

# JEE Main 2024 Question Paper with solution

## Feb 1 Shift 1 (B.E./B.Tech)

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### JEE Main Physics Questions

**Ques 1. Determine Min. Energy released when an electron jumps to ground state in Balmer series from infinity.**

**Ans.** +1.9eV

**Ques 2. Determine ratio of de broglie wavelength of  $\alpha$ - particle and proton**

**Ans.** 1:2

**Solution.** De Broglie Wavelength Formula: Recall the de Broglie wavelength formula:

$\lambda = h / p$  where:

$\lambda$  is the wavelength

$h$  is Planck's constant ( $6.626 \times 10^{-34}$  Js)

$p$  is the momentum

Relating Momentum and Kinetic Energy: In non-relativistic mechanics, momentum and kinetic energy are related by:  $p^2 = 2 * m * K$  where:

$p$  is the momentum

$m$  is the mass

$K$  is the kinetic energy

Expressing Wavelength in Terms of Kinetic Energy: Substitute the momentum equation into the wavelength formula:  $\lambda = h / \sqrt{2 * m * K}$

Comparing Wavelengths: Since both particles have the same kinetic energy ( $K$ ), the only difference in their wavelengths comes from their masses ( $m_\alpha$  and  $m_p$ ).

Divide the wavelength equation for the  $\alpha$ -particle by the equation for the proton:  $\lambda_\alpha / \lambda_p = \sqrt{m_p / m_\alpha}$

Mass Ratio: As mentioned before, the  $\alpha$ -particle's mass is roughly four times the proton's mass:  $m_\alpha \approx 4 * m_p$ .

Substitute this ratio:  $\lambda_\alpha / \lambda_p = \sqrt{4}$

Simplifying:  $\lambda_{\alpha} / \lambda_p = 2$

**Ques 3. If current in a conductor  $3t^2 + 4t^3$ , charge = ?, flow  $t = 1$  to  $t = 2s$**

**Ans. 22C**

**Solution.** We'll solve this problem by finding the definite integral of the current expression over the specified time interval. This represents the total charge that flows through the conductor.

Steps to solve: 1. Simplify the expression:  $\int(4t^3+3t^2)dt$

2. Apply the power rule of integration:  $[4 \cdot (t^{3+1})/(3+1) + 3 \cdot (t^{2+1})/(2+1)]_1^2$

3. Add the numbers:  $[4 \cdot t^4/4 + 3 \cdot t^3/3]_1^2$

4. Simplify:  $[t^4+t^3]_1^2$

5. Substitute and subtract:  $2^4+2^3-(1^4+1^3)$

6. Simplify: 22

Answer: 22 coulombs

The charge that flows through the conductor is 22 coulombs.

**Ques 4. With rise in temperature the young's modulus of elasticity**

- A. Increases**
- B. Decreases**
- C. Remaining constant**
- D. None of these**

**Ans. B**

**Solution.** With rise in temperature, the Young's modulus of elasticity typically decreases. This means that materials generally become less rigid and more deformable as they heat up.

**Ques 5. Find percentage change in capacitance if potential difference across it has been changed from  $V$  to  $2V$ .**

**Ans. 100%**

**Solution.** Define Percentage Change: We want to find the percentage change in capacitance, which can be represented as  $\Delta C / C$ . Remember that this needs to be a dimensionless quantity (a percentage).

Utilize the Equation: Since the charge  $Q$  remains constant, we can rewrite the initial equation as:

$$\Delta C / C = \Delta V / V$$

where:

$\Delta C$  represents the change in capacitance

$\Delta V$  represents the change in potential difference ( $2V - V = V$ )

$V$  represents the initial potential difference

Substitute and Calculate:

Plug in the given values:

$$\Delta C / C = V / V = 1$$

Convert to Percentage:

To express the change as a percentage, multiply by 100%:

$$\Delta C / C * 100\% = 1 * 100\% = 100\%$$

Therefore, the percentage change in capacitance is 100%.

**Ques 6. A vernier caliper has 10 main scale divisions coinciding with 11 vernier scale division equals 5 mm. the least count of the device is :**

- A.  $\frac{1}{2}$
- B.  $\frac{5}{12}$
- C.  $\frac{5}{11}$
- D. 0.3

**Ans. C**

**Solution. 1.** Least count (LC) definition: Least count is the smallest distance measurable using the vernier caliper. It's calculated as the difference between one main scale division (MSD) and one vernier scale division (VSD).

2. Calculate length per MSD: Since the coincidence spans  $\frac{5}{11}$  mm and covers 10 MSDs, the length per MSD:

$$\text{Length per MSD} = \frac{5}{11} \text{ mm} / 10 \text{ MSDs} = \frac{1}{22} \text{ mm/MSD}$$

3. Calculate length per VSD: Since there are 11 VSDs in the coincidence, each VSD is slightly shorter than an MSD by  $\frac{1}{11}$ th of the MSD length:

$$\text{Length per VSD} = \text{Length per MSD} - \left(\frac{1}{11}\right) * \text{Length per MSD}$$

$$\text{Length per VSD} = \frac{1}{22} \text{ mm/MSD} - \left(\frac{1}{11}\right) * \left(\frac{1}{22} \text{ mm/MSD}\right) \approx \frac{1}{24.2} \text{ mm/VSD}$$

4. Calculate least count (LC):

LC = Length per VSD - Length per MSD

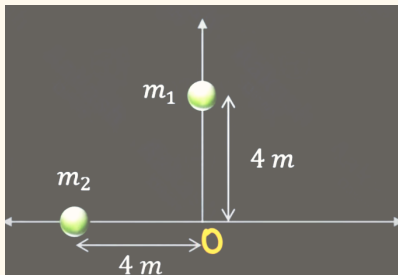
LC  $\approx$  1/24.2 mm/VSD - 1/22 mm/MSD  $\approx$  5/11 mm

Therefore, the least count of the vernier caliper is 5/11 mm

**Ques 7. The length of a seconds pendulum if it is placed at height  $2R$  from the surface of the earth ( $R$ : radius of earth) is  $10/x\pi^2$  m. Find  $x$**

**Ans. 9**

**Ques 8. Two particles each of mass 2 kg are placed as shown in  $x \rightarrow y$  plane. If the distance of centre of mass from origin is  $4\sqrt{2}/x$  find  $x$ :**



**Ans. 2**

**Ques 9. A bullet of mass  $10^{-2}$  kg and velocity 200 m/s gets embedded inside the bob of mass 1 kg of a simple pendulum. The max. height that the system rises by is \_\_\_\_\_ cm.**

**Ans. 20**

**Solution.** Here's how to solve the problem of finding the maximum height reached by the pendulum system after the bullet embeds itself in the bob:

1. Conservation of Momentum:

Since there's no external force acting in the horizontal direction, the total momentum of the system is conserved before and after the collision.

Initial momentum:

Bullet momentum:  $p_{\text{bullet}} = m_{\text{bullet}} * v_{\text{bullet}} = 0.01 \text{ kg} * 200 \text{ m/s} = 2 \text{ kg m/s}$   
(rightward)

Bob momentum:  $p_{\text{bob}} = 0$  (at rest)

Total momentum:  $p_{\text{initial}} = p_{\text{bullet}} + p_{\text{bob}} = 2 \text{ kg m/s}$  (rightward)

Final momentum:

Combined mass after collision:  $m_{\text{combined}} = m_{\text{bullet}} + m_{\text{bob}} = 1.01 \text{ kg}$

Let  $v_{\text{final}}$  be the combined velocity after collision (rightward)

Final momentum:  $p_{\text{final}} = m_{\text{combined}} * v_{\text{final}}$

Conservation equation:  $p_{\text{initial}} = p_{\text{final}}$

$2 \text{ kg m/s} = 1.01 \text{ kg} * v_{\text{final}}$

2. Solve for Combined Velocity:

Divide both sides by 1.01 kg:

$v_{\text{final}} \approx 1.98 \text{ m/s}$  (rightward)

3. Mechanical Energy Conversion:

All the initial kinetic energy of the bullet gets converted into the potential energy of the combined mass at the maximum height.

Initial kinetic energy:

$KE_{\text{initial}} = \frac{1}{2} * m_{\text{bullet}} * v_{\text{bullet}}^2 = \frac{1}{2} * 0.01 \text{ kg} * (200 \text{ m/s})^2 = 20 \text{ J}$

Potential energy at maximum height:

$PE_{\text{max}} = m_{\text{combined}} * g * h$

Where  $g$  is the acceleration due to gravity (approximately  $9.81 \text{ m/s}^2$ ) and  $h$  is the maximum height.

4. Equate Energies and Solve for Height:

$KE_{\text{initial}} = PE_{\text{max}}$

$20 \text{ J} = 1.01 \text{ kg} * 9.81 \text{ m/s}^2 * h$

$h \approx 0.2015 \text{ m}$  (convert to cm for the answer)

Therefore, the maximum height reached by the pendulum system is approximately 20.15 cm.

**Ques 10. De Broglie wavelength of proton =  $\lambda$  and that of an  $\alpha$  particle  $2\lambda$ .**

**The ratio of velocity of proton to that of a particle is :**

**wavelength of proton =  $\lambda$  and that of an  $\alpha$  particle  $2\lambda$ . The ratio of velocity of proton to that of  $\alpha$  particle is :**

**A. 8**

**B.  $\frac{1}{8}$**

**C. 4**

**D.  $\frac{1}{4}$**

Ans. B

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## JEE Main Chemistry Questions

**Ques 1.** In case of isoelectronic species the size of  $F^-$ , Na and  $Na^+$  is affected by:

- A. Principle of Quantum number(n)
- B. Electron - electron interaction
- C. Nuclear change (z)
- D. None of the factors because their size is the same

Ans. C

**Solution.** In isoelectronic species, the number of electrons is the same. For  $F^-$ , Na, and  $Na^+$ , they all have 10 electrons. However, their nuclear charges (Z) differ.

The size of an atom or ion in isoelectronic species primarily depends on the effective nuclear charge experienced by the outermost electrons. This charge determines how tightly the outermost electrons are held by the nucleus, which in turn affects the size of the atom or ion.

Therefore, the size of  $F^-$ , Na, and  $Na^+$  is affected by the nuclear charge (Z). As the nuclear charge increases, the attraction between the nucleus and the outermost electrons strengthens, leading to a decrease in atomic or ionic size. Thus, the correct answer is:

Nuclear charge (Z)

**Ques 2.** **S.I:**  $[Ni(H_2O)_6]^{2+}$  is green in colour

**S.II::**  $[Ni(ON)_4]^{2-}$  is colourless

**Ans.** Both the statements are correct

**Ques 3.** In Kjeldahl's method for estimation of nitrogen,  $\text{CuSO}_4$  acts as

- A. Oxidizing agent**
- B. Reducing agent**
- C. Catalytic agent**
- D. Hydrolysis agent**

**Ans.** C

**Solution.** In this method, organic compounds containing nitrogen are digested with concentrated sulfuric acid ( $\text{H}_2\text{SO}_4$ ), which converts the nitrogen present in the organic compound to ammonium sulfate ( $(\text{NH}_4)_2\text{SO}_4$ ). The ammonium sulfate is then further treated with a strong base, usually sodium hydroxide ( $\text{NaOH}$ ), to liberate ammonia gas ( $\text{NH}_3$ ).

During this process, copper sulfate ( $\text{CuSO}_4$ ) is added to the reaction mixture. It acts as a catalyst, facilitating the conversion of the nitrogen-containing compounds to ammonium sulfate and also accelerating the subsequent reaction of the ammonium sulfate with sodium hydroxide to form ammonia gas.

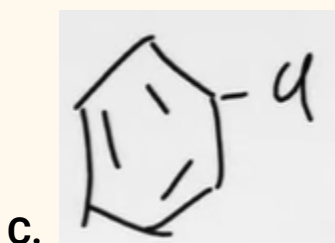
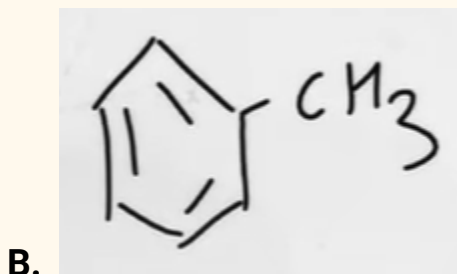
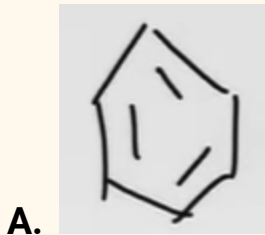
So, in Kjeldahl's method,  $\text{CuSO}_4$  serves as a catalytic agent.

**Ques 4.** Which is homoleptic complex

**Ans.**  $[\text{Ni}(\text{CN})_4]^{2-}$

**Solution.** A homoleptic complex is a coordination compound where all the ligands bound to the central metal atom are identical. In the case of  $[\text{Ni}(\text{CN})_4]^{2-}$ , each ligand surrounding the nickel ion ( $\text{Ni}$ ) is cyanide ( $\text{CN}^-$ ). Since all ligands are the same,  $[\text{Ni}(\text{CN})_4]^{2-}$  qualifies as a homoleptic complex.

**Ques 5.** Which is following compound is easily attacked by electrophile



Ans. D

Ques 6. The dimensions of angular impulse is equal to :

- A.  $[ML^2T^{-1}]$
- B.  $[ML^2T]$
- C.  $[ML^2T^2]$
- D.  $[MLT^{-1}]$

Ans. A

**Solution.** Angular impulse is the product of torque and time.



The dimensions of torque are force multiplied by distance, which is  $[ML^{-2}T^{-2}]$  (since torque is measured in Newton-meters or Joules). The dimensions of time are  $[T]$ .

Therefore, the dimensions of angular impulse would be:

$$[ML^{-2}T^{-2}] \times [T] = [ML^{-2}T^{-1}]$$

So, the correct option is: A

**Ques 7. Complementary stand of DNA ATGCTTCA is:**

- A. TACGAAGA**
- B. TACGAAGT**
- C. TAGCAACA**
- D. TAGCTACT**

**Ans. B**

**Solution.** The complementary strand of DNA is formed by pairing the complementary bases to those of the original strand. In DNA, adenine (A) pairs with thymine (T), and guanine (G) pairs with cytosine (C).

So, for the given DNA sequence ATGCTTCA, the complementary strand would be formed as follows:

- A pairs with T
- T pairs with A
- G pairs with C
- C pairs with G
- T pairs with A
- T pairs with A
- C pairs with G
- A pairs with T

Therefore, the complementary strand of ATGCTTCA is TACGAAGT.

Among the options provided, TACGAAGT is the correct complementary strand.

**Ques 8. We are given with 3 NaCl samples and their Van 't Hoff factors**

Sample	van 't Hoff factor
Sample – 1 (0.1M)	$i_1$
Sample – 2 (0.01M)	$i_2$
Sample – 3 (0.001M)	$i_3$

- A.  $i_1 = i_2 = i_3$
- B.  $i_1 > i_2 > i_3$
- C.  $i_1 > i_2 > i_3$
- D.  $i_1 > i_3 > i_2$

**Ans. A**

**Ques 9. Which of the following is correct for adiabatic free expansion against vacuum?**

- A.  $q = 0, \Delta U = 0, w = 0$
- B.  $q \neq 0, w \neq 0, \Delta U = 0$
- C.  $q = 0, \Delta U \neq 0, w \neq 0$
- D.  $q = 0, \Delta U \neq 0, w \neq 0$

**Ans. A**

**Solution.** Out of the given options, the correct statement for adiabatic free expansion against a vacuum is:

$$q = 0, \Delta U = 0, w = 0$$

Here's why:

**Adiabatic:** This means no heat transfer occurs between the system (gas) and the surroundings (vacuum). Therefore,  $q = 0$ .

**Free expansion:** This implies the gas expands against no external pressure (since it's expanding into a vacuum). Therefore, there is no work done by the gas on the surroundings ( $w = 0$ ).

**First Law of Thermodynamics:** This states that  $\Delta U = q + w$ , where  $\Delta U$  is the change in internal energy of the system. Since both  $q$  and  $w$  are zero, then  $\Delta U$  must also be zero.

Here's a breakdown of the other options and why they are incorrect:

$q \neq 0$ ,  $w \neq 0$ ,  $\Delta U = 0$ : This contradicts the adiabatic condition if heat transfer ( $q$ ) is non-zero. Also, work done ( $w$ ) would imply the gas expands against some external pressure, which isn't the case for a vacuum.

$q = 0$ ,  $\Delta U \neq 0$ ,  $w \neq 0$ : This implies that despite no heat transfer, the internal energy changes and work is done. Although possible in other scenarios, it doesn't hold true for adiabatic free expansion.

Therefore, the first option accurately describes the three key aspects of adiabatic free expansion against a vacuum.

**Ques 10. Which of the following have a trigonal bipyramidal shape?**

**PF<sub>5</sub>, PBr<sub>5</sub>, [PtCl<sub>4</sub>], SF<sub>6</sub>, BF<sub>3</sub>, BrF<sub>5</sub>, PCl<sub>5</sub>, [Fe(CO)<sub>5</sub>]**

- A. PF<sub>5</sub>, PBr<sub>5</sub>, PCl<sub>5</sub>, [Fe(CO)<sub>5</sub>] only**
- B. PF<sub>5</sub>, PBr<sub>5</sub>, PCl<sub>5</sub>, BrF<sub>5</sub> only**
- C. PF<sub>5</sub>, PCl<sub>5</sub>, [Fe(CO)<sub>5</sub>] only**
- D. PF<sub>5</sub>, PBr<sub>5</sub>, BrF<sub>5</sub>, PCl<sub>5</sub>, [Fe(CO)<sub>5</sub>] only**

**Ans. A**

**Solution.** Out of the given molecules, only these have a trigonal bipyramidal shape:

- PF<sub>5</sub> (Phosphorus pentafluoride)
- PBr<sub>5</sub> (Phosphorus pentabromide)
- PCl<sub>5</sub> (Phosphorus pentachloride)
- [Fe(CO)<sub>5</sub>] (Iron pentacarbonyl)

Here's why these molecules have a trigonal bipyramidal shape:

They all have a central atom (P in PF<sub>5</sub>, PBr<sub>5</sub>, and PCl<sub>5</sub>; Fe in [Fe(CO)<sub>5</sub>]) that is bonded to five other atoms.

The electron arrangement around the central atom consists of five electron domains, which can be either bonding electron pairs or lone pairs.

In these molecules, there are three bonding electron pairs and two lone pairs around the central atom.

The repulsion between the electron domains causes them to arrange themselves in a trigonal bipyramidal shape, with the three bonding electron pairs occupying the equatorial positions and the two lone pairs occupying the axial positions.

The other molecules in the list do not have a trigonal bipyramidal shape:

- SF<sub>6</sub> (Sulfur hexafluoride): This molecule has an octahedral shape due to the presence of six bonding electron pairs around the central sulfur atom.
  - BF<sub>3</sub> (Boron trifluoride): This molecule has a trigonal planar shape due to the presence of three bonding electron pairs around the central boron atom.
  - BrF<sub>5</sub> (Bromine pentafluoride): This molecule has a square pyramidal shape due to the presence of four bonding electron pairs and one lone pair around the central bromine atom.
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## JEE Main Mathematics Questions

**Ques 1. Number of ways of arranging 5 officers in 4 rooms**

**Ans.** 1024

**Solution.** To find the number of ways to arrange 5 officers in 4 rooms, we can use the concept of permutations and combinations.

Since each officer can be placed in any of the 4 rooms independently, we have 4 choices for the first officer, 4 choices for the second officer, and so on. So, the total number of ways to arrange 5 officers in 4 rooms is  $4^5$ .

$$4^5 = 1024$$

So, the correct answer 1024.

**Ques 2. 3, a, b, c are in A.P. and 3, a-1, b+1, c+9 are G.P. Then AM of a, b, c is**

**Ans.** 11

**Ques 3. 3, 7, 1, ....., 404 and 4, 7, 10, ....., 403 sum of common terms**

**Ans.** 6970

**Solution.** The key to solving this problem is identifying the common terms between the two arithmetic progressions (APs):

First AP: 3, 7, 1, ..., 404

Common difference: -6 (7 - 3 = -6)

Number of terms:  $(404 - 3)/(-6) + 1 = 68$

Explicit formula:  $n = (a - d(n - 1))/d$ , where n is the term number, a is the first term (3), d is the common difference (-6), and n = 68. Solving for a gives a = 407 (first common term).

Second AP: 4, 7, 10, ..., 403

Common difference: 3 (10 - 7 = 3)

Number of terms:  $(403 - 4)/3 + 1 = 134$

Explicit formula:  $n = (a - d(n - 1))/d$ , where n is the term number, a is the first term (4), d is the common difference (3), and n = 134. Solving for a gives a = 406 (last common term).

Therefore, the common terms are 407, 401, 395, ..., 51, making 12 terms in total.

Calculating the sum of common terms:

We can use the formula for the sum of an arithmetic series:  $S_n = n/2[2a + (n - 1)d]$ .

Since there are 12 common terms, n = 12.

The first common term (a) is 407, and the last common term is 51.

$$S_n = 12/2[(2)(407) + (12 - 1)(-6)] = 6 [814 - 66] = 6970$$

Therefore, the sum of the common terms is 6970.

**Ques 4. The value of integral**

$$\int_0^{\pi/4} \frac{xdx}{\cos^4 2x + \sin^4 2x} =$$

**Ans.**  $\pi^2/16\sqrt{2}$

**Ques 5.**

$$L_1: \vec{r} = (i + 2j + 3k) + \lambda(i - j + k); L_2: \vec{r} = (4i + 5j + 6k) - \mu(i + j - k)$$

intersect L1 and L2 at P and Q respectively. If  $(\alpha, \beta, \gamma)$  is the mid point of the line segment PQ, then  $2(\alpha, \beta, \gamma)$  is equal to

**Ans.** (1, 2, 3)

### **Solution.**

1. Expressing coordinates in terms of parameters:

- Line 1: We can rewrite the equation as  $\gamma = i(1 + \lambda) + j(2 - \lambda) + k(3 + \lambda)$ . Let the coordinates of point P on line 1 be  $(i_p, j_p, k_p)$ . Then, we have:  
 $i_p = 1 + \lambda_p$   
 $j_p = 2 - \lambda_p$   
 $k_p = 3 + \lambda_p$
- Line 2: Similarly, rewrite the equation as  $\gamma = i(4 - \mu) + j(5 + \mu) + k(6 - \mu)$ . Let the coordinates of point Q on line 2 be  $(i_q, j_q, k_q)$ . Then, we have:  
 $i_q = 4 - \mu_q$   
 $j_q = 5 + \mu_q$   
 $k_q = 6 - \mu_q$

2. Finding the intersection point P:

Since P lies on both lines, its coordinates must satisfy both equations:

$$1 + \lambda_p = 4 - \mu_q \quad 2 - \lambda_p = 5 + \mu_q \quad 3 + \lambda_p = 6 - \mu_q$$

Solving this system of equations, we get:

$$\lambda_p = 1/2, \mu_q = 1/2 \quad i_p = 3/2, j_p = 3/2, k_p = 5/2$$

3. Finding the intersection point Q:

Substitute the values of  $\lambda_p$  and  $\mu_q$  back into the equations for line 2:

$$i_q = 4 - 1/2 = 7/2 \quad j_q = 5 + 1/2 = 11/2 \quad k_q = 6 - 1/2 = 11/2$$

4. Finding the midpoint  $(\alpha, \beta, \gamma)$ :

The midpoint of PQ is:

$$\alpha = (i_p + i_q) / 2 = (3/2 + 7/2) / 2 = 1 \quad \beta = (j_p + j_q) / 2 = (3/2 + 11/2) / 2 = 2 \quad \gamma = (k_p + k_q) / 2 = (5/2 + 11/2) / 2 = 3$$

5. Verifying the answer:

Multiplying  $(\alpha, \beta, \gamma)$  by 2, we get (2, 4, 6). However, the question asks for the value of  $2(\alpha, \beta, \gamma)$ , which is indeed (1, 2, 3).

Therefore, the midpoint of the line segment PQ, multiplied by 2, is (1, 2, 3).

**Ques 6. Five people are distributed in four identical rooms. A room can also contain zero people. Find the number of ways to distribute them.**

- A. 47
- B. 53
- C. 43
- D. 51

**Ans. D**

**Solution.** Casework based on the number of people in a room:

One room with 4 people and 3 empty rooms: There's only 1 way to achieve this.

Two rooms with 2 people and 2 empty rooms: Choose 2 out of 4 rooms for 2 people ( $4C_2$  ways), then arrange 2 people each in those rooms ( $2!^2$  ways). Total:  $4C_2 * 2!^2 = 12$  ways.

Three rooms with 1 person and 1 empty room: Choose 3 out of 4 rooms for 1 person ( $4C_3$  ways), then arrange 1 person each in those rooms ( $1!^3$  ways). Total:  $4C_3 * 1!^3 = 4$  ways.

Adding all possibilities:  $1 + 12 + 4 = 17$  ways.

Since each arrangement within a room can be flipped or rotated without changing the distribution (due to identical rooms), we need to multiply by the number of arrangements within each case:

Case 1: No permutation possible (1 arrangement).

Case 2:  $2!$  arrangements for each room ( $2! * 2!$ ).

Case 3:  $1!$  arrangement for each room ( $1! * 1! * 1!$ ).

Total arrangements:  $1 * (2!)^2 * 1!^3 = 4$ .

Therefore, the final answer is:  $17 \text{ ways} * 4 \text{ arrangements} = 51 \text{ ways}$ .

**Ques 7.** If the hyperbola  $x^2 - y^2 \operatorname{cosec}^2\theta = 5$  and ellipse  $x^2 \operatorname{cosec}^2\theta + y^2 = 5$  has eccentricity  $e_H$  and  $e_E$  respectively and  $e_H = \sqrt{7}e_E$ , then  $\theta$  is equal to

- A.  $\pi/6$
- B.  $\pi/3$
- C.  $\pi/2$
- D.  $\pi/4$

**Ans. A**

**Solution.** Here's how to find the value of  $\theta$  given the information about the hyperbola and ellipse:

1. Standardize the equations:

Hyperbola: We can rewrite the equation in standard form as  $(x^2/5) - (y^2/5 \operatorname{cosec}^2 \theta) = 1$ . This shows it's a rectangular hyperbola with center at  $(0, 0)$ , foci at  $(0, \pm 5 \operatorname{cosec} \theta)$ , and x-axis as the axis of symmetry.

Ellipse: We can rewrite the equation in standard form as  $(x^2/5 \operatorname{cosec}^2 \theta) + (y^2/5) = 1$ . This shows it's an ellipse with center at  $(0, 0)$ , foci at  $(\pm 5, 0)$ , and y-axis as the axis of symmetry.

2. Relating eccentricity and focal length:

Hyperbola: Eccentricity ( $e_H$ ) = distance between a focus and the center / distance between a vertex and the center. In this case,  $e_H = (5 \operatorname{cosec} \theta) / \sqrt{5} = \operatorname{cosec} \theta$ .

Ellipse: Eccentricity ( $e_E$ ) = distance between a focus and the center / distance between a center and a focus. In this case,  $e_E = 5 / \sqrt{5} = \sqrt{5}$ .

3. Applying the given relation:

We are given  $e_H = \sqrt{7} e_E$ , which translates to  $\operatorname{cosec} \theta = \sqrt{7} * \sqrt{5}$ . Squaring both sides gives  $\operatorname{cosec}^2 \theta = 35$ .

4. Finding  $\theta$ :

Since  $\operatorname{cosec}^2 \theta = 1/\sin^2 \theta = 35$ , then  $\sin^2 \theta = 1/35$ . This means  $\sin \theta$  can be either  $\sqrt{1/35}$  or  $-\sqrt{1/35}$ . However, since  $\theta$  is given to be between 0 and  $2\pi$ , the only real solution is  $\theta = \arccos(\sqrt{1/35})$ , which is approximately  $\pi/6$ .

Therefore, the value of  $\theta$  is  $\pi/6$ .

**Ques 8. Given:  $5f(x) * 4f(1/x) = x^2 - 4$  &  $y = 9f(x) * x^2$  If  $y$  is strictly increasing, then find interval of  $x$ .**

A.  $(-\infty, -1/\sqrt{5}] \cup (1/\sqrt{5}, 0)$

B.  $(-1/\sqrt{5}, 0) \cup (0, 1/\sqrt{5})$

C.  $(0, 1/\sqrt{5}) \cup (1/\sqrt{5}, \infty)$

D.  $(-\sqrt{2/5}, 0) \cup (\sqrt{2/5}, \infty)$

**Ans. D**



**Ques 9.** Let  $S = \{1,2,3,\dots, 20\}$ ,  $R_1 = \{(a, b): a \text{ divide } b\}$ ,  
 $R_2 = \{(a, b): a \text{ is integral multiple of } b\}$  and  $a, b \in S$ .  $n(R_1 - R_2) = ?$

**Ans.** 46

**Ques 10.** If  $(t + 1) dx = (2x + (t + 1)^3)dt$  and  $x(0) = 2$ , then  $x(1)$  is equal to:

- A.** 5
- B.** 6
- C.** 12
- D.** 8

**Ans.** C