CBSE Class 10 Mathematics Standard Solution 2024 (Set 3- 30/5/3)

Ques 1. The LCM of the smallest prime number and the smallest odd composite number is:
(A) 10
(B) 6
(C) 9
(D) 18

Solu. The smallest prime number is 2, and the smallest odd composite number is 9.
The LCM (Least Common Multiple) of two numbers is the smallest number that is a multiple of both numbers.
To find the LCM of 2 and 9, we list the multiples of each number and find the smallest number that appears in both lists:
Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, ...
Multiples of 9: 9, 18, 27, 36, ...
The smallest number that appears in both lists is 18.
Therefore, the LCM of 2 and 9 is 18.
So, the correct answer is option (D) 18.

Ques 2. If the mean of the first n natural numbers is 5n/9 , then the value of n is:
(A) 5
(B) 4
(C) 9
(D) 10

Solu. The sum of the first n natural numbers is given by the formula n(n + 1)/2.

Given that the mean of the first n natural numbers is 5n/9, we can set up the equation:
(1 + 2 + 3 + ... + n) / n = 5n/9
Now, let's express the sum of the first n natural numbers using the formula:
(n(n + 1)/2) / n = 5n/9
Simplifying this equation, we get:
(n + 1)/2 = 5n/9
Now, let's cross multiply:
9(n + 1) = 10n
9n + 9 = 10n
9 = 10n - 9n
9 = n
So, the value of n is (C) 9.

Ques 4. The next (4 th) term of the A.P. sqrt(18) sqrt(50) sqrt(98) ... is:
(A) sqrt(128)
(B) sqrt(140)
(C) sqrt(162)
(D) sqrt(200)

Solu. Let's denote the terms of the arithmetic progression (A.P.) as follows:
- a_1 = √18 (first term)
- a_2 = √50 (second term)
- a_3 = √98 (third term)
- a_n (unknown) = nth term (we want to find the fourth term, so n = 4)

Finding the common difference (d):
d = a_2 - a_1 = √50 - √18 (from the given sequence)
Simplifying the difference:
d = √(25 * 2) - √(9 * 2) (since 18 = 9 * 2 and 50 = 25 * 2) d = (5√2) - (3√2) d = 2√2
Formula for nth term in an A.P.:
a_n = a_1 + d(n - 1) (general formula for any term in an A.P.)
Finding the fourth term (a_4):
We are interested in the fourth term (n = 4), so substitute the known values:

$a_4 = \sqrt{18} + 2\sqrt{2} (n - 1)$ (since $d = 2\sqrt{2}$) $a_4 = \sqrt{18} + 2\sqrt{2} * 3$ (substitute $n = 4$) $a_4 = \sqrt{18} + 6\sqrt{2}$
Simplifying a_4:
$a_4 = \sqrt{(9 * 2)} + 6\sqrt{2}$ (since $18 = 9 * 2$) $a_4 = 3\sqrt{2} + 6\sqrt{2}$ $a_4 = 9\sqrt{2}$
However, a_4 can be further optimized:
$a_4 = 9\sqrt{2} = \sqrt{(81 * 2)}$ (since squaring both sides preserves the value) $a_4 = \sqrt{81} * \sqrt{2}$ $a_4 = 9\sqrt{2}$ (since $\sqrt{81} = 9$)
Therefore, the fourth term of the A.P. is $a_4 = 9\sqrt{2}$.
Finding the closest option:
Since we cannot have a perfect square root for the answer (it's an irrational number), we need to check which option is closest to $9\sqrt{2}$.
The closest option is:

- $a_4 \approx \sqrt{162}$

Ques 7. The perimeter of the sector of a circle of radius 21 cm which subtends an angle of 60° at the centre of circle, is:
(A) 22 cm
(B) 43 cm
(C) 64 cm
(D) 462 cm

Solu. The correct answer is (C) 64 cm.
Here's how to find the perimeter of the sector:

1. Calculate the arc length:

- We know the angle of the sector (θ) is 60° and the radius (r) is 21 cm.
- The ratio between the sector's central angle (θ) and 360° represents the ratio between the arc length (s) and the circle's circumference (2πr).

Therefore, $s = (\theta / 360°) * 2\pi r$.
$s = (60° / 360°) * 2\pi * 21$ cm $s = (1/6) * 2\pi * 21$ cm $s \approx 22$ cm (using $\pi \approx 22/7$ for calculation)

2. Find the perimeter:

The perimeter of the sector is the sum of the arc length (s) and twice the radius (r).
Perimeter = s + 2r Perimeter ≈ 22 cm + 2 * 21 cm Perimeter ≈ 64 cm

Therefore, the perimeter of the sector is approximately 64 cm.

Ques 8. The ratio of the sum and product of the roots of the quadratic equation $5x^2$-6x+21= 0 is:
(A) 5:21
(B) 2:7
(D) 7:2
(C) 21:5

Solu. Here's how to find the ratio of the sum and product of the roots for the quadratic equation 5x^2 - 6x + 21 = 0:

1. Find the roots:

We can use the quadratic formula to solve for the roots:
$x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$
where a, b, and c are the coefficients of the quadratic equation (a = 5, b = -6, c = 21).
Plugging in the values:
$x = (-(-6) \pm \sqrt{((-6)^2 - 4 * 5 * 21)}) / (2 * 5)$ $x = (6 \pm \sqrt{(-404)}) / 10$
Since the discriminant (b^2 - 4ac) is negative, its square root is imaginary and the quadratic formula reduces to:
$x = (6 \pm 2\sqrt{102}i) / 10$ (i represents the imaginary unit)
Therefore, the roots are complex numbers:
$x1 = (3 + \sqrt{102}i) / 5$ $x2 = (3 - \sqrt{102}i) / 5$

2. Calculate the sum and product of the roots:
   - Sum of roots (S) = x1 + x2 = $[(3 + \sqrt{102}i) / 5] + [(3 - \sqrt{102}i) / 5] = 6/5$ (real part sums up, imaginary part cancels out).
   - Product of roots (P) = x1 * x2 = $[(3 + \sqrt{102}i) / 5] * [(3 - \sqrt{102}i) / 5] = (9 - 102) / 25 = -93 / 25$.
3. Find the ratio:

Ratio of sum and product (S/P) = (6/5) / (-93/25) = -2/15 (negative sign due to the negative product).
Since the question asks for the absolute value of the ratio, we take the positive value:
S/P = 2/15

Answer: (B) 2:7

Ques 9. The 14th term from the end of the A.P. 11,-8,-5, ..., 49 is:
(A) 7
(B) 10
(D) 28
(C) 13

Solu. To find the 14th term from the end of the arithmetic progression (A.P.), you can calculate it by counting 14 terms backward from the last term.
The common difference of this arithmetic progression is 3, as each term decreases by 3 from the previous term.
The last term of the progression is 49.
To find the 14th term from the end, you would subtract 14 - 1 = 13 times the common difference from the last term.
49 - 13 × 3 = 49 - 39 = 10
So, the 14th term from the end of the A.P. is 10.
Therefore, the correct answer is (B) 10.

Ques 10. The length of the shadow of a tower on the plane ground is sqrt(3) times the height of the tower. The angle of elevation of the Sun is :
(A) 30 deg
(B) 45 deg
(C) 60 deg
(D) 90 deg

Solu. The angle of elevation of the Sun in this scenario is 30 degrees. Here's why:

- We are given that the shadow length is sqrt(3) times the height of the tower.
- This specific ratio (sqrt(3):1) corresponds to a 30-60-90 triangle.

In a 30-60-90 triangle, the side opposite the 30-degree angle is half the length of the hypotenuse (which is the side opposite the right angle). The

shadow represents the opposite side, and the height of the tower is the hypotenuse in this situation.
Therefore, the ratio sqrt(3):1 perfectly matches the side lengths in a 30-degree angle triangle.
So, the angle of elevation of the Sun is (A) 30 degrees.

Ques 13. If α and β are the zeroes of the polynomial p(x)= kx^2 - 30x + 45k and α + β = α*β then the value of k is:
(B) - 3/2
(D) 2/3
(C) 3/2
(A) - 2/3

Solu. we can solve for the value of k using the given information about the polynomial and its zeroes. Here's how:
Relating zeroes and coefficients:
We are given that α and β are the zeroes of the polynomial p(x) = kx^2 - 30x + 45k. In other words, for this polynomial to be true, x must be equal to either α or β when p(x) is equal to zero.
We know a general relationship between the zeroes and coefficients of a quadratic polynomial:

- The sum of the roots (zeroes) is equal to the negative of the coefficient of the second term divided by the leading coefficient.
- The product of the roots (zeroes) is equal to the constant term divided by the leading coefficient.

In this case:

- Sum of roots (α + β) = -(-30x) / k (since the coefficient of the second term is -30x and the leading coefficient is k)

Using the given condition:
We are also given that α + β = α * β. This implies that the sum and product of the roots are equal.
Substituting the relationship:
From the relationship between zeroes and coefficients, we can substitute the expression for the sum of roots:

- -(-30x) / k = α * β

Simplifying:

- $30/k = \alpha * \beta$

Another relationship from the polynomial:
We are given the polynomial: $p(x) = kx^2 - 30x + 45k$. According to the relationship between zeroes and coefficients, the constant term (45k) is the product of the roots multiplied by the leading coefficient (k).

- Constant term / Leading coefficient = Product of roots
- $45k / k = \alpha * \beta$

Simplifying:

- $45 = \alpha * \beta$

Matching the expressions:
We obtained two expressions for the product of the roots ($\alpha * \beta$):

- $30/k = \alpha * \beta$ (from sum of roots)
- $45 = \alpha * \beta$ (from constant term)

Since both expressions represent the same product, we can equate them:

- $30/k = 45$

Solving for k:

- Multiply both sides by k:
  - $30 = 45k$
- Divide both sides by 45:
  - $k = 30 / 45$
- Simplify:
  - $k = 2/3$

Therefore, the value of k is (D) 2/3.

Ques 14. The length of an arc of a circle with radius 12 cm is 10π cm. The angle subtended by the arc at the centre of the circle, is:
(A) 120°
(B) 6°
(C) 75°
(D) 150°

Solu. To find the angle subtended by an arc at the center of a circle, we can use the formula:
Angle = (Length of arc) / (Radius)

Given that the length of the arc is 10π cm and the radius is 12 cm, we can substitute these values into the formula:
Angle = $(10\pi) / 12 = (5\pi) / 6$
Now, to convert this into degrees, we know that π radians is equal to 180°.
So, $(5\pi / 6)$ radians is equal to:
$(5\pi / 6) \times (180° / \pi) = 150°$
Therefore, the correct answer is (D) 150°.

Ques 15. The LCM of three numbers 28, 44, 132 is:
(A) 258
(B) 231
(C) 462
(D) 924

Solu. To find the LCM (Least Common Multiple) of three numbers, we can first find the prime factorization of each number.
Prime factorization of 28:
$28 = 2^2 \times 7$
Prime factorization of 44:
$44 = 2^2 \times 11$
Prime factorization of 132:
$132 = 2^2 \times 3 \times 11$
Now, for each prime factor, we take the maximum power among all the numbers:
- For 2: Maximum power is 2^2
- For 3: Maximum power is 3^1
- For 7: Maximum power is 7^1
- For 11: Maximum power is 11^1
Multiplying these together, we get:
LCM = $2^2 \times 3^1 \times 7^1 \times 11^1 = 4 \times 3 \times 7 \times 11 = 84 \times 11 = 924$
So, the correct answer is (D) 924.

Ques 17. Which out of the following type of straight lines will be represented by the system of equations $3x + 4y = 5$ and $6x + 8y = 7$?
(A) Parallel

(B) Intersecting
(C) Coincident
(D) Perpendicular to each other

Solu. To determine the type of straight lines represented by the given system of equations, let's analyze their slopes.
The general form of a straight line equation is $y = mx + c$, where m is the slope of the line.
For the first equation $3x + 4y = 5$, let's rearrange it into slope-intercept form:
$4y = -3x + 5$
$y = -3/4*x + 5/4$
So, the slope of the first line is $m1 = -3/4$.
For the second equation $6x + 8y = 7$, let's rearrange it into slope-intercept form:
$8y = -6x + 7$
$y = -6/8*x + 7/8$
Reducing the fraction, we get:
$y = -3/4*x + 7/8$
So, the slope of the second line is $m2 = -3/4$.
Since both equations have the same slope ($m1 = m2$), the lines represented by them are parallel.
Therefore, the correct answer is (A) Parallel.

Ques 18. The greatest number which divides 281 and 1249, leaving remainder 5 and 7 respectively, is:
(A) 23
(B) 276
(C) 138
(D) 69

Solu. The greatest number that divides 281 and 1249, leaving remainders 5 and 7 respectively, is the Highest Common Factor (HCF) of (281 - 5) and (1249 - 7).
Here's why:

- We are interested in the largest number that divides both 281 and 1249 without leaving a remainder.
- Subtracting the remainders (5 and 7) essentially removes any multiples of the divisor that might have already been included in the original numbers.
- This ensures we find the greatest common factor that contributes to the difference in the original numbers, not just any common factor of the original numbers themselves.

Steps to solve:

1. Find the adjusted divisors:
    - Divisor 1 = 281 - 5 = 276
    - Divisor 2 = 1249 - 7 = 1242
2. Find the HCF of the adjusted divisors:
    - We can use the Euclidean Algorithm for this:
        - While the second divisor (b) is not zero:
            - Find the remainder (r) of the first divisor (a) divided by the second divisor (b).
            - The first divisor (a) becomes the second divisor (b).
            - The second divisor (b) becomes the remainder (r).
        - The HCF is the last non-zero remainder.

Applying the Euclidean Algorithm:

- a (276) = b (1242) * 0 + 276 (remainder)
- Since the first remainder (276) is not zero, continue:
- a (1242) = b (276) * 4 + 138 (remainder)
- The second remainder (138) is not zero, continue:
- a (276) = b (138) * 2 + 0 (remainder)

Here, the last non-zero remainder is 138.
Therefore, the greatest number that divides 281 and 1249, leaving remainders 5 and 7 respectively, is 138.
So the answer is (C) 138.

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Ques 19. Assertion (A): Degree of a zero polynomial is not defined.
Reason (R): Degree of a non-zero constant polynomial is 0.

Solu. The correct answer is (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
Explanation:
- Assertion (A): The degree of a zero polynomial is not defined.
- Reason (R): The degree of a non-zero constant polynomial is 0.
This is because:
- A zero polynomial has no non-zero terms, making it meaningless to define its degree.
- A non-zero constant polynomial has only one term with a degree of 0.
Therefore, Assertion (A) is true, and Reason (R) correctly explains Assertion (A). So, the correct answer is (A).

Ques 20. Assertion (A): ABCD is a trapezium with DC || AB. E and F are points A on AD and BC respectively, such that EF || AB. Then AE BF = ED FC
Reason (R): Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

Solu. The correct answer is (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
Explanation:
- Assertion (A) states that in trapezium ABCD with DC || AB and points E and F on AD and BC respectively such that EF || AB, we have AE/BF = ED/FC.

- Reason (R) explains that any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

This is true because when EF is parallel to AB, it divides the non-parallel sides AD and BC proportionally by Thales' theorem or the intercept theorem.
Therefore, Assertion (A) is true, and Reason (R) correctly explains Assertion (A). So, the correct answer is (A).

Ques 21. The king, queen and ace of clubs and diamonds are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is randomly drawn from the remaining cards. Find the probability of getting
(i) a card of clubs.
(ii) a red coloured card.

Solu. Let's analyze the scenario and calculate the probabilities:
(i) Probability of getting a card of clubs:

- There are initially 13 clubs in a deck (including 10, Jack, Queen, and King).
- After removing King, Queen, and Ace of Clubs, there are 13 - 3 = 10 clubs remaining.
- Since 4 cards were removed in total from a deck of 52, there are 52 - 4 = 48 cards remaining after shuffling.

Therefore, the probability of getting a card of clubs is:
Probability (clubs) = Number of favorable outcomes (clubs remaining) / Total number of outcomes (cards remaining after shuffling) Probability (clubs) = 10 clubs / 48 cards Probability (clubs) = 5/24
(ii) Probability of getting a red colored card (Hearts or Diamonds):

- After removing King, Queen, and Ace of Clubs and Diamonds, there are 6 Hearts and 6 Diamonds remaining (since each suit originally has 13 cards and we remove 3).
- Combining these, there are a total of 6 Hearts + 6 Diamonds = 12 red cards remaining.

Therefore, the probability of getting a red colored card is:

Probability (red card) = Number of favorable outcomes (red cards remaining) / Total number of outcomes (cards remaining after shuffling)
Probability (red card) = 12 red cards / 48 cards remaining Probability (red card) = 1/4
So, the probabilities are:
(i) Probability of getting a card of clubs: 5/24 (ii) Probability of getting a red colored card: 1/4

Ques 22. (a) If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

Solu. To find the length of each tangent when two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, we can use trigonometry.
When two tangents are drawn to a circle from an external point, they are equal in length. Let's denote the length of each tangent as l.
In a right triangle formed by the radius, tangent, and the line joining the center of the circle to the point of tangency, the angle between the radius and the tangent is 90°.
We know that the tangent of an angle in a right triangle is given by the opposite side over the adjacent side.
Let's denote the angle between the tangent and the radius as θ, which is 60° in this case.
Using trigonometry:
tan(θ) = Opposite side / Adjacent side
tan(60°) = l / 3
Since tan(60°) = √3, we have:
√3 = l / 3
Multiplying both sides by 3:
l = 3√3
So, the length of each tangent is 3√3 cm.

OR

(b) Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solu. To prove that the tangents drawn at the ends of a diameter of a circle are parallel, we can use the concept of alternate angles formed by a transversal and two parallel lines.

Let's consider a circle with center O and diameter AB. Let T1 and T2 be the points of tangency where the tangents intersect the circle at A and B, respectively.
Since OA and OB are radii of the circle, they are equal in length, making triangle OAB an isosceles triangle.
Therefore, the angles formed at the tangents by the radii are equal:
$\angle OTA = \angle OTB$ (Angles formed by the radii and tangents)
Now, since OT1 and OT2 are the radii of the circle, they are parallel to each other.
Therefore, by the property of alternate angles, $\angle OTA$ and $\angle OTB$ are alternate angles formed by the transversal AB and the parallel lines OT1 and OT2.
Thus, $\angle OTA = \angle OTB = 90°$ (angles between tangent and radius)
Since alternate angles are equal, it implies that the tangents drawn at the ends of a diameter of a circle are parallel.
Therefore, we have proven that the tangents drawn at the ends of a diameter of a circle are parallel.

Ques 23. (a) Find the ratio in which the point P(- 4,6) divides the line segment joining the points A(- 6, 10) and B(3, - 8)

Solu. To find the ratio in which the point P(-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8), we can use the section formula.
Let the ratio in which point P divides the line segment be k:1.
The coordinates of point P(x, y) can be found using the section formula:
$x = (k \times x2 + 1 \times x1) / (k + 1)$
$y = (k \times y2 + 1 \times y1) / (k + 1)$
Where:

$(x1, y1)$ = Coordinates of point A(-6, 10)
$(x2, y2)$ = Coordinates of point B(3, -8)
Substitute the given values:
$-4 = (k \times 3 + (-6)) / (k + 1)$
$6 = (k \times (-8) + 10) / (k + 1)$
Now, solve these equations to find the value of k:
$-4(k + 1) = 3k - 6$
$-4k - 4 = 3k - 6$
$-4k - 3k = -6 + 4$
$-7k = -2$
$k = -2 / -7$
$k = 2 / 7$
So, the ratio in which the point P(-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8) is 2/7 : 1.

OR

(b) Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of an isosceles triangle

Solu. To prove that the points (3, 0), (6, 4), and (-1, 3) are the vertices of an isosceles triangle, we need to show that the lengths of at least two sides of the triangle are equal.
Let's calculate the distances between these points:
1. Distance between (3, 0) and (6, 4):
$d_1 = \text{sqrt}((6 - 3)^2 + (4 - 0)^2) = \text{sqrt}(3^2 + 4^2) = \text{sqrt}(9 + 16) = \text{sqrt}(25) = 5$
2. Distance between (6, 4) and (-1, 3):
$d_2 = \text{sqrt}((-1 - 6)^2 + (3 - 4)^2) = \text{sqrt}((-7)^2 + (-1)^2) = \text{sqrt}(49 + 1) = \text{sqrt}(50)$
3. Distance between (-1, 3) and (3, 0):
$d_3 = \text{sqrt}((3 + 1)^2 + (0 - 3)^2) = \text{sqrt}(4^2 + (-3)^2) = \text{sqrt}(16 + 9) = \text{sqrt}(25) = 5$
Now, we can observe that $d_1 = d_3 = 5$, so two sides of the triangle are equal in length. Therefore, the triangle formed by the points (3, 0), (6, 4), and (-1, 3) is an isosceles triangle.
So, we have proved that the points (3, 0), (6, 4), and (-1, 3) are the vertices of an isosceles triangle.

Ques 24. If a, ß are zeroes of the polynomial p(x) = $5x^2 - 6x + 1$ then find the value of a + β + αβ

Solu. Here's how to find the value of a + β + αβ for the given polynomial:
We can utilize the relationship between the zeroes (a and β) and the coefficients of a quadratic polynomial.

1. Sum of zeroes: The sum of the roots (zeroes) of a quadratic polynomial is equal to the negative of the coefficient of the second term (the x term in this case) divided by the leading coefficient (the coefficient of the x^2 term).

In this case:

- Sum of roots (a + β) = - (-6x) / 5 (remember, negative sign flips because we're negating)
- a + β = 6/5

2. Product of zeroes: The product of the roots (zeroes) of a quadratic polynomial is equal to the constant term divided by the leading coefficient.

In this case:

- Product of roots (a * β) = 1 / 5 (constant term is 1 and leading coefficient is 5)

3. What to find: We need to find a + β + αβ. This combines both the sum and product of the roots.

Solution:
Substitute the expressions we found for the sum and product of roots:

- a + β + αβ = 6/5 + 1/5
- Combine like terms:
    - a + β + αβ = 7/5

Therefore, the value of a + β + αβ is 7/5.

Ques 27. A sector is cut from a circle of radius 21 cm. The central angle of the sector is 150°. Find the length of the arc of this sector and the area of the sector.

Solu. We can find both the arc length (s) and the area (A) of the sector using the following formulas:

1. Arc Length (s):

s = (θ / 360°) * 2πr
where:

- θ is the central angle of the sector (in degrees)
- r is the radius of the circle
- π (pi) is a mathematical constant approximately equal to 3.14159

In this case:

- $\theta = 150°$
- $r = 21$ cm

s = (150° / 360°) * 2π * 21 cm s ≈ (5/12) * 2π * 21 cm (using π ≈ 22/7 for calculation) s ≈ 55 cm (rounded to two decimal places)

Therefore, the arc length of the sector is approximately 55 cm.

2. Area of the Sector (A):

$A = (\theta / 360°) * \pi r^2$

where:

- θ and r are the same as defined previously

$A = (150° / 360°) * \pi * (21 \text{ cm})^2$ $A \approx (5/12) * \pi * 441 \text{ cm}^2$ (since $21^2 = 441$) $A \approx 577.5 \text{ cm}^2$ (rounded to one decimal place)

Therefore, the area of the sector is approximately 577.5 cm^2.

Ques 28. (a) Prove that sqrt(3) is an irrational number.

Solu. proof for the irrationality of sqrt(3)

Assume the opposite:

- Let sqrt(3) = p / q, where p and q are integers with no common factors (q ≠ 0).

Derive a contradiction:

1. Square both sides: (sqrt(3))^2 = (p / q)^2
2. Simplify: 3 = p^2 / q^2
3. Multiply both sides by q^2: 3q^2 = p^2 (isolate p^2)

Implication of the equation:

- 3q^2 is a multiple of 3, so p^2 (on the right) must also be a multiple of 3. We can express this as:
  - p^2 = 3k (where k is another integer)

Contradiction:

- Take the square root of both sides (remembering positive/negative implications):
  - $p = \pm\sqrt{3k}$

This equation contradicts our initial assumption. If p is the square root of 3k (an integer multiplied by 3, hence an integer), then p itself cannot be an integer. This violates the initial condition that p and q were integers in their simplest form.
Conclusion:
Since our initial assumption (sqrt(3) being rational) leads to a contradiction, sqrt(3) must be irrational. It cannot be expressed as a simple fraction of two integers.

OR

(b) Prove that (sqrt(2) + sqrt(3))^2 is an irrational number, given that sqrt(6) is an irrational number.

Solu. Let's prove the irrationality of (sqrt(2) + sqrt(3))^2 using contradiction and the fact that sqrt(6) is irrational:
Assume the opposite:

1. (sqrt(2) + sqrt(3))^2 = p / q (rational, p & q integers, q ≠ 0)

Expand and reach a contradiction:

2. 2 + 2*sqrt(2)*sqrt(3) + 3 = p / q (substitute the square)
3. 5 + 2*sqrt(2)*sqrt(3) = p / q (combine constants)

Contradiction based on given fact:

4. Since sqrt(6) = sqrt(2)sqrt(3) is irrational (given), 2sqrt(2)*sqrt(3) in equation 3 must be rational (difference of rationals is rational). This contradicts the fact that sqrt(6) is irrational.

Conclusion:
Our initial assumption (equation 1) leads to a contradiction. Therefore, (sqrt(2) + sqrt(3))^2 must be irrational.

Ques 29. Three unbiased coins are tossed simultaneously. Find the probability of getting:
(i) at least one head.
(ii) exactly one tail.
(iii) two heads and one tail.

Solu. Here's how to find the probability of each scenario when tossing three unbiased coins:
(i) At least one head:

1. Favorable cases: This includes getting one head (HTT, THH, HTH), two heads (THT, HTT), or three heads (HHH).
2. Total cases: There are 2^3 (8) total possible outcomes (HHH, HHT, HTH, THH, HTT, THT, TTH, TTT) since each coin has two possibilities (heads or tails).
3. Probability: Probability = Favorable cases / Total cases

- Probability (at least one head) = (Number of outcomes with heads) / Total outcomes
- Probability = (5 out of 8) = 5/8

(ii) Exactly one tail:

1. Favorable cases: This includes getting one tail (HTT, THH, HTH).
2. We already calculated this in part (i): The favorable cases for exactly one tail are the same as getting one head in (i).
3. Probability: Probability (exactly one tail) = 5/8 (from part (i))

(iii) Two heads and one tail:

1. Favorable cases: There are three possible arrangements for two heads and one tail (THT, HTT, THT).
2. Total cases (mentioned earlier): There are 8 total possible outcomes.
3. Probability: Probability (two heads and one tail) = Favorable cases / Total cases

- Probability = 3/8

Summary:

- Probability (at least one head) = 5/8
- Probability (exactly one tail) = 5/8
- Probability (two heads and one tail) = 3/8

Ques 30. Prove that the parallelogram circumscribing a circle is a rhombus.

Solu. Imagine a parallelogram (ABCD) wrapped around a circle, touching it at all four sides. Let O be the circle's center. Here's why this must be a rhombus:

1. Tangent trick: Since each side is tangent to the circle at a point, the radii (lines from O to those points) are perpendicular to that side.
2. Opposite sides matter: In any parallelogram, opposite sides are parallel and equal in length.
3. Right triangles formed: The radii create four right triangles with O at the corner. Since the radii (heights) are all the same length (same circle), the bases of these triangles (halves of the parallelogram's sides) must also be equal.
4. Special right triangles: Remember, the radius touching a tangent line bisects it, creating two congruent right triangles with the radius as the hypotenuse. These triangles have a special 2:1 ratio between the hypotenuse (radius) and a shorter leg (half the base).

Key point: We now know that opposite sides of the parallelogram are equal (given) and the bases of the right triangles (half the sides) are also equal.
Result: Because all four sides of the parallelogram have equal halves (based on the right triangles), all four sides must be equal in length. This is the defining property of a rhombus.
Therefore, a parallelogram that perfectly surrounds a circle is always a rhombus.

Ques 31 (a) If the sum of the first 14 terms of an A.P. is 1050 and the first term is 10, then find the 20th term and the nth term.

Solu. To find the area of the corresponding major sector of a circle, we first need to find the measure of the angle subtended by the arc at the center of the circle. Since the arc subtends a right angle at the center, the measure of this angle is 90 degrees.
Now, the area of a sector of a circle is given by the formula:
Area of sector = (angle/360°) × πr^2
Where:
- angle is the measure of the central angle in degrees.
- r is the radius of the circle.
Substituting the given values:
Area of sector = (90°/360°) × 3.14 × (10 cm)^2
Simplify and calculate:

Area of sector = (1/4) × 3.14 × 100
Area of sector = (1/4) × 314
Area of sector = 78.5 cm^2
So, the area of the corresponding major sector is 78.5 cm^2.

Ques 32 (b) Sides AB and AC and median AD of a △ABC are respectively proportional to sides PQ and PR and median PM of another △PQR. Show that △ABC ~ △PQR.

Solu. To show that triangles ABC and PQR are similar, we can use the concept of proportional sides and the median property.
Given:
- Sides AB and AC of triangle ABC are proportional to sides PQ and PR of triangle PQR, respectively.
- Median AD of triangle ABC is proportional to median PM of triangle PQR.
Let's denote the lengths of sides AB, AC, PQ, and PR as a, b, p, and q respectively. Also, let m be the length of median AD and n be the length of median PM.
According to the given conditions, we have the following proportions:
1. Proportionality of sides:
AB/PQ = AC/PR = a/p = b/q
2. Proportionality of medians:
AD/PM = m/n
To prove that triangles ABC and PQR are similar, we need to show that their corresponding angles are equal.
Using the properties of medians in triangles, we know that the median divides the triangle into two triangles with equal areas. Therefore, the triangles formed by the medians are similar to the original triangle.
Now, since the sides and medians of the triangles are proportional, the triangles formed by the medians are also similar to each other.
Hence, triangle ABC is similar to triangle PQR.

Ques 33 (a) Find the value of 'k' for which the quadratic equation $(k + 1)x^2 - 6(k + 1)x + 3(k + 9) = 0$, $k \neq -1$ has real and equal roots.

Solu. We can find the value of 'k' for which the quadratic equation has real and equal roots by using the concept of the discriminant.
Here's how:

1. Discriminant and equal roots:
    - In a quadratic equation ax^2 + bx + c = 0, the discriminant (b^2 - 4ac) determines the nature of the roots.
    - When the discriminant is equal to zero (b^2 - 4ac = 0), the quadratic equation has real and equal roots.
2. Identify the coefficients:
    - In the given equation (k + 1)x^2 - 6(k + 1)x + 3(k + 9) = 0:
        - a = k + 1
        - b = -6(k + 1)
        - c = 3(k + 9)
3. Set the discriminant to zero and solve for k:
    - b^2 - 4ac = 0
    - (-6(k + 1))^2 - 4 * (k + 1) * 3(k + 9) = 0 (substitute coefficients)
    - Expand and solve for k:
        - 36(k + 1)^2 - 12(k + 1)(k + 9) = 0
        - Factor out (k + 1):
            - (k + 1) (36(k + 1) - 12(k + 9)) = 0
        - We are given k ≠ -1, so we can ignore the factor (k + 1).
        - This leaves us with: 36(k + 1) - 12(k + 9) = 0
        - Solve for k:
            - 12(3k + 3 - k - 9) = 0
            - 12(2k - 6) = 0
            - k - 3 = 0
            - k = 3

Therefore, the value of 'k' for which the quadratic equation has real and equal roots is k = 3.

OR

(b) The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find their present ages.

Solu. Let's denote the son's current age as x. We can then use the given information to set up two equations and solve for x and the man's age.

1. Current Ages:
   - Man's age: The man's current age is twice the square of his son's age. We can express this as:
     - Man's age = 2 * x^2
2. Ages Eight Years Hence:
   - Man's age: Eight years from now, the man's age will be 4 years more than three times his son's age. We can express this as:
     - Man's age (8 years later) = 3 * (x + 8) + 4
3. Relating the Equations (Present Ages):

We know that the man's age remains the same, so his current age and his age eight years later must be equal.

   - Equate the two expressions from equations 1 and 2:
     - 2 * x^2 = 3 * (x + 8) + 4
4. Solve for x (Son's Age):
   - Expand the equation from step 3:
     - 2x^2 = 3x + 24 + 4
     - 2x^2 - 3x - 28 = 0
   - This is a quadratic equation. You can solve it by factoring, using the quadratic formula, or using a graphing calculator. Here, we'll use factoring:
     - Factor the equation: (2x + 7)(x - 4) = 0
     - Since we cannot have a negative age, the valid solution is: x =4
5. Find the Man's Age:
   - Now that you know the son's age (x = 4), substitute it back into equation 1 to find the man's current age:
     - Man's age = 2 * (4)^2
     - Man's age = 2 * 16
     - Man's age = 32

Therefore, the son is currently 4 years old, and the man is 32 years old.

Ques 34. From a window 15 metres high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30° and 45° respectively. Find the height of the opposite house. (Use sqrt(3) = 1.732)

Solu. Let's solve this step-by-step to find the height of the opposite house:

1. Define the variables:

- Let h be the height of the opposite house (what we need to find).
- Let P be the window point (15 meters high).
- Let D be the foot of the opposite house.
- Let C be the top of the opposite house.

2. Identify the right triangles:

- Triangle PCD: This is a right triangle with right angle at D. We know the angle of depression from P to D is 45° and PD (height of the window) is 15 meters.
- Triangle PCQ: This is another right triangle with right angle at C. We know the angle of elevation from P to C is 30°. We also know PQ (height of the window) is 15 meters.

3. Solve for unknown sides:

- Triangle PCD:
  - Since the angle of depression is 45°, PD (opposite to the angle) is equal to the distance between P and D (hypotenuse) divided by the square root of 2.
  - Therefore, PD (15 meters) = distance between P and D / √2.
  - Solving for the distance between P and D: distance between P and D = PD * √2 = 15 meters * √2.
- Triangle PCQ:
  - We know the tangent of 30° is equal to the opposite side (PQ - 15 meters) divided by the adjacent side (distance between P and D).
  - tan(30°) = 1/√3 (since tangent of 30° is a fixed value).
  - Substituting the values: (PQ - 15 meters) / (distance between P and D) = 1/√3.

- We already found the distance between P and D from the previous step (15 meters * √2).
- Substituting again: (PQ - 15 meters) / (15 meters * √2) = 1/√3.
- To simplify the equation, multiply both sides by 15 meters * √2 * √3: √3 * (PQ - 15 meters) = 15 meters * √2

4. Solve for the height of the opposite house (h):

- Now we have two equations with two unknowns (PQ and distance between P and D).
- We can solve for PQ from either equation and then find h (height of the opposite house).
- From the second equation: PQ = (15 meters * √2) / √3 + 15 meters.
- Substitute this value of PQ in the equation h = PQ + QC (where QC is the height of the window, 15 meters).
- h = [(15 meters * √2) / √3 + 15 meters] + 15 meters.
- Simplify the equation: h ≈ 15 meters + 15 meters + (15 meters * √2) / √3.
- Using the approximation √3 ≈ 1.732, h ≈ 30 meters + (15 meters * 1.732).
- h ≈ 30 meters + 25.98 meters.
- Therefore, the height of the opposite house (h) is approximately 55.98 meters.

Rounding to a more reasonable answer in the context of house heights, the height of the opposite house is approximately 56 meters.
