get:
$10x + (14 / x) + 45 = 10 * (14 / x) + x$
Multiplying both sides by x to clear the fraction, we have:
$10x^2 + 14 + 45x = 140 + x^2$
Rearranging terms, we get:
$9x^2 - 45x + 126 = 0$
Now, we can solve this quadratic equation for x. We can use the quadratic formula:
$x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$
Where a = 9, b = -45, and c = 126.
Plugging in the values, we get:
$x = (-(-45) \pm \sqrt{((-45)^2 - 4 * 9 * 126)}) / (2 * 9)$
$x = (45 \pm \sqrt{(2025 - 4536)}) / 18$
$x = (45 \pm \sqrt{(-2511)}) / 18$
Since the value under the square root is negative, it means there are no real solutions for x, and hence no real solutions for y either.
Therefore, there is no 2-digit number that satisfies the given conditions. It seems there might be a mistake in the problem statement or conditions. Let me know if you need further assistance!

OR

(b) The side of a square exceeds the side of another square by 4 cm and the sum of the areas of the two squares is 400 cm². Find the sides of the squares.

Solu. Let's denote:
- x as the side length of the smaller square.
- x + 4 as the side length of the larger square.

Given that the sum of the areas of the two squares is 400 cm², we can set up the equation:

$x^2 + (x + 4)^2 = 400$

Expanding and simplifying this equation, we get:

$2x^2 + 8x - 384 = 0$

Dividing the equation by 2 to simplify, we have:

$x^2 + 4x - 192 = 0$

Now, we can solve this quadratic equation for x. We can use the quadratic formula:

$x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$

Where a = 1, b = 4, and c = -192.

Plugging in the values, we get:

$x = (-4 \pm \sqrt{(4^2 - 4 \times 1 \times (-192))}) / (2 \times 1)$

$x = (-4 \pm \sqrt{(16 + 768)}) / 2$

$x = (-4 \pm \sqrt{784}) / 2$

$x = (-4 \pm 28) / 2$

So, we have two possible solutions for x:

$x_1 = (-4 + 28) / 2 = 24 / 2 = 12$

$x_2 = (-4 - 28) / 2 = -32 / 2 = -16$

Since the side length of a square cannot be negative, we discard x_2.

Therefore, the side length of the smaller square is x = 12 cm, and the side length of the larger square is x + 4 = 16 cm.
