WARNING	Any malpractice or any attempt to commit any kind of malpractice in the Examination will DISQUALIFY THE CANDIDATE.

PAPER – II MATHEMATICS-2020

Version Code	**B1**	Question Booklet Serial Number :	9126960

Time: 150 Minutes	Number of Questions: 120	Maximum Marks: 480

Name of the Candidate	
Roll Number	
Signature of the Candidate	

INSTRUCTIONS TO CANDIDATES

1. Please ensure that the VERSION CODE shown at the top of this Question Booklet is same as that shown in the Admit Card issued to you. If you have received a Question Booklet with a different Version Code please get it replaced with a Question Booklet with the same Version Code as in the Admit Card from the Invigilator. **THIS IS VERY IMPORTANT.**

2. Please fill the items such as Name, Roll Number and Signature in the columns given above. Please also write Question Booklet Serial Number given at the top of this page against item 3 in the OMR Answer Sheet.

3. This Question Booklet contains 120 questions. For each question five answers are suggested and given against (A), (B), (C), (D) and (E) of which only one will be the 'Most Appropriate Answer.' Mark the bubble containing the letter corresponding to the 'Most Appropriate Answer' in the OMR Answer Sheet, by using either **Blue or Black Ball Point Pen only.**

4. **Negative Marking:** In order to discourage wild guessing the score will be subjected to penalization formula based on the number of right answers actually marked and the number of wrong answer marked. Each correct answer will be awarded FOUR marks. ONE mark will be deducted for each incorrect answer. More than one answer marked against a question will be deemed as incorrect answer and will be negatively marked.

5. Please read the instructions in the OMR Answer Sheet for marking the answers. Candidates are advised to strictly follow the instruction contained in the OMR Answer Sheet.

IMMEDIATELY AFTER OPENING THE QUESTION BOOKLET, THE CANDIDATE SHOULD VERIFY WHETHER THE QUESTION BOOKLET CONTAINS ALL THE 120 QUESTIONS IN SERIAL ORDER. IF NOT, REQUEST FOR REPLACEMENT.

DO NOT OPEN THE SEAL UNTIL THE INVIGILATOR ASKS YOU TO DO SO.

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1. The domain of the function f given by $f(x) = \sqrt{x-1}$ is

    (A) $(-\infty, \infty)$
    (B) $(1, \infty)$
    (C) $[1, \infty)$
    (D) $[0, \infty)$
    (E) $(0, \infty)$

2. Let $f(x) = -2x^2 + 1$ and $g(x) = 4x - 3$, then $(g \circ f)(-1)$ is equal to

    (A) 9
    (B) -9
    (C) 7
    (D) -7
    (E) -8

3. Let A and B be finite sets such that $n(A-B) = 18$, $n(A \cap B) = 25$ and $n(A \cup B) = 70$. Then $n(B)$ is equal to

    (A) 52
    (B) 25
    (C) 27
    (D) 43
    (E) 45

4. In a group of 100 persons, 80 people can speak Malayalam and 60 can speak English. Then the number of people who speak English only is

    (A) 40
    (B) 30
    (C) 20
    (D) 25
    (E) 35

Space for rough work

5. If $*$ is a binary operation defined by $a*b = \dfrac{a}{b} + \dfrac{b}{a} + \dfrac{1}{ab}$ for positive integers a and b, then $2*5$ is equal to

(A) 4   (B) 3   (C) 2   (D) 1   (E) 5

6. If $A = \{1,2,3,4,5\}$ and $B = \{2,4,6\}$, then $A - B =$

(A) $\{1,3,5,6\}$ (B) $\{0,1,3,5,6\}$ (C) $\{1,3,5\}$

(D) $\{1,2,3,4,5,6\}$ (E) $\{2,4\}$

7. Let $A = \{2,3,4,5\}$, $B = \{36,45,49,60,77,90\}$ and let R be the relation 'is factor of' from A to B. Then the range of R is the set

(A) $\{60\}$ (B) $\{36,45,60,90\}$ (C) $\{49,77\}$

(D) $\{49,60,77\}$ (E) $\{36,45,49,60,77,90\}$

8. The real part of $e^{(3+4i)x}$ is

(A) e^{3x} (B) $\cos 7x$ (C) $e^{3x}\cos 4x$

(D) $e^{3x}\sin 4x$ (E) 0

9. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\dfrac{1}{p^2+q^2}\left(\dfrac{x}{p} + \dfrac{y}{q}\right)$ is equal to

(A) -2 (B) -1 (C) 1   (D) 2   (E) 0

Space for rough work

10. Let $z = x + iy$ be a complex number such that $|z + i| = 2$. Then the locus of z is a circle whose centre and radius are

(A) $(0, -1)$; 2   (B) $(0, 2)$; 2   (C) $(1, -1)$; 2

(D) $(0, -1)$; $\sqrt{3}$ (E) $(0, 2)$; $\sqrt{3}$

11. If $2 + i$ is a root of $x^2 - 4x + c = 0$, where c is a real number, then the value of c is

(A) 2   (B) 3   (C) 4   (D) 5   (E) 0

12. Let z_1 and z_2 be complex numbers satisfying $|z_1| = |z_2| = 2$ and $|z_1 + z_2| = 3$.

Then $\left| \dfrac{1}{z_1} + \dfrac{1}{z_2} \right| =$

(A) $\dfrac{3}{2}$ (B) 2   (C) $\dfrac{3}{4}$ (D) $\dfrac{1}{2}$ (E) 4

13. The principal argument of the complex number $z = \dfrac{1 + \sin \pi - i \cos \pi}{1 + \sin \pi + i \cos \pi}$ is

(A) $\dfrac{\pi}{3}$ (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{5}$ (D) $\dfrac{\pi}{2}$ (E) $\dfrac{\pi}{4}$

Space for rough work

14. If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, then $|z_1 + z_2|$ is equal to

    (A) 50   (B) 10   (C) $5\sqrt{2}$ (D) 25   (E) $2\sqrt{5}$

15. $\dfrac{10i}{1+2i}$ is equal to

    (A) $-2i$ (B) $2i$ (C) $-4+2i$ (D) $4+2i$ (E) $6i$

16. The value of $\displaystyle\sum_{k=1}^{10}\left(3k^2 + 2k - 1\right)$ is

    (A) 1120   (B) 1200   (C) 1230   (D) 1265   (E) 1255

17. The numbers $a_1, a_2, a_3, \ldots$ form an arithmetic sequence with $a_1 \neq a_2$. The three numbers a_1, a_2 and a_6 form a geometric sequence in that order. Then the common difference of the arithmetic sequence is

    (A) a_1 (B) $2a_1$ (C) $3a_1$ (D) $4a_1$ (E) $5a_1$

18. In an arithmetic sequence, the sum of first and third terms is 6 and the sum of second and fourth terms is 20. Then the 11th term is

    (A) 67   (B) 62   (C) 57   (D) 73   (E) 66

Space for rough work

19. In an A.P., the first term is 3 and the last term is 17. The sum of all the terms in the sequence is 70. Then the number of terms in the arithmetic sequence is

(A) 7   (B) 5   (C) 9   (D) 6   (E) 8

20. Consider the set of all positive rational numbers that are less than 1 and that have denominators as 30 in their lowest terms. Their sum is equal to

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

21. If p, q and 23 is an increasing arithmetic sequence and p and q are prime numbers, then $p + q =$

(A) 22   (B) 24   (C) 26   (D) 28   (E) 30

22. The 5th and 7th terms of a G.P. are 12 and 48 respectively. Then the 9th term is

(A) 162   (B) 96   (C) 192   (D) 144   (E) 182

23. The number of positive integers less than 1000 having only odd digits is

(A) 155   (B) 177   (C) 55   (D) 205   (E) 85

Space for rough work

24. Five points are marked on a circle. The number of distinct polygons of three or more sides can be drawn using some (or all) of the five points as vertices is

(A) 10   (B) 12   (C) 14   (D) 16   (E) 18

25. The middle term in the expansion of $\left(1+\dfrac{1}{5}\right)^{20}$ is

(A) $\left(\dfrac{1}{5}\right)^{10}$ (B) $\left(\dfrac{1}{5}\right)^{11}$ (C) $^{20}C_{11}\left(\dfrac{1}{5}\right)^{11}$ (D) $^{20}C_{9}\left(\dfrac{1}{5}\right)^{9}$ (E) $^{20}C_{10}\left(\dfrac{1}{5}\right)^{10}$

26. $^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + {}^{11}C_3 + {}^{11}C_4 + {}^{11}C_5 =$

(A) 2^6 (B) 2^8 (C) 2^{10} (D) 2^{11} (E) 2^9

27. If $^nP_r = 840$ and $^nC_r = 35$, then the value of r is equal to

(A) 2   (B) 4   (C) 6   (D) 3   (E) 5

28. The sum of the coefficients in the expansion of $\left(1 + 2x - x^2\right)^{20}$ is

(A) 2^{20} (B) 2^{21} (C) 2^{19} (D) 2^{40} (E) 2

Space for rough work

29. The number of ways a committee of 4 people can be chosen from a panel of 10 people is

 (A) 315   (B) 240   (C) 210   (D) 720   (E) 120

30. If $A = \begin{pmatrix} 6 & 2 \\ 7 & -5 \end{pmatrix}$ and $A - B = \begin{pmatrix} -2 & 1 \\ 4 & -9 \end{pmatrix}$, then $B =$

 (A) $\begin{pmatrix} -8 & -1 \\ 3 & 4 \end{pmatrix}$ (B) $\begin{pmatrix} 8 & 1 \\ -3 & -4 \end{pmatrix}$ (C) $\begin{pmatrix} 4 & 3 \\ 11 & -14 \end{pmatrix}$ (D) $\begin{pmatrix} 8 & 1 \\ 3 & 4 \end{pmatrix}$ (E) $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$

31. The value of the determinant $\begin{vmatrix} bc & ca & ab \\ a^3 & b^3 & c^3 \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix}$ is

 (A) $a^5 - 1$ (B) $a^2bc + ab^2c + abc^2$ (C) $ab(a+b+c)$

 (D) $a^4b^4c^4(a+b+c)$ (E) 0

32. If the matrix $\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & k \\ -4 & 2 & 6 \end{bmatrix}$ is singular, then the value of k is equal to

 (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

Space for rough work

33. If $\begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 19 \\ \alpha & -27 \\ 0 & 14 \end{bmatrix}$, then the value of α is

(A) 5   (B) 4   (C) 7   (D) -14 (E) -5

34. If $A^{-1} = \dfrac{1}{11}\begin{pmatrix} -3 & 4 \\ 5 & -3 \end{pmatrix}$, then $A =$

(A) $\dfrac{-1}{11}\begin{pmatrix} 3 & 4 \\ 5 & 3 \end{pmatrix}$ (B) $\dfrac{1}{11}\begin{pmatrix} 3 & 4 \\ 5 & 3 \end{pmatrix}$ (C) $\begin{pmatrix} 3 & -4 \\ -5 & 3 \end{pmatrix}$

(D) $\begin{pmatrix} 3 & 4 \\ 5 & 3 \end{pmatrix}$ (E) $\begin{pmatrix} -3 & 4 \\ 5 & -3 \end{pmatrix}$

35. The system of equations

$$x + y + 2z = 4$$
$$3x + 3y + 6z = 17$$
$$5x - 3y + 2z = 27$$

has

(A) no solution   (B) finitely many solutions

(C) infinitely many solutions   (D) unique and trivial solution

(E) unique and non-trivial solution

Space for rough work

36. The smallest prime number satisfying the inequality $\dfrac{2n-3}{3} \geq \dfrac{n-1}{6}+1$ is

   (A) 2   (B) 3   (C) 5   (D) 7   (E) 11

37. The number of integers satisfying the inequality $|n^2 - 100| < 50$ is

   (A) 5   (B) 6   (C) 12   (D) 8.   (E) 10

38. The solution set of the rational inequality $\dfrac{x+9}{x-6} \leq 0$ is

   (A) $(-\infty, 9) \cup (6, \infty)$ (B) $(-\infty, 9] \cup (6, \infty)$ (C) $(-\infty, 9] \cup [6, \infty)$

   (D) $[-9, 6)$ (E) $(-9, 6]$

39. Which of the following sentences is/are statement(s)?

   (i) 10 is less than 5.

   (ii) All rational numbers are real numbers.

   (iii) Today is a sunny day.

   (A) (i), (ii) and (iii)   (B) (i) and (ii) only   (C) (i) and (iii) only

   (D) (ii) and (iii) only   (E) (i) only

Space for rough work

40. The value of θ with $0 \leq \theta \leq 90°$ and $\sin^2 \theta + 2\cos^2 \theta = \dfrac{7}{4}$ is equal to

(A) 15°   (B) 30°   (C) 45°   (D) 60°   (E) 75°

41. The value of $\sin^2 1° + \sin^2 2° + \sin^2 3° + \cdots + \sin^2 88° + \sin^2 89°$ is equal to

(A) $\dfrac{45}{2}$ (B) $\dfrac{49}{2}$ (C) $\dfrac{89}{2}$ (D) 45   (E) 89

42. The value of $\sin^4 \dfrac{\pi}{8} + \sin^4 \dfrac{3\pi}{8}$ is equal to

(A) $\dfrac{5}{8}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{3}{\sqrt{2}}$ (D) $\dfrac{3}{8}$ (E) $\dfrac{5}{4}$

43. The value of $\sin(45° + \theta) - \cos(45° - \theta)$ is equal to

(A) 1   (B) $\cos \theta$ (C) $\sin \theta$ (D) $2\cos \theta$ (E) 0

44. The values of x in $0 \leq x \leq \pi$ such that $\cos 2x = \cos x$ are

(A) 0 and $\dfrac{2\pi}{3}$ (B) $\dfrac{\pi}{3}$ and $\dfrac{2\pi}{3}$ (C) 0 and $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{4}$ and $\dfrac{\pi}{3}$ (E) 0 and $\dfrac{\pi}{2}$

Space for rough work

45. The value of $10 \tan(\cot^{-1} 3 + \cot^{-1} 7)$ is equal to

    (A) 3   (B) 5   (C) 7   (D) 9   (E) 10

46. If $\tan x + \tan y = \dfrac{5}{6}$ and $\cot x + \cot y = 5$, then $\tan(x+y)$ is

    (A) $\dfrac{6}{5}$ (B) $\dfrac{5}{6}$ (C) 5   (D) 6   (E) 1

47. $\dfrac{\sin 91° + \sin 1°}{\sin 91° - \sin 1°} =$

    (A) $\tan 46°$ (B) $\cot 46°$ (C) $\sin 46°$ (D) $\cos 46°$ (E) 1

48. The value of $\cos\left(\cos^{-1}\dfrac{1}{5} + 2\sin^{-1}\dfrac{1}{5}\right)$ is equal to

    (A) $\dfrac{4}{5}$ (B) $\dfrac{-4}{5}$ (C) $\dfrac{3}{5}$ (D) $\dfrac{-1}{5}$ (E) $\dfrac{1}{5}$

Space for rough work

49. The equation of the line passing through the point $(-3, 7)$ with slope zero is

(A) $x = 7$ (B) $y = 7$ (C) $x = -3$ (D) $y = -3$ (E) $x = 0$

50. The line $y = mx + 2$ intersects the parabola $y = ax^2 + 5x - 2$ at $(1, 5)$. Then the value of $a + m$ is equal to

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

51. If the points $P(7, 5)$, $Q(a, 2a)$ and $R(12, 30)$ are collinear, then the value of a is equal to

(A) 5   (B) 6   (C) 8   (D) 9   (E) 10

52. If the straight lines $4x + 6y = 5$ and $6x + ky = 3$ are parallel, then the value of k is equal to

(A) $\dfrac{-2}{3}$ (B) 8   (C) 9   (D) 10   (E) $\dfrac{3}{2}$

53. If $(a, 2)$ is the point of intersection of the straight lines $y = 2x - 4$ and $y = x + c$, then the value of c is equal to

(A) -1 (B) 3   (C) -2 (D) -3 (E) 1

Space for rough work

54. The maximum value of $z = 7x + 5y$ subject to $2x + y \leq 100$, $4x + 3y \leq 240$, $x \geq 0$, $y \geq 0$ is

(A) 350   (B) 380   (C) 400   (D) 410   (E) 420

55. A circle with centre at $(3, 6)$ passes through $(-1, 1)$. Its equation is

(A) $x^2 + y^2 - 6x - 12y + 3 = 0$ (B) $x^2 + y^2 + 6x - 10y + 3 = 0$

(C) $x^2 + y^2 - 3x - 6y + 1 = 0$ (D) $x^2 + y^2 + 5x + 9y + 5 = 0$

(E) $x^2 + y^2 - 6x - 12y + 4 = 0$

56. The centre and radius of the circle $x^2 + y^2 - 4x + 2y = 0$ are

(A) $(2, -1)$ and 5 (B) $(4, 2)$ and $\sqrt{20}$ (C) $(2, -1)$ and $\sqrt{5}$

(D) $(-2, 1)$ and 5 (E) $(-2, 1)$ and $\sqrt{5}$

57. The equation of the circle whose radius is $\sqrt{7}$ and concentric with the circle $x^2 + y^2 - 8x + 6y - 11 = 0$ is

(A) $x^2 + y^2 - 8x + 6y + 7 = 0$ (B) $x^2 + y^2 - 8x + 6y + 18 = 0$

(C) $x^2 + y^2 - 8x + 6y - 4 = 0$ (D) $x^2 + y^2 - 8x + 6y - 18 = 0$

(E) $x^2 + y^2 - 8x + 6y - 7 = 0$

Space for rough work

58. The vertex of the parabola $y = x^2 - 2x + 4$ is shifted p units to the right and then q units up. If the resulting point is (4, 5), then the values of p and q respectively are

(A) 2 and 3   (B) 3 and 5   (C) 5 and 2   (D) 3 and 2   (E) 1 and 2

59. The vertex of the parabola $y = (x - 2)(x - 8) + 7$ is

(A) (5, 2)   (B) (5, −2)   (C) (−5, −2)   (D) (−5, 2)   (E) (2, 8)

60. The major and minor axis of the ellipse $400x^2 + 100y^2 = 40000$ respectively are

(A) 100 and 20   (B) 20 and 10   (C) 40 and 20

(D) 400 and 100   (E) 16 and 8

61. The eccentricity of the ellipse $x^2 + \dfrac{y^2}{4} = 1$ is

(A) $\sqrt{3}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{\sqrt{3}}{4}$ (D) $\dfrac{\sqrt{3}}{2}$ (E) $\dfrac{1}{\sqrt{3}}$

62. The latus rectum of the hyperbola $3x^2 - 2y^2 = 6$ is

(A) $\dfrac{3}{\sqrt{2}}$ (B) $\dfrac{4}{\sqrt{3}}$ (C) $\dfrac{2}{\sqrt{3}}$ (D) 3   (E) $3\sqrt{2}$

Space for rough work

63. If $\vec{u} = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{v} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, then $\left|\vec{u} \times \vec{v}\right|^2 + \left|\vec{u} \cdot \vec{v}\right|^2 =$

    (A) 640   (B) 630   (C) 690   (D) 740   (E) 730

64. The direction cosines of the vector $\hat{i} - 5\hat{j} + 8\hat{k}$ are

    (A) $\left(\dfrac{1}{\sqrt{10}}, \dfrac{-5}{\sqrt{10}}, \dfrac{8}{\sqrt{10}}\right)$ (B) $\left(\dfrac{1}{3\sqrt{10}}, \dfrac{-5}{3\sqrt{10}}, \dfrac{8}{3\sqrt{10}}\right)$ (C) $\left(\dfrac{1}{3}, \dfrac{-5}{3}, \dfrac{8}{3}\right)$

    (D) $\left(\dfrac{1}{3\sqrt{10}}, \dfrac{-1}{3\sqrt{10}}, \dfrac{1}{3\sqrt{10}}\right)$ (E) $\left(\dfrac{1}{3\sqrt{10}}, \dfrac{5}{3\sqrt{10}}, \dfrac{8}{3\sqrt{10}}\right)$

65. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ and θ is the angle between them, then $\tan\theta =$

    (A) $\dfrac{\sqrt{38}}{4}$ (B) $\dfrac{\sqrt{26}}{4}$ (C) $\dfrac{\sqrt{26}}{5}$ (D) $\dfrac{\sqrt{26}}{6}$ (E) $\dfrac{\sqrt{38}}{6}$

66. The value of λ such that the vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + 2\lambda\hat{j}$ are perpendicular is

    (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

Space for rough work

67. The values of α so that $\left|\alpha\hat{i} + (\alpha+1)\hat{j} + 2\hat{k}\right| = 3$, are

(A) 2, -4   (B) 1, 2   (C) -1, 2   (D) -2, 4   (E) 1, -2

68. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, then the value of $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$ is equal to

(A) 8   (B) 7   (C) 9   (D) 11   (E) 13

69. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = \lambda\hat{j} + 3\hat{k}$. If the projection of $\vec{a}$ on $\vec{b}$ is equal to the projection of $\vec{b}$ on $\vec{a}$, then the values of λ are

(A) $\pm\sqrt{7}$ (B) $\pm\sqrt{3}$ (C) ± 5 (D) ± 3 (E) $\pm\sqrt{5}$

70. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then $|\vec{a} - \vec{b}|$ is equal to

(A) $\sqrt{5}$ (B) $\sqrt{7}$ (C) $\sqrt{6}$ (D) 5   (E) 6

71. Which one of the following points lies on the straight line $\dfrac{x-1}{2} = \dfrac{y+1}{4} = \dfrac{z-2}{-2}$?

(A) $(2, 6, -2)$ (B) $(4, 3, 1)$ (C) $(3, 4, -1)$ (D) $(3, 3, 0)$ (E) $(6, 2, -1)$

Space for rough work

72. A plane passes through the point $(0, 1, 1)$ and has normal vector $\hat{i} + \hat{j} + \hat{k}$. Its equation is

(A) $x + y + z = 1$ (B) $x + y + z = 2$ (C) $2x + 2y + 2z = 1$

(D) $y + z = 2$ (E) $y + z = 1$

73. The distance of the point $(4, 2, 3)$ from the plane $\vec{r} \cdot (6\hat{i} + 2\hat{j} - 9\hat{k}) = 46$ is

(A) $\dfrac{23}{5}$ (B) $\dfrac{46}{11}$ (C) $\dfrac{45}{11}$ (D) $\dfrac{11}{45}$ (E) $\dfrac{5}{23}$

74. The sum of the intercepts made by the plane $\vec{r} \cdot \left(3\hat{i} + \hat{j} + 2\hat{k}\right) = 18$ on the co-ordinate axes is

(A) 30   (B) 18   (C) 33   (D) 36   (E) 27

75. The point at which the line $\dfrac{x-2}{1} = \dfrac{y-4}{-5} = \dfrac{z+3}{4}$ intersects the xy-plane is

(A) $\left(\dfrac{11}{4}, \dfrac{1}{4}, 0\right)$ (B) $\left(\dfrac{5}{4}, \dfrac{1}{4}, 0\right)$ (C) $\left(\dfrac{11}{4}, \dfrac{3}{4}, 0\right)$ (D) $\left(\dfrac{7}{4}, \dfrac{1}{4}, 0\right)$ (E) $\left(\dfrac{11}{4}, \dfrac{7}{4}, 0\right)$

Space for rough work

76. The Cartesian equation of the line passing through the points $(1, -1, 2)$ and $(7, 0, 5)$ is

(A) $\dfrac{x-1}{4} = \dfrac{y+1}{1} = \dfrac{z-2}{2}$ 　　(B) $\dfrac{x-7}{1} = \dfrac{y}{-1} = \dfrac{z-5}{2}$ 　　(C) $\dfrac{x-1}{7} = \dfrac{y+1}{1} = \dfrac{z-2}{5}$

(D) $\dfrac{x-1}{6} = \dfrac{y+1}{1} = \dfrac{z-2}{3}$ 　　(E) $\dfrac{x-7}{6} = \dfrac{y}{-1} = \dfrac{z-5}{3}$

77. The angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$ is

(A) $\cos^{-1}\left(\dfrac{2}{\sqrt{42}}\right)$ 　　(B) $\cos^{-1}\left(\dfrac{5}{\sqrt{42}}\right)$ 　　(C) $\cos^{-1}\left(\dfrac{3}{\sqrt{42}}\right)$

(D) $\cos^{-1}\left(\dfrac{1}{\sqrt{42}}\right)$ 　　(E) $\cos^{-1}\left(\dfrac{4}{\sqrt{42}}\right)$

78. The equation of the plane passing through the intersection of the planes

$x + 2y - z = 3$ and $x + y - 3z = 5$ and passing through the point $(1, -1, 0)$ is

(A) $x + 7y + 6z + 6 = 0$ 　　(B) $x - 6y - 7z + 5 = 0$ 　　(C) $x + 7y + 6z + 5 = 0$

(D) $x + 6y - 7z - 5 = 0$ 　　(E) $x + 6y + 7z + 5 = 0$

Space for rough work

79. The average marks of 30 students in a class was 80. After two students left out of the class, the average marks of the remaining students was 82. Then the average marks of the two left out students is

(A) 62   (B) 72   (C) 70   (D) 52   (E) 60

80. Two dice are rolled. If each die has six faces which are numbered 2, 3, 5, 7, 11, 13, then the probability that sum of the numbers on the top faces being a prime number is

(A) $\dfrac{1}{6}$ (B) $\dfrac{5}{36}$ (C) $\dfrac{1}{18}$ (D) $\dfrac{1}{9}$ (E) $\dfrac{1}{12}$

81. Three different numbers are chosen at random from the set {1, 2, 3, 4, 5} and arranged in increasing order. The probability that the resulting sequence is an A.P. is

(A) $\dfrac{1}{2}$ (B) $\dfrac{3}{10}$ (C) $\dfrac{1}{5}$ (D) $\dfrac{1}{10}$ (E) $\dfrac{2}{5}$

82. In an examination, 20% of the students scored 70 marks, 40% scored 80 marks, 30% scored 90 marks and the rest scored 100 marks. Then the mean score of the students is

(A) 82   (B) 85   (C) 83   (D) 90   (E) 93

Space for rough work

83. If A and B are mutually exclusive events such that $p(A) = 0.5$ and $p(A \cup B) = 0.75$, then $P(B)$ is equal to

(A) 0.4 (B) 0.25 (C) 0.5 (D) 0.6 (E) 0.75

84. A jar contains 7 black balls, 6 yellow balls, 4 green balls and 3 red balls. All of them are of same size and weight. If a ball is drawn at random, then the probability of the ball being red is

(A) $\dfrac{1}{5}$ (B) $\dfrac{3}{20}$ (C) $\dfrac{1}{10}$ (D) $\dfrac{3}{10}$ (E) $\dfrac{1}{20}$

85. Let the probability distribution of a random variable X be given by

X	-1	0	1	2	3
$p(X)$	a	$2a$	$3a$	$4a$	$5a$

Then the expectation of X is

(A) $\dfrac{1}{5}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{4}{15}$ (E) $\dfrac{5}{3}$

Space for rough work

86. Let $f(x) = \begin{cases} 1-5x, & \text{if } x < -2 \\ x^2 - 2x, & \text{if } -2 \le x \le 1 \\ -1+2x, & \text{if } x > 1. \end{cases}$

Then the value of $f(-1)$ is equal to

(A) -3 (B) 3 (C) -1 (D) 1 (E) 0

87. The general solution of $\dfrac{dy}{dx} = \dfrac{2x-y}{x+2y}$ is given by

(A) $x^2 - y^2 - xy = C$ (B) $x^2 + y^2 + xy = C$

(C) $x^2 + 2y^2 + y + x = C$ (D) $2x^2 + y^2 + xy + y = C$

(E) $x^2 - y^2 - xy + x = C$

88. $\displaystyle\lim_{x \to 3} \dfrac{e^{x-3} - x + 1}{x^2 - \log(x-2)}$ is equal to

(A) $\dfrac{-1}{3}$ (B) $\dfrac{-2}{9}$ (C) $\dfrac{-1}{2}$ (D) $\dfrac{-1}{4}$ (E) $\dfrac{-1}{9}$

Space for rough work

89. $\lim\limits_{x \to 4} \dfrac{\sqrt{x^2+9}-5}{x-4}$ is equal to

(A) $\dfrac{2}{5}$ (B) $\dfrac{8}{25}$ (C) 0   (D) $\dfrac{8}{5}$ (E) $\dfrac{4}{5}$

90. Let $f(x) = \begin{cases} cx^2 + 2x, & \text{if } x < 2 \\ 2x + 4, & \text{if } x \geq 2 \end{cases}$

If the function f is continuous on $(-\infty, \infty)$, then the value of c is equal to

(A) 4   (B) 2   (C) 3   (D) 1   (E) 5

91. $\lim\limits_{x \to 0} \dfrac{x^{100} \sin 7x}{(\sin x)^{101}}$ is equal to

(A) 7   (B) $\dfrac{1}{7}$ (C) 14   (D) 1   (E) 0

92. Let $f(x) = \dfrac{5}{2}x^2 - e^x$. Then the value of c such that $f''(c) = 0$ is

(A) 1   (B) $\log 5$ (C) $5e$ (D) e^5 (E) 0

Space for rough work

93. If $y = (\cos x)^{2x}$, then $\dfrac{dy}{dx}$ is equal to

(A) $2(\cos x)^{2x}(\sin x - x\tan x)$

(B) $2(\cos x)^{2x}\left[\log(\cos x) + x\tan x\right]$

(C) $2(\sin x)^{2x}\left[\log(\cos x) - x\tan x\right]$

(D) $2(\sin x)^{2x}x\cot x$

(E) $2(\cos x)^{2x}\left[\log(\cos x) - x\tan x\right]$

94. If $x^3 + 2xy + \dfrac{1}{3}y^3 = \dfrac{11}{3}$, then $\dfrac{dy}{dx}$ at $(2,-1)$ is

(A) -2 (B) 2 (C) 5 (D) -5 (E) -10

95. Let $f(x) = \begin{cases} x^2, & \text{for } x \le 1 \\ 1, & \text{for } 1 < x \le 3 \\ 5 - 2x, & \text{for } x > 3 \end{cases}$

Then $f'(6)$ is equal to

(A) -7 (B) 3 (C) -2 (D) -3 (E) 2

96. Given $F(x) = \left(f(g(x))\right)^2$, $g(1) = 2$, $g'(1) = 3$, $f(2) = 4$ and $f'(2) = 5$. Then the value of $F'(1)$ is equal to

(A) 25 (B) 100 (C) 75 (D) 50 (E) 120

Space for rough work

97. If $y = 2 + \sqrt{u}$ and $u = x^3 + 1$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{x^2}{2\sqrt{x^3+1}}$ (B) $\dfrac{3x^2}{\sqrt{x^3+1}}$ (C) $\dfrac{3x^2}{2\sqrt{x^3+1}}$

(D) $3x^2\sqrt{x^3+1}$ (E) $x^2\sqrt{x^3+1}$

98. The equation of the tangent to $y = -2x^2 + 3$ at $x = 1$ is

(A) $y = -4x$ (B) $y = -4x + 5$ (C) $y = 4x$

(D) $y = 4x + 5$ (E) $y = -4x + 3$

99. The function f given by $f(x) = x^3 e^x$ is increasing on the interval

(A) $(0, \infty)$ (B) $(3, \infty)$ (C) $(-3, \infty)$ (D) $(-3, 3)$ (E) $(-\infty, -3)$

100. Let $f(x) = \sqrt{x}$, $4 \le x \le 16$. If the point $c \in (4, 16)$ is such that the tangent line to the graph of f at $x = c$ is parallel to the chord joining $(16, 4)$ and $(4, 2)$, then the value of c is

(A) 7   (B) 9   (C) 10   (D) 11   (E) 14

101. The function f given by $f(x) = (x^2 - 3)e^x$ is decreasing on the interval

(A) $(-3, \infty)$ (B) $(1, \infty)$ (C) $(-\infty, 1)$ (D) $(-\infty, -3)$ (E) $(-3, 1)$

Space for rough work

102. The equation of normal to the curve $y = \dfrac{2}{x^2}$ at the point on the curve where $x = 1$, is

(A) $4y - x - 7 = 0$

(B) $y - 4x + 2 = 0$

(C) $4y + x - 9 = 0$

(D) $y - x - 1 = 0$

(E) $4y + x + 7 = 0$

103. The local minimum value of the function f given by $f(x) = x^2 - x$, $x \in \mathbb{R}$, is

(A) $\dfrac{1}{2}$

(B) $\dfrac{1}{4}$

(C) $\dfrac{-1}{4}$

(D) $\dfrac{3}{4}$

(E) $\dfrac{-1}{2}$

104. $\displaystyle\int 3x^2 (x^3 + 1)^{10}\, dx =$

(A) $\dfrac{(x^3 + 1)^{11}}{11} + C$

(B) $\dfrac{(x^3 + 1)^{9}}{9} + C$

(C) $\dfrac{(x^3 + 1)^{11}}{33} + C$

(D) $\dfrac{(x^3 + 1)^{11}}{11} + x^3 + C$

(E) $\dfrac{(x^3 + 1)^{11}}{10} + C$

105. $\displaystyle\int \dfrac{2x + \sin 2x}{1 + \cos 2x}\, dx =$

(A) $x^2 \sec x + C$

(B) $x + \tan x + C$

(C) $x^2 \tan x + C$

(D) $x \sec x + C$

(E) $x \tan x + C$

Space for rough work

106. $\int \dfrac{1}{x^2 - 25}\, dx =$

(A) $\log \left| \dfrac{x-5}{x+5} \right| + C$

(B) $\log \left| \dfrac{x+5}{x-5} \right| + C$

(C) $\dfrac{1}{5} \log \left| \dfrac{x-5}{x+5} \right| + C$

(D) $\dfrac{1}{10} \log \left| \dfrac{x-5}{x+5} \right| + C$

(E) $\dfrac{1}{5} \log \left| \dfrac{x+5}{x-5} \right| + C$

107. $\int \dfrac{1}{x(\log x)}\, dx =$

(A) $\log \left| \log x \right| + C$

(B) $\dfrac{\left(\log |x| \right)^2}{2} + C$

(C) $\log |x| + C$

(D) $\dfrac{1}{\log |x|} + C$

(E) $\dfrac{1}{\left(\log |x| \right)^2} + C$

108. $\int e^x \sec x (1 + \tan x)\, dx =$

(A) $e^x \tan x + C$

(B) $e^x + \sec x + C$

(C) $e^{-x} \sec x + C$

(D) $e^x + \tan x + C$

(E) $e^x \sec x + C$

109. $\int \dfrac{1}{x + \sqrt{x}}\, dx =$

(A) $\log \left| 1 + \sqrt{x} \right| + C$

(B) $2 \log \left| 1 - \sqrt{x} \right| + C$

(C) $\log \left| 1 - \sqrt{x} \right| + C$

(D) $2 \log \left| 1 + \sqrt{x} \right| + C$

(E) $2 \log \left| x + \sqrt{x} \right| + C$

Space for rough work

110. $\int \sec^2(5x-1)\, dx =$

(A) $\dfrac{1}{5}\tan(5x-1)+C$ (B) $5\tan(5x-1)+C$ (C) $\tan(5x-1)+C$

(D) $\cot(5x-1)+C$ (E) $\dfrac{1}{5}\cot(5x-1)+C$

111. $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{1}{1+\cot^4 x}\, dx =$

(A) $\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{4}$ (C) π (D) $\dfrac{\pi}{8}$ (E) 2π

112. The value of $\displaystyle\int_{-10}^{10} (0.0002x^3 - 0.3x + 20)\, dx$ is equal to

(A) 423 (B) 400 (C) 378 (D) 410 (E) 390

113. The area enclosed by the curve $x = 3\cos\theta$, $y = 2\sin\theta$, $0 \le \theta \le \pi$, is (in square units)

(A) 9π (B) 6π (C) 4π (D) 3π (E) 2π

Space for rough work

114. The area of the region bounded by $y = |x|$, $y = 0$, $x = 3$ and $x = -3$ is (in square units)

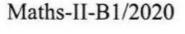(A) 3   (B) 6   (C) 7   (D) 9   (E) 10

115. The value of $\int_{e}^{e^2} \frac{1}{x}\, dx$ is equal to

(A) e (B) 1   (C) e^2 (D) $e^2 - e$ (E) 0

116. $\int_{-3}^{3} |x + 2|\, dx =$

(A) 17   (B) 9   (C) 14   (D) 13   (E) 12

117. The order and degree of the differential equation $\dfrac{d^2 y}{dx^2} + \sqrt{x^2 + \left(\dfrac{dy}{dx}\right)^{3/2}} = 0$

are respectively

(A) 2, 4   (B) 2, 3   (C) 2, 2   (D) 3, 4   (E) 4, 3

Space for rough work

118. The general solution of the differential equation $xy' + y = x^2$, $x > 0$ is

   (A) $y = \dfrac{x^2}{2} + Cx$ (B) $y = \dfrac{x^3}{3} + C$ (C) $y = \dfrac{x^2}{3} + C$

   (D) $y = \dfrac{x^3}{3} + \dfrac{C}{x}$ (E) $y = \dfrac{x^2}{3} + \dfrac{C}{x}$

119. The integrating factor of the differential equation $3xy' - y = 1 + \log x$, $x > 0$ is

   (A) $\log x$ (B) $\dfrac{1}{x}$ (C) $x^{-1/3}$ (D) $\dfrac{1}{x^3}$ (E) $x^{1/3}$

120. Elimination of arbitrary constants A and B from $y = \dfrac{A}{x} + B$, $x > 0$ leads to the differential equation

   (A) $x \dfrac{d^2 y}{dx^2} + 2 \dfrac{dy}{dx} = 0$ (B) $x^2 \dfrac{d^2 y}{dx^2} + 2 \dfrac{dy}{dx} = 0$ (C) $x^2 \dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} = 0$

   (D) $x \dfrac{d^2 y}{dx^2} - 2 \dfrac{dy}{dx} = 0$ (E) $x \dfrac{d^2 y}{dx^2} - \dfrac{dy}{dx} = 0$

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