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GMAT

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WORD PROBLEMS

BUILD UNDERSTANDING OF STATISTICS, RATES, WORK, & MORE

LEARN TO CLASSIFY AND SOLVE WORD PROBLEMS EFFECTIVELY

REVIEW PRACTICE PROBLEMS WITH DETAILED EXPLANATIONS

6TH EDITION

Angela Guido, Manhattan Prep Instructor

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Word Problems

GMAT Strategy Guide

This comprehensive guide analyzes the GMAT's complex word problems and provides structured frameworks for attacking each question type. Master the art of translating challenging word problems into organized data.

guide **3**

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December 2nd, 2014

Dear Student,

Thank you for picking up a copy of *Word Problems*. I hope this book gives you just the guidance you need to get the most out of your GMAT studies.

A great number of people were involved in the creation of the book you are holding. First and foremost is Zeke Vanderhoek, the founder of Manhattan Prep. Zeke was a lone tutor in New York City when he started the company in 2000. Now, well over a decade later, the company contributes to the successes of thousands of students around the globe every year.

Our Manhattan Prep Strategy Guides are based on the continuing experiences of our instructors and students. The overall vision of the 6th Edition GMAT guides was developed by Stacey Koprince, Whitney Garner, and Dave Mahler over the course of many months; Stacey and Dave then led the execution of that vision as the primary author and editor, respectively, of this book. Numerous other instructors made contributions large and small, but I'd like to send particular thanks to Josh Braslow, Kim Cabot, Dmitry Farber, Ron Purewal, Emily Meredith Sledge, and Ryan Starr. Dan McNaney and Cathy Huang provided design and layout expertise as Dan managed book production, while Liz Krisher made sure that all the moving pieces, both inside and outside of our company, came together at just the right time. Finally, we are indebted to all of the Manhattan Prep students who have given us feedback over the years. This book wouldn't be half of what it is without your voice.

At Manhattan Prep, we aspire to provide the best instructors and resources possible, and we hope that you will find our commitment manifest in this book. We strive to keep our books free of errors, but if you think we've goofed, please post to manhattanprep.com/GMAT/errata. If you have any questions or comments in general, please email our Student Services team at gmat@manhattanprep.com. Or give us a shout at 212-721-7400 (or 800-576-4628 in the US or Canada). I look forward to hearing from you.

Thanks again, and best of luck preparing for the GMAT!

Sincerely,

Chris Ryan
Vice President of Academics
Manhattan Prep

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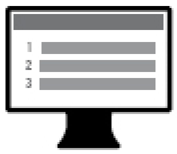

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Official Guide Problem Sets

As you work through this strategy guide, it is a very good idea to test your skills using official problems that appeared on the real GMAT in the past. To help you with this step of your studies, we have classified all of the problems from the three main *Official Guide* books and devised some problem sets to accompany this book.

These problem sets live in your Manhattan GMAT Student Center so that they can be updated whenever the test makers update their books. When you log into your Student Center, click on the link for the *Official Guide Problem Sets*, found on your home page. Download them today!

The problem sets consist of four broad groups of questions:

1. A mid-term quiz: Take this quiz after completing **Chapter 5** of this guide.
2. A final quiz: Take this quiz after completing this entire guide.
3. A full practice set of questions: If you are taking one of our classes, this is the home-work given on your syllabus, so just follow the syllabus assignments. If you are not taking one of our classes, you can do this practice set whenever you feel that you have a very solid understanding of the material taught in this guide.
4. A full reference list of all *Official Guide* problems that test the topics covered in this strategy guide: Use these problems to test yourself on specific topics or to create larger sets of mixed questions.

As you begin studying, try one problem at a time and review it thoroughly before moving on. In the middle of your studies, attempt some mixed sets of problems from a small pool of topics (the two quizzes we've devised for you are good examples of how to do this). Later in your studies, mix topics from multiple guides and include some questions that you've chosen randomly out of the *Official Guide*. This way, you'll learn to be prepared for anything!

Study Tips:

1. DO time yourself when answering questions.
2. DO cut yourself off and make a guess if a question is taking too long. You can try it again later without a time limit, but first practice the behavior you want to exhibit on the real test: let go and move on.
3. DON'T answer all of the *Official Guide* questions by topic or chapter at once. The real test will toss topics at you in random order, and half of the battle is figuring out what each new question is testing. Set yourself up to learn this when doing practice sets.

Chapter 1
of
Word Problems

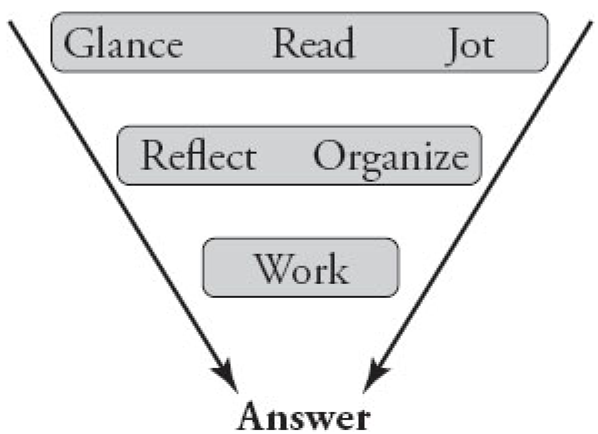
Translations

In This Chapter…

Chapter 1

Translations

Story problems are prevalent on the GMAT and can come in any form: Word Problems, Fractions, Percents, Algebra, and so on. Tackle story problems using your standard three-step approach to solving:



Step 1: Glance, Read, Jot: What's the story?

Glance at the problem: is it Problem Solving or Data Sufficiency? Do the answers or statements give you any quick clues? (Example: variables in the answers might lead you to choose smart numbers.)

Often, on story problems, it's best to finish reading the entire problem before you begin to write.

Step 2: Reflect, Organize: Translate

Your task is to turn the story into math. You can use either the Algebraic method or one of the special strategy methods (work backwards, choose smart numbers, or draw it out, all of which are discussed in this book).

Step 3: Work: Solve

Now that you have the story laid out, you can go ahead and solve.

Try out the three-step process on this problem:

> A candy company sells premium chocolate candies at $5 per pound and regular chocolate candies at $4 per pound in increments of whole pounds only. If Barrett buys a 7-pound box of chocolate candies that costs him $31, how many pounds of premium chocolate candies are in the box?
>
> (A) 1
> (B) 2
> (C) 3
> (D) 4
> (E) 5

Try the algebraic approach first.

Step 1: Glance, Read, Jot

The problem contains a bunch of numbers, but hold off writing them down. Get oriented on the story first so that you can organize the information in a way that makes sense.

Step 2: Reflect, Organize

The problem asks for the number of pounds of premium chocolate candies. Since this is an unknown, assign a variable. Choose variables that tell you what they mean. The variables x and y, while classic choices, do not indicate whether x is premium and y is regular or vice versa. The following labels are more useful:

p = pounds of premium chocolate candies
r = pounds of regular chocolate candies

Note that, while the problem asks only for the premium figure, you also want to assign a variable for the regular figure, since this is another unknown in the problem. You would also want to write down something similar to this:

p = _____?

What else can you write down? Barrett bought a 7-pound box of the candies. Both premium and regular make up that 7 pounds, so you can write an equation:

$p + r = 7$

The other given concerns the total cost of the box, \$31. The total cost is equal to the cost of the premium chocolates plus the cost of the regular chocolates.

This relationship is slightly more complicated than it appears, because it involves a relationship the GMAT expects you to know: *Total Cost* = *Unit Price* × *Quantity*. Just as you want to minimize the number of variables you create, you want to minimize the number of equations you have to create. You can express all three terms in the above equation using information you already have:

Total Cost of Box = \$31

Cost of Premiums = (5 \$/pound) × (p pounds) = $5p$

Cost of Regulars = (4 \$/pound) × (r pounds) = $4r$

Note that you can translate "dollars per pound" to "\$/pound." In general, the word "per" is translated as "divided by."

Put that all together, and you have your second equation:

$31 = 5p + 4r$

Step 3: Work

Here's your current scrap paper; how can you solve?

p = # prem choc

r = # reg choc

$$p + r = 7$$
$$31 = 5p + 4r$$
$$p = \_\_\_\_\_?$$

When you have two equations with two variables, the most efficient way to find the desired value is to eliminate the unwanted variable in order to solve for the desired variable.

You’re looking for p. To eliminate r, first isolate it in one of the equations. It is easier to isolate r in the first equation:

$$r + p = 7 \quad \rightarrow \quad r = 7 - p$$

Now replace r with $(7 - p)$ in the second equation and solve for p:

$$31 = 5p + 4(7 - p)$$
$$31 = 5p + 28 - 4p$$
$$3 = p$$

The correct answer is **(C)**.

The Work Backwards Method

What if you didn’t want to write a bunch of formulas? How else could you solve?

Step 1: Glance, Read, Jot

Glance: you have a story problem. Read the whole thing—including the answer choices—before you start to solve.

Step 2: Reflect, Organize

Notice anything? The answer choices are very “nice” numbers! You don’t need to do algebra; instead, you can work backwards from the answers.

Step 3: Work

Start laying out the information you were given and try answer choice (B) first:

#s of p	Cost of p (\$5/#)	Cost of r ($p + r =$ \$31)	Is r an integer? (cost/4)
(A)			
(B) 2	(2)(5) = \$10	\$31 − \$10 = \$21	\$21/4 = No

That didn't work, so try (D):

(C)			
(D) 4	(4)(5) = \$20	\$31 − \$20 = 11	\$11/4 = No
(E)			

Answer (D) also doesn't work. Are you noticing any patterns?

In order for *R* to be an integer, what has to happen? In this case, \$31 minus the cost of *P* must be a multiple of 4. Run through the beginning of the calculation, looking for something that will produce a multiple of 4 at the right stage:

#s of p	Cost of p (\$5/#)	Cost of r ($p + r =$ \$31)	Is r an integer? (cost/4)
(A) 1	(1)(5) = \$5	Mult of 4? No.	
(B) 2	(2)(5) = \$10	\$31 − \$10 = \$21	\$21/4 = No
(C) 3	(3)(5) = \$15	\$31 − \$15 = \$16	\$16/4 = 4 Yes!
(D) 4	(4)(5) = \$20	\$31 − \$20 = \$11	\$11/4 = No
(E) 5			

The correct answer is **(C)**.

The GMAT has many ways of making various stages of a Word Problem more difficult, which is why it is so important to have a good process. Train yourself to use these three steps to help assess what you have, figure out an approach, and only then perform the necessary work to get to the solution.

Pay Attention to Units

Unlike problems that test pure algebra, Word Problems have a context. The values, both unknown and known, have a meaning. Practically, this means that every value in a Word Problem has units.

Every equation that correctly represents a relationship has units that make sense. Most relationships are either additive or multiplicative.

Additive Relationships

In the chocolates problem, there were two additive relationships:

$$r + p = 7$$
$$31 = 5p + 4r$$

For each equation, the units of every term are the same; for example, pounds plus pounds equals pounds. Adding terms with the same units does not change the units. Here are the same equations with the units added in parentheses:

$$\underset{\text{(pounds)}}{r} \quad + \quad \underset{\text{(pounds)}}{p} \quad = \quad \underset{\text{(pounds)}}{7}$$

$$\underset{\text{(dollars)}}{31} \quad = \quad \underset{\text{(dollars)}}{5p} \quad + \quad \underset{\text{(dollars)}}{4r}$$

You may be wondering how you can know the units for $5p$ and $4r$ are dollars. That brings us to the second type of relationship.

Rate Relationships

Remember the relationship you used to find those two terms?

Total Cost = *Unit Price* × *Quantity*

Look at them again with units in parentheses:

$$5\left(\frac{\text{dollars}}{\text{pound}}\right) \times p \text{ (pounds)} = 5p \text{ (dollars)}$$

$$4\left(\frac{\text{dollars}}{\text{pound}}\right) \times r \text{ (pounds)} = 4r \text{ (dollars)}$$

For multiplicative relationships, treat units like numerators and denominators. Units that are multiplied together *do* change.

In the equations above, pounds in the denominator of the first term cancel out pounds in the numerator of the second term, leaving dollars as the final units:

$$5\left(\frac{\text{dollars}}{\cancel{\text{pounds}}}\right) \times p\ (\cancel{\text{pounds}}) = 5p\ (\text{dollars})$$

Look at the formula for area to see what happens to the same units when they appear on the same side of the fraction:

l (feet) × w (feet) = lw (feet^2)

Keep track of the units to stay on track in the calculation.

Common Relationships

The GMAT will assume that you have mastered the following relationships. Notice that for all of these relationships, the units follow the rules laid out in the previous section:

- Total Cost ($) = Unit Price ($/unit) × Quantity Purchased (units)
- Profit ($) = Revenue ($) − Cost ($)
- Total Earnings ($) = Wage Rate ($/hour) × Hours Worked (hours)
- Miles = Miles per Hour × Hours
- Miles = Miles per Gallon × Gallons

Units Conversion

When values with units are multiplied or divided, the units change. This property is the basis of using **conversion factors** to convert units. A conversion factor is a fraction whose numerator and denominator have different units but the same value.

For instance, how many seconds are in 7 minutes? If you said 420, you are correct. You were able to make this calculation because you know there are 60 seconds in a minute. In this case, $\frac{60 \text{ seconds}}{1 \text{ minute}}$ is a conversion factor. Because the numerator and denominator are the same, multiplying by a conversion factor is just a sneaky way of multiplying by 1. The multiplication looks like this:

$$7 \text{ minutes} \times \frac{60 \text{ seconds}}{1 \text{ minute}} = 420 \text{ seconds}$$

Because you are multiplying, you can cancel minutes, leaving you with your desired units (seconds).

Questions will occasionally center around your ability to convert units. Try the following example:

> A certain medicine requires 4 doses per day. If each dose is 150 milligrams, how many milligrams of medicine will a person have taken after the end of the third day, if the medicine is used as directed?

For any question that involves unit conversion, there will have to be some concrete value given. In this case, you were told that the time period is three days, that there are 4 doses/day, and that 1 dose equals 150 milligrams.

Now you need to know what the question wants. It's asking for the number of milligrams of medicine that will be taken in that time. How can you combine all of those givens so that the only units that remain are milligrams?

Combine the calculations into one big expression:

$$3 \text{ days} \times \frac{4 \text{ doses}}{1 \text{ day}} \times \frac{150 \text{ milligrams}}{1 \text{ dose}} = 1{,}800 \text{ milligrams}$$

During the GMAT, you may not actually write out the units for each piece of multiplication. If you don't, however, make sure that your conversion factors are set up properly to cancel out the units you don’t want and to leave the units you do want.

Finally, keep an eye out for more of these relationships! For instance, rate and work problems are also built on a common relationship that you’re expected to know for the test; you’ll learn about that relationship in chapter 3.

Problem Set

Solve the following problems using the three-step method outlined in this chapter.

1. United Telephone charges a base rate of $10.00 for service, plus an additional charge of $0.25 per minute. Atlantic Call charges a base rate of $12.00 for service, plus an additional charge of $0.20 per minute. For what number of minutes would the bills for each telephone company be the same?

2. Caleb spends $72.50 on 50 hamburgers for the marching band. If single burgers cost $1.00 each and double burgers cost $1.50 each, how many double burgers did he buy?

3. On the planet Flarp, 3 floops equal 5 fleeps, 4 fleeps equal 7 flaaps, and 2 flaaps equal 3 fliips. How many floops are equal to 35 fliips?

4. Carina has 100 ounces of coffee divided into 5- and 10-ounce packages. If she has 2 more 5-ounce packages than 10-ounce packages, how many 10-ounce packages does she have?

5. A circus earned $150,000 in ticket revenue by selling 1,800 V.I.P. and Standard tickets. They sold 25% more Standard tickets than V.I.P. tickets. If the revenue from Standard tickets represents one-third of the total ticket revenue, what is the price of a V.I.P. ticket?

Solutions

1. **40 minutes:**

Let x = the number of minutes.

A call made by United Telephone costs \$10.00 plus \$0.25 per minute: $10 + 0.25x$.

A call made by Atlantic Call costs \$12.00 plus \$0.20 per minute: $12 + 0.20x$.

Set the expressions equal to each other:

$$10 + 0.25x = 12 + 0.20x$$
$$0.05x = 2$$
$$x = 40$$

2. **45 double burgers:**

Let s = the number of single burgers purchased.

Let d = the number of double burgers purchased.

Caleb bought 50 burgers:	Caleb spent \$72.50 in all:
$s + d = 50$	$s + 1.5d = 72.50$

Combine the two equations by subtracting equation 1 from equation 2.

$$\begin{array}{r} s + 1.5d = 72.50 \\ -\,(s + \quad d = 50) \\ \hline 0.5d = 22.5 \\ d = 45 \end{array}$$

3. **8 floops:** All of the objects in this question are completely made up, so you can't use intuition to help you convert units. Instead, you need to use

the conversion factors given in the question. Start with 35 fliips, and keep converting until you end up with floops as the units:

$$35\ \cancel{\text{fliips}} \times \frac{2\ \cancel{\text{flaaps}}}{3\ \cancel{\text{fliips}}} \times \frac{4\ \cancel{\text{fleeps}}}{7\ \cancel{\text{flaaps}}} \times \frac{3\ \text{floops}}{5\ \cancel{\text{fleeps}}} = 8\ \text{floops}$$

4. **6:**

Let a = the number of 5-ounce packages.

Let b = the number of 10-ounce packages.

Carina has 100 ounces of coffee:	She has two more 5-ounce packages than 10-ounce packages:
$5a + 10b = 100$	$a = b + 2$

Combine the equations by substituting the value of a from equation 2 into equation 1:

$$\begin{aligned} 5(b+2)+10b &= 100 \\ 5b+10+10b &= 100 \\ 15b+10 &= 100 \\ 15b &= 90 \\ b &= 6 \end{aligned}$$

5. **\$125:** To answer this question correctly, you need to make sure to differentiate between the price of tickets and the *quantity* of tickets sold.

Let V = # of V.I.P. tickets sold.

Let S = # of Standard tickets sold.

The question tells you that the circus sold a total of 1,800 tickets, and that the circus sold 25% more Standard tickets than V.I.P. tickets. You can create two equations:

$V + S = 1{,}800$ $\qquad\qquad$ $1.25V = S$

You can use these equations to figure out how many of each type of ticket was sold:

$$
\begin{aligned}
V + S &= 1{,}800 \\
V + (1.25V) &= 1{,}800 \\
2.25V &= 1{,}800 \\
V &= 800
\end{aligned}
$$

Thus, 800 V.I.P. tickets were sold. Next, subtract 800 from the total number of tickets (1,800 − 800) to find that 1,000 Standard tickets were sold.

Now you need to find the cost per V.I.P. ticket. The question states that the circus earned $150,000 in ticket revenue, and that Standard tickets represented one-third of the total revenue. Therefore, Standard tickets accounted for 1/3 × $150,000 = $50,000. V.I.P. tickets then accounted for $150,000 − $50,000 = $100,000 in revenue.

Now, you know that the circus sold 800 V.I.P. tickets for a total of $100,000. Thus, $100,000/800 = $125 per V.I.P. ticket.

Chapter 2
of
Word Problems

Strategy: Work Backwards

In This Chapter…

Chapter 2

Strategy: Work Backwards

Work backwards literally means to start with the answers and do the math in the reverse order described in the problem. You're essentially plugging the answers into the problem to see which one makes the math work.

Try this problem, using any solution method you like:

> Four brothers are splitting a sum of money between them. The first brother receives 50% of the total, the second receives 25% of the total, the third receives 20% of the total, and the fourth receives the remaining $4. How many dollars are the four brothers splitting?
>
> (A) $60
> (B) $70
> (C) $80
> (D) $90
> (E) $100

How to Work Backwards

Here's how to Work Backwards to solve the above problem.

Step 1: Start with answer (B) or answer (D). (In many cases, you'll do less work if you start with one of these two; if you're curious as to why, you'll learn later in the chapter!)

Plug that answer into the problem to see whether it works. Use a chart to track your work because you may need to try more than one answer. (Remember that the GMAT gives you graph paper, so you won't have to draw the gridlines.)

The first column is labeled Total because the question stem indicates that the answers represent possible Total amounts of money. Let's say that you start with answer (B). Assume it's correct and start calculating according to the problem:

Total	B#1 (50%)	B#2 (25%)	B#3 (20%)	B#4 (4)	Sum	Match?
(A) 60						
(B) 70	35	17.5	14	4	70.5	No
(C) 80						
(D) 90						
(E) 100						

Assuming that $70 is the total amount of money, the first brother would get 50%, or $35. The second brother, at 25%, would get $17.5, and the third brother, at 20%, would get $14. The fourth brother is given a set amount: $4. Finally, add up the individual amounts. Does it match your starting point of $70?

Close! But not good enough. Answer (B) is incorrect.

Which answer should you try next?

Step 2: Narrow your answers. If the first answer you try works, pick it. If not, cross it off and figure out what to try next.

If you can tell that the starting number has to be smaller than (B), then the answer must be (A) because, in this problem, (A) is the only smaller number.

If you can tell that the starting number has to be larger, then cross off both (A) and (B) and try answer (D) next.

If you can't tell, try answer (D) next. Let's say that you can't tell.

Total	B#1 (50%)	B#2 (25%)	B#3 (20%)	B#4 (4)	Sum	Match?
(A) 60						
(B) 70	30	15	12	4	70.5	No
(C) 80						
(D) 90	45	22.5	18	4	89.5	No
(E) 100						

Answer (D) is also not a match, so it's incorrect. What should you try next?

Step 3: Pick! Actually, you don't have to try another answer. You can pick the correct answer right now! Try to figure out how before you keep reading.

Compare the given answers to your calculated sums. In answer (B), the calculated sum, 70.5, was a bit *higher* than your starting point of 70. In answer (D), by contrast, the calculated sum, 89.5, was *lower* than your starting point of 90. The correct answer, then, should be in between—answer (C).

For these problems, the answers will always be in increasing or decreasing order, so if you try (B) and (D) and neither work, then you can almost always figure out which of the remaining answers must be right without actually checking them. (You'll see more examples of this later in the chapter.)

Here's how the math works for correct answer (C):

Total	B#1 (50%)	B#2 (25%)	B#3 (20%)	B#4 (4)	Sum	Match?
(C) 80	40	20	16	4	80	Yes!

Try another:

> Machine X produces cartons at a uniform rate of 90 every 3 minutes, and Machine Y produces cartons at a uniform rate of 100 every 2 minutes. Working simultaneously, how many minutes would it take for the two machines to produce a total of 560 cartons?

(A) 7
(B) 6
(C) 5
(D) 4
(E) 3

Step 1: Start with answer (B) or answer (D). Set up your chart and solve:

Minutes	X (90c in 3 min)	Y (100c in 2 min)	Total	= 560?
(B) 6	180	300	480	No

Step 2: Narrow your answers. Answer (B) is incorrect. The total is too low, so you need a higher number; therefore, only answer (A) can work.

Step 3: Pick! If you're confident in your reasoning, pick (A). If not, try answer (A) to confirm.

If you didn't notice that you needed a higher number, you'd try answer (D) next:

Minutes	X (90c in 3 min)	Y (100c in 2 min)	Total	= 560?
(B) 6	180	300	480	No
(D) 4	90 + 30 = 120	200	320	No

The answers are moving in the wrong direction—you're getting even further away from the desired 560! The answer definitely has to be greater than 6. Only answer choice (A) is greater.

When to Work Backwards

The two examples shown above possess a couple of characteristics in common that make working backwards a viable method.

First, the answer choices are numerical and they are what are called "nice" numbers. In the second problem, the answers were small integers. In the first one, the numbers were larger, but they were still integers and they all ended in zero. "Nice" numbers make working backwards easier.

Second, the question stems ask for a discrete number—in the first case, the total, and, in the second case, the number of minutes. A problem that asks for something that you could label with a single variable (for example, T or m) is more likely to work well using this technique than a problem that asks for something more complicated, such as the difference between two numbers.

In sum, look for "nice" numbers and a question that asks for a single variable. When these characteristics exist, it may be easier to work backwards than forwards!

How to Get Better at Working Backwards

First, practice the problems at the end of this chapter. Try each problem two times: once working backwards and once using the "textbook" method. (Time yourself separately for each attempt.)

When you're done, ask yourself which way you prefer to solve *this* problem and why. The key to mastering strategies such as working backwards, and others, is developing an instinct for when to use them. On the real test, you won't have time to try both methods; you'll have to make a decision and go with it.

Learn *how* to make that decision while studying; then, the next time a new problem pops up in front of you that could be solved by working backwards, you'll be able to make a quick (and good!) decision.

One important note: at first, you may find yourself always choosing the textbook approach. You've practiced algebra for years, after all, and you've only been trying the work backwards technique for a short period of time. Keep practicing; you'll get better! Every high-scorer on the Quant section will tell you that this technique is one of the essential techniques for getting through Quant on time and with a high enough performance to reach a top score.

Try this problem:

> Boys and girls in a class are writing letters. There are twice as many girls as boys in the class, and each girl writes 3 more letters than each boy. If boys write 24 of the 90 total letters written by the class, how many letters does each boy write?
>
> (A) 3
> (B) 4
> (C) 5
> (D) 6

(E) 8

Step 0: How do you know that you can work backwards on this problem?

The answers are fairly nice. The question asks for a discrete variable (the number of letters written by each boy).

Step 1: Start with answer (B) or answer (D). Set up your chart and solve.

Letters per boy	# boys (24 letters)	# girls (2 × boys)	L per girl (+3)	Total letters	90?
(B) 4	6	12	7	(6)(4) + (12)(7)	No

Step 2: Narrow your answers: 24 + 84 is more than 90, so (B) can't be correct. Try (D) next.

Letters per boy	# boys (24 letters)	# girls (2 × boys)	L per girl (+3)	Total letters	90?
(B) 4	6	12	7	(6)(4) + (12)(7)	No
(D) 6	4	8	9	(6)(4) + (8)(9)	No

So 24 + 72 is still larger than 90, though not by much. Answer (D) is also incorrect.

Step 3: Because both (B) and (D) are too large, answer (E) must be correct.

If you're not confident in that reasoning, check the math.

Letters per boy	# boys (24 letters)	# girls (2 × boys)	L per girl (+3)	Total letters	90?
(E) 8	3	6	11	(8)(3) + (6)(11)	Yes!

By the time you get to the test, though, make sure you have enough practice with this method that you will be confident in your reasoning. Then, most of the time, you won't need to check more than two answers!

Here's an algebraic solution:

Call the number of boys b and the number of girls g. Call the number of letters for one boy L. Start translating the problem:
There are twice as many girls: $g = 2b$
If each boy writes L letters, then each girl writes $L + 3$ letters.
All of the boys, then, write a total of $bL = 24$ letters.
Together, the boys and girls write a total of $bL + g(L + 3)= 90$ letters.

How to put that all together? Start substituting. Try to reduce the number of variables:

$$bL + gL + 3g = 90 \qquad \text{substitute } g = 2b$$

$$bL + 2bL + 3(2b) = 90 \qquad \text{substitute } bL = 24$$

$$24 + 2(24) + 6b = 90$$

$$6b = 90 - 24 - 48$$

$$6b = 18$$

$$b = 3$$

There are 3 boys, so they write $3L = 24$, or 8 letters each. The correct answer is **(E)**.

Both solution methods are valid. Which do you prefer?

With the second method, you have to be capable of thinking through a pretty tricky situation in order to set up the math correctly. If you find that straightforward, great! If not, then you may want to work backwards when you can on these kinds of problems.

When *Not* to Work Backwards

There are two scenarios in which working backwards can get messy. The first one is obvious: what if the numbers are really large or ugly? In that case, starting with those numbers doesn't sound like the best idea.

The second is a little more subtle. Take a look at this problem (careful—it's a bit different than the first version you saw!):

Four brothers are splitting a sum of money between them. The first brother receives 50% of the total, the second receives 25% of the total, the third receives 20% of the total, and the fourth receives the remaining \$4. How much more does the first brother receive than the third brother?

(A) 4
(B) 16
(C) 20
(D) 24
(E) 36

The first version of this problem asked for a discrete variable: the total sum of money. This time, though, the problem asks for the difference between the amounts that two of the brothers receive. How could you do that backwards? Try it out.

$B_1 - B_3$	B#1 (50%)	B#2 (25%)	B#3 (20%)	B#4 (4)	Sum	$B_1 - B_3$	Match?
(B) 16	x?	?	$x - 16$?	4	?		

The value 16 represents the first brother's amount minus the third brother's amount. But how do you find the actual values for brother #1 and brother #3? Maybe you can just pick a number for brother #1 and subtract 16 for brother #2?

This isn't actually making your life any easier. You shouldn't need to pick random numbers; the math should work from the numbers that you were given in the first place. When the question stem asks for a combination of variables, such as $B_1 - B_3$, rather than one discrete variable, such as the total, then solving the normal way is likely a better bet than working backwards.

Why can't I start with answer (C)?

You can, actually. In fact, you might want to when answer (C) is a much nicer number than answers (B) or (D).

In general, though, there is one good reason to use (B) or (D) as the default starting point. If you're really curious what that reason is, read on; if not, feel free to skip this section.

Assume that you start with answer (B)—as opposed to answer (D)—and that the answers go in ascending order, from the smallest at (A) to the largest at (E).

You have a 20% chance that choice (B) will be the correct answer. If (B) is incorrect, then what? If you had started with answer (C), then you could only know whether to try one of the two higher answers or one of the two lower answers—so you'd have only a 20% chance of answering the problem after your first try, when (C) is correct.

If, on the other hand, you start with answer (B), and you realize you need a smaller number, then (A) has to be correct. In other words, you have a 40% chance of getting the right answer, even though you've tried only one answer choice so far!

This is really the only difference between starting with choice (B) and starting with choice (C). Once you try the second answer choice, the odds are all equivalent. Still, it's better to have a 40% chance that you can be done with the problem after the first try rather than just a 20% chance. Let's work through another problem:

> Train X is traveling at a constant speed of 30 miles per hour and Train Y is traveling at a constant speed of 40 miles per hour. If the two trains are traveling in the same direction along the same route but Train X is 25 miles ahead of Train Y, how many hours will it be until Train Y is 10 miles ahead of Train X?
>
> (A) 1.5
> (B) 2.0
> (C) 2.5
> (D) 3.0

(E) 3.5

Step 0: How do you know that you can work backwards on this problem?

The answers are fairly nice. The question asks for a discrete variable (the number of hours it takes Train *Y* to travel a certain distance).

Step 1: Start with answer (B) or answer (D). Set up your chart and solve:

Hours	**Y (40 mph)**	**X (30 mph) + 25 miles**	**Where is Y?**	**Is Y 10 miles ahead?**
(B) 2.0	80 miles	60 + 25 = 85 miles	5 miles behind	No

Step 2: Narrow your answers. Answer (B) is incorrect. Train Y is still behind Train X, so you need a higher number. Try (D) next:

Hours	**Y (40 mph)**	**X (30 mph) + 25 miles**	**Where is Y?**	**Is Y 10 miles ahead?**
(B) 2.0	80 miles	60 + 25 = 85 miles	5 miles behind	No
(D) 3.0	120 miles	90 + 25 = 115 miles	5 miles ahead	No

Thus, answer (D) is incorrect.

Step 3: Pick! Train Y has passed Train X, though, so you're moving in the right direction. Because Train Y is not yet 10 miles ahead, though, the answer must be larger. Only answer (E) is larger, so it must be correct.

Here's the math:

Hours	Y (40 mph)	X (30 mph) + 25 miles	Where is Y?	Is Y 10 miles ahead?
(E) 3.5	140 miles	105 + 25 = 130 miles	10 miles ahead	Yes!

Here's one algebraic solution:

The two trains are currently 25 miles apart, with X ahead of Y. The problem asks you to solve for the time when Y had moved 10 miles ahead of X. Therefore, Y has to catch up to X to erase that initial 25-mile deficit and then move an additional 10 miles beyond X, for a total of 35 extra miles.

For every hour that the two trains travel, Y goes 10 miles per hour faster (since it travels 40 miles per hour to X's 30 miles per hour). Plug these numbers into your *RTD* formula:

$$\frac{35 \text{ miles}}{10 \text{ miles per hour}} = 3.5 \text{ hours}$$

Here's what you might find in a textbook:

Plug the scenario into an RTD chart:

	R	T	D
Train X	30	t	D
Train Y	40	t	$D + 35$

Note that the time is the same for the two trains. In order for Train Y to catch up to Train X, it must cover an additional 25 miles. In order for Train Y to pull 10 miles ahead of Train X, it must cover an additional 25 + 10 = 35 miles.

Write two equations:

$30t = D$
$40t = D + 35$

Substitute and solve:

$$40t = 30t + 35$$
$$10t = 35$$
$$t = \frac{35}{10} = 3.5$$

All three solution methods are valid. Which do you prefer?

With the second and third methods, you have to be capable of thinking through a pretty tricky situation in order to set up the math correctly. If you find that straightforward, great! If not, then you may want to work backwards when you can on these kinds of problems.

Chapter 3
of
Word Problems

Rates & Work

In This Chapter…

Chapter 3

Rates & Work

Rate problems come in a variety of forms on the GMAT, but all are marked by three primary components: *rate*, *time*, and *distance* or *work*.

These three elements are related by the following equations:

Rate × Time = Distance
Rate × Time = Work

These equations can be abbreviated as $RT = D$ or as $RT = W$.

This chapter will discuss the ways in which the GMAT makes rate situations more complicated. Often, $RT = D$ problems will involve more than one person or vehicle traveling. Similarly, many $RT = W$ problems will involve more than one worker.

Let's get started with a review of some fundamental properties of rate problems.

Basic Motion: The RTD Chart

All basic motion problems involve three elements: rate, time, and distance.

Rate is expressed as a ratio of distance and time, with two corresponding units. Some examples of rates include: 30 miles per hour, 10 meters/second, 15 kilometers/day.

Time is expressed using a unit of time. Some examples of times include: 6 hours, 23 seconds, 5 months.

Distance is expressed using a unit of distance. Some examples of distances include: 18 miles, 20 meters, 100 kilometers.

You can make an "RTD chart" to solve a basic motion problem. Read the problem and fill in two of the variables. Then use the $RT = D$ formula to find the missing variable. For example:

> If a car is traveling at 30 miles per hour, how long does it take to travel 75 miles?

An RTD chart is shown to the right. Fill in your RTD chart with the given information. Then solve for the time:

	Rate (miles/hr)	×	*Time* (hr)	=	*Distance* (miles)
Car	30	×		=	75

> $30t = 75$, or $t = 2.5$ hours

Matching Units in the RTD Chart

All the units in your RTD chart must match up with one another. The two units in the rate should match up with the unit of time and the unit of distance. For example:

> It takes an elevator 4 seconds to go up one floor. How many floors will the elevator rise in 2 minutes?

The rate is 1 floor every 4 seconds, or 1/4, which simplifies to 0.25 floors/second. Note: the rate is NOT 4 seconds per floor! This is an extremely frequent error. Always express rates as "distance over time," not as "time over distance."

The desired time is 2 minutes. The distance is unknown.

Watch out! There is a problem with this RTD chart on the right. The rate is expressed in floors per second, but the time is expressed in minutes. This will yield an incorrect answer.

	R (floors/sec)	$\times$	T (min)	$=$	D (floors)
Elevator	0.25	$\times$	2	$=$	?

To correct this table, change the time into seconds. To convert minutes to seconds, multiply 2 minutes by 60 seconds per minute, yielding 120 seconds, as shown in the chart on the right.

	R (floors/sec)	$\times$	T (sec)	$=$	D (floors)
Elevator	0.25	$\times$	120	$=$	?

Once the time has been converted from 2 minutes to 120 seconds, the time unit will match the rate unit, and you can solve for the distance using the $RT = D$ equation:

$$0.25(120) = d$$
$$d = 30 \text{ floors}$$

Thus, the elevator will go up 30 floors in 2 minutes.

Try another example:

> A train travels 90 kilometers/hr. How many hours does it take the train to travel 450,000 meters? (1 kilometer = 1,000 meters)

First, divide 450,000 meters by 1,000 to convert this distance to 450 km. By doing so, you match the distance unit (kilometers) with the rate unit (kilometers per hour). Set up an RTD chart like the one to the right.

	R (km/hr)	$\times$	T (hr)	$=$	D (km)
Train	90	$\times$	?	$=$	450

You can now solve for the time: $90t = 450$. Thus, t is equal to 5 hours. Note that this time is the "stopwatch" time: if you started a stopwatch at the start of the trip, what would the stopwatch read at the end of the trip? This is not what a clock on the wall would read, but if you take the *difference* of the start and end clock times (say, 1pm and 6pm), you will get the stopwatch time of 5 hours.

The RTD chart may seem like overkill for a relatively simple problem such as this one. In fact, for such problems, you can simply set up the equation $RT = D$ or $RT = W$ and then substitute. However, the RTD chart comes into its own when you have more complicated scenarios that contain more than one RTD relationship, as you'll see in the next section.

Multiple Rates

Some rate questions on the GMAT will involve *more than one trip or traveler*. To deal with this, you will need to deal with multiple $RT = D$ relationships. For example:

> Harvey runs a 30-mile course at a constant rate of 4 miles per hour. If Clyde runs the same track at a constant rate and completes the course in 90 fewer minutes, how fast did Clyde run?

An RTD chart for this question would have two rows, one for Harvey and one for Clyde, as shown below:

	R (miles/hr)	$\times$	T (hr)	$=$	D (miles)
Harvey					
Clyde					

Pay attention to the relationships between these two equations. Try to use the minimum necessary number of variables.

For instance, both Harvey and Clyde ran the same course, so the distance they both ran was 30 miles. Additionally, you know Clyde ran for 90 fewer minutes. To make units match, you can convert 90 minutes to 1.5 hours. If Harvey ran t hours, then Clyde ran $(t - 1.5)$ hours. Fill in this information on your chart:

	R (miles/hr)	$\times$	T (hr)	$=$	D (miles)
Harvey	4		t		30
Clyde	?		$t - 1.5$		30

Now solve for t:

$$4t = 30$$
$$t = 7.5$$

If $t = 7.5$, then Clyde ran for $7.5 - 1.5 = 6$ hours. You can now solve for Clyde's rate. Let r equal Clyde's rate:

$$r \times 6 = 30$$
$$r = 5$$

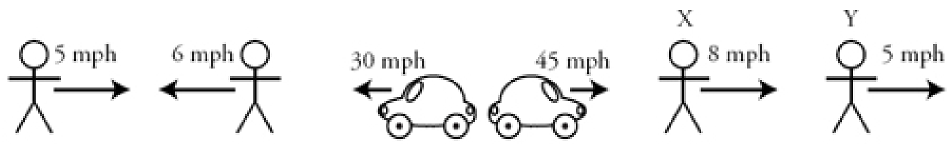
For questions that involve multiple rates, remember to set up multiple $RT = D$ equations and look for relationships between the equations. These relationships will help you reduce the number of variables you need and allow you to solve for the desired value.

Relative Rates

Relative rate problems are a subset of multiple rate problems. The defining aspect of relative rate problems is that two bodies are traveling *at the same time*. There are three possible scenarios:

1. The bodies move towards each other.
2. The bodies move away from each other.
3. The bodies move in the same direction on the same path.

These questions can be dangerous because they can take a long time to solve using the conventional multiple rates strategy (discussed in the last section). You can save valuable time and energy by creating a third $RT = D$ equation for the rate at which the distance between the bodies changes:



Two people decrease the distance between themselves at a rate of 5 + 6 = 11 mph.

Two cars increase the distance between themselves at a rate of 30 + 45 = 75 mph.

Persons X and Y decrease the distance between themselves at a rate of 8 – 5 = 3 mph.

Try an example:

> Two people are 14 miles apart and begin walking towards each other. Person A walks 3 miles per hour, and Person B walks 4 miles per hour. How long will it take them to reach each other?

To answer this question using multiple rates, you would need to make two important inferences: the time that each person walks is exactly the same (t hours) and the total distance they walk is 14 miles. If one person walks d miles, the other walks $(14 - d)$ miles. The chart would look like this:

	R (miles/hr)	×	T (hr)	=	D (miles)
Person A	3		t		d
Person B	4		t		$14 - d$

Alternatively, you can create an $RT = D$ equation for the rate at which they're getting closer to each other.

The rate at which they're getting closer to each other is 3 + 4 = 7 miles per hour. In other words, after every hour they walk, they are 7 miles closer to each other. Now you can create one $RT = D$ equation:

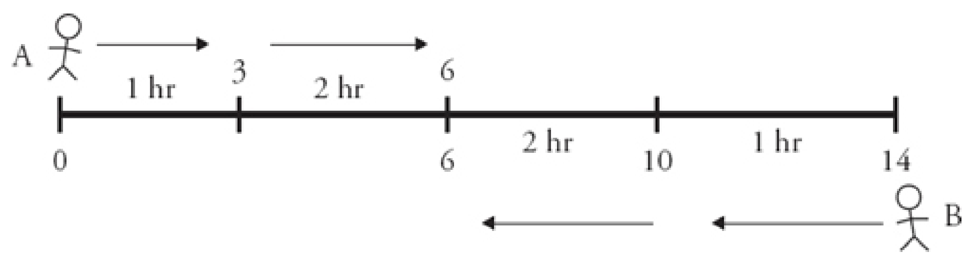
	R (miles/hr)	×	T (hr)	=	D (miles)
$A + B$	7		t		14

$$7t = 14$$
$$t = 2$$

Alternatively, draw it out!

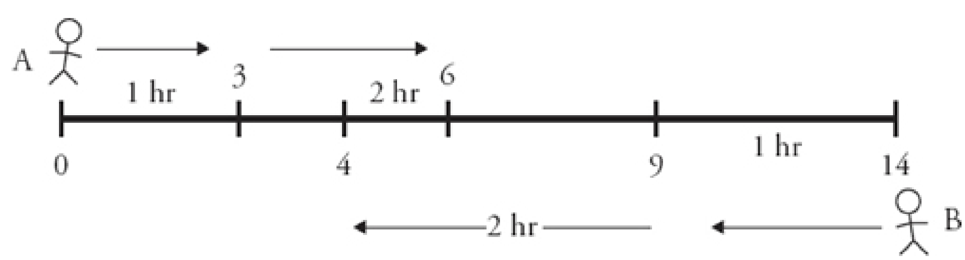
The distance is 14 miles; label one end 0 and the other end 14. After 1 hour, A has traveled 3 miles. B has traveled 4 (so subtract B's distance from 14). Have they met yet?

No. Keep going:

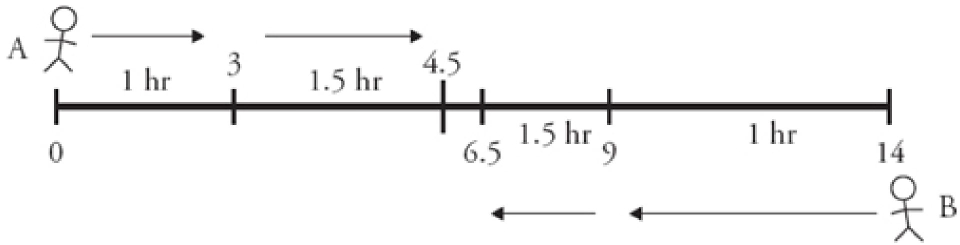


After 2 hours, A has traveled another 3 miles, bringing him up to 6. B has traveled another 4 miles, bringing her all the way to…6! The two people meet at the 2-hour mark.

Note that this technique will still work even when the answer isn't an integer. Let's say that person B is walking at a rate of 5 mph:



They haven't passed at 1 hour, but they have at 2 hours. You would typically be able to eliminate two or three multiple-choice answers at this stage. Next, try 1.5 hours:



They haven't passed yet. This is often enough for you to narrow the choices down to a single answer, though it depends on the exact mix of answer choices.

Average Rate: Find the Total Time

Consider the following problem:

> If Lucy walks to work at a rate of 4 miles per hour, and she walks home by the same route at a rate of 6 miles per hour, what is Lucy's average walking rate for the round trip?

It is very tempting to find an average rate as you would find any other average: add and divide. Thus, you might say that Lucy's average rate is 5 miles per hour ($4 + 6 = 10$ and $10 \div 2 = 5$). However, this is incorrect!

If an object moves the same distance twice, but at different rates, then *the average rate will NEVER be the average of the two rates given for the two legs of the journey*. In fact, because the object spends more time traveling at the slower rate, *the average rate will ALWAYS be closer to the slower of the two rates than to the faster.* On DS problems, that knowledge may be enough to answer the question.

In order to find the average rate, first find the *total* combined time for the trips and the *total* combined distance for the trips. Use this formula:

$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

The problem never establishes a specific distance. Because she walks the same route to work and back home, the average does not depend upon the specific distance. If Lucy walks 1 mile or 15, the average will be the same.

Pick your own number for the distance. Since 12 is a multiple of the two rates in the problem, 4 and 6, 12 is an ideal choice. (You'll learn more about this technique, choose smart numbers, in the next chapter.)

Set up a multiple RTD chart:

	Rate (miles/hr)	×	Time (hr)	=	Distance (miles)
Going	4	×	t_g	=	12
Return	6	×	t_r	=	12
Total	?	×		=	24

The times can be found using the RTD equation. For the GOING trip, $4t_g = 12$, so $t_g = 3$ hours. For the RETURN trip, $6t_r = 12$, so $t_r = 2$ hours. Thus, the combined time for the two trips is 5 hours.

	Rate (miles/hr)	×	Time (hr)	=	Distance (miles)
Going	4	×	**3**	=	12
Return	6	×	**2**	=	12
Total	?	×	5	=	24

Now that you have the total time and the total distance, find the average rate using the RTD formula:

$$r(5) = 24$$
$$r = 4.8 \text{ miles per hour}$$

You can test different numbers for the distance (try 24 or 36) to prove that you will get the same answer, regardless of the number you choose for the distance.

Basic Work Problems

Work problems are just another type of rate problem. Instead of distances, however, these questions are concerned with the amount of "work" done.

Work: Work takes the place of distance. Instead of $RT = D$, use the equation $RT = W$. The amount of work done is often a number of jobs completed or a number of items produced.

Time: This is the time spent working.

Rate: In work problems, the rate expresses the amount of work done in a given amount of time. Rearrange the equation to isolate the rate:

$$R = \frac{W}{T}$$

Be sure to express a rate as work per time (W/T), NOT time per work (T/W). For example, if a machine produces pencils at a constant rate of 120 pencils every 30 seconds, the rate at which the machine works is $\frac{120 \text{ pencils}}{30 \text{ seconds}} = 4$ pencils/second.

Many work problems will require you to calculate a rate. Try the following problem:

> Martha can paint $\frac{3}{7}$ of a room in $4\frac{1}{2}$ hours. If Martha finishes painting the room at the same rate, how long will it have taken Martha to paint the entire room?
>
> (A) $8\frac{1}{3}$ hours (B) 9 hours (C) $9\frac{5}{7}$ hours (D) $10\frac{1}{2}$ hours (E) $11\frac{1}{7}$ hours

Your first step in this problem is to calculate the rate at which Martha paints the room. Set up an RTW chart:

	R (rooms/hr)	$\times$	T (hr)	$=$	W (rooms)
Martha	r		$\frac{9}{2}$		$\frac{3}{7}$

Now you can solve for the rate:

$$r \times \frac{9}{2} = \frac{3}{7}$$

$$r = \frac{3}{7} \times \frac{2}{9} = \frac{2}{21}$$

The division would be messy, so leave it as a fraction. Martha paints $\frac{2}{21}$ of the room every hour. Now you have what you need to answer the question. You can say that painting the whole room is the same as doing 1 unit of work. Set up another RTW chart:

	R (rooms/hr)	×	T (hr)	=	W (rooms)
Martha	$\frac{2}{21}$		t		1

$$\left(\frac{2}{21}\right)t = 1$$

$$t = \frac{21}{2} = 10\frac{1}{2}$$

The correct answer is **(D)**. Notice that the rate and the time in this case were reciprocals of each other. This will always be true when the amount of work done is 1 unit (because reciprocals are defined as having a product of 1).

Working Together: Add the Rates

More often than not, work problems will involve more than one worker. When two or more workers are performing the same task, their rates can be added together. For instance, if Machine A can make 5 boxes in an hour, and Machine B can make 12 boxes in an hour, then working together the two machines can make 5 + 12 = 17 boxes per hour.

Likewise, if Lucas can complete $\frac{1}{3}$ of a task in an hour and Serena can complete $\frac{1}{2}$ of that task in an hour, then working together they can complete $\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$ of the task every hour.

If, on the other hand, one worker is undoing the work of the other, subtract the rates. For instance, if one hose is filling a pool at a rate of 3 gallons per minute, and another hose is draining the pool at a rate of 1 gallon per minute, the pool is being filled at a rate of $3 - 1 = 2$ gallons per minute.

Try the following problem:

> Machine A fills soda bottles at a constant rate of 60 bottles every 12 minutes and Machine B fills soda bottles at a constant rate of 120 bottles every 8 minutes. How many bottles can both machines working together at their respective rates fill in 25 minutes?

To answer these questions quickly and accurately, it is a good idea to begin by expressing rates in equivalent units:

$$\text{Rate}_{\text{Machine A}} = \frac{60 \text{ bottles}}{12 \text{ minutes}} = 5 \text{ bottles/minute}$$

$$\text{Rate}_{\text{Machine B}} = \frac{120 \text{ bottles}}{8 \text{ minutes}} = 15 \text{ bottles/minute}$$

Working together, they fill $5 + 15 = 20$ bottles every minute. Now you can fill out an RTW chart. Let b be the number of bottles filled:

	R (bottles/min)	×	T (min)	=	W (bottles)
$A + B$	20		25		b

$b = 20 \times 25 = 500$ bottles

Remember that, even as work problems become more complex, there are still only a few relevant relationships: $RT = W$ and $R_A + R_B = R_{A+B}$.

Try another example:

> Alejandro, working alone, can build a doghouse in 4 hours. Betty can build the same doghouse in 3 hours. If Betty and Carmelo, working together, can build the doghouse twice as fast as Alejandro can alone, how long would it take Carmelo, working alone, to build the doghouse?

Begin by solving for the rate that each person works. Let *c* represent the number of hours it takes Carmelo to build the doghouse.

Alejandro can build $\frac{1}{4}$ of the doghouse every hour, Betty can build $\frac{1}{3}$ of the doghouse every hour, and Carmelo can build $\frac{1}{c}$ of the doghouse every hour.

The problem states that Betty and Carmelo, working together, can build the doghouse twice as fast as Alejandro. In other words, their rate is twice Alejandro's rate:

$$\text{Rate}_B + \text{Rate}_C = 2(\text{Rate}_A)$$

$$\frac{1}{3} + \frac{1}{c} = 2\left(\frac{1}{4}\right)$$

$$\frac{1}{c} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$c = 6$$

It takes Carmelo 6 hours working by himself to build the doghouse.

When dealing with multiple rates, be sure to express rates in equivalent units. When the the work involves completing a task, remember to treat completing the task as doing one “unit” of work. Once you know the rates of every worker, add the rates of workers who work together to complete the same task.

Problem Set

Solve the following problems using the strategies you have learned in this section. Use RTD or RTW charts as appropriate to organize information.

1. An empty bucket being filled with paint at a constant rate takes 6 minutes to be filled to $\frac{7}{10}$ of its capacity. How much more time will it take to fill the bucket to full capacity?

2. Two hoses are pouring water into an empty pool. Hose 1 alone would fill up the pool in 6 hours. Hose 2 alone would fill up the pool in 4 hours. How long would it take for both hoses to fill up two-thirds of the pool?

3. Did it take a certain ship less than 3 hours to travel 9 kilometers? (1 kilometer = 1,000 meters)

   (1) The ship's average speed over the 9 kilometers was greater than 55 meters per minute.
   (2) The ship's average speed over the 9 kilometers was less than 60 meters per minute.

4. Twelve identical machines, running continuously at the same constant rate, take 8 days to complete a shipment. How many additional machines, each running at the same constant rate, would be needed to reduce the time required to complete a shipment by 2 days?

   (A) 2 (B) 3 (C) 4 (D) 6 (E) 9

Save the below problem set for review, either after you finish this book or after you finish all of the Quant books that you plan to study.

5. Al and Barb shared the driving on a certain trip. What fraction of the total distance did Al drive?

   (1) Al drove for $\frac{3}{4}$ as much time as Barb did.

(2) Al's average driving speed for the entire trip was $\frac{4}{5}$ of Barb's average driving speed for the trip.

6. Nicky and Cristina are running a race. Since Cristina is faster than Nicky, she gives him a 36-meter head start. If Cristina runs at a pace of 5 meters per second and Nicky runs at a pace of only 3 meters per second, how many seconds will Nicky have run before Cristina catches up to him?

(A) 15 seconds (B) 18 seconds (C) 25 seconds (D) 30 seconds (E) 45 seconds

7. Mary and Nancy can each perform a certain task in m and n hours, respectively. Is $m < n$?

(1) Twice the time it would take both Mary and Nancy to perform the task together, each working at their respective constant rates, is greater than m.
(2) Twice the time it would take both Mary and Nancy to perform the task together, each working at their respective constant rates, is less than n.

Solutions

1. $2\frac{4}{7}$ **minutes:** Use the $RT = W$ equation to solve for the rate, with $t = 6$ minutes and $w = \frac{7}{10}$:

$$r(6 \text{ minutes}) = \frac{7}{10}$$
$$r = \frac{7}{10} \div 6 = \frac{7}{60} \text{ buckets per minute.}$$

R (bucket/min)	×	T (min)	=	W (bucket)
r	×	6	=	7/10

Next, substitute this rate into the equation again, using $\frac{3}{10}$ for w (the remaining work to be done):

$$\left(\frac{7}{60}\right)t = \frac{3}{10}$$
$$t = \frac{3}{1\cancel{0}} \times \frac{6\cancel{0}}{7} = \frac{18}{7} = 2\tfrac{4}{7} \text{ minutes}$$

R (bucket/min)	×	T (min)	=	W (bucket)
7/60	×	t	=	3/10

2. $1\frac{3}{5}$ **hours:** If Hose 1 can fill the pool in 6 hours, its rate is $\frac{1}{6}$ "pool per hour," or the fraction of the job it can do in one hour. Likewise, if Hose 2 can fill the pool in 4 hours, its rate is $\frac{1}{4}$ pool per hour. Therefore, the combined rate is $\frac{5}{12}$ pool per hour $\left(\frac{1}{4} + \frac{1}{6} = \frac{5}{12}\right)$:

$$RT = W$$
$$(5/12)t = 2/3$$
$$t = \left(\frac{2}{3}\right)\left(\frac{12}{5}\right) = \frac{8}{5} = 1\frac{3}{5} \text{ hours}$$

R (pool/hr)	×	T (hr)	=	W (pool)
5/12	×	t	=	2/3

3. **(A):** Notice that the statements provide rates in meters per minute. A good first step here is to figure out how fast the ship would have to travel to cover 9

kilometers in 3 hours. Create an RTD chart, and convert kilometers to meters and hours to minutes:

R (meters/min)	$\times$	T (min)	$=$	D (meters)
r		180		9,000

$$180r = 9{,}000$$
$$r = 50$$

The question asks whether the ship traveled 9 kilometers in *less than* 3 hours. The ship must travel faster than 50 meters/min to make it in less than 3 hours. Therefore, the question is really asking, is $r > 50$?

(1): SUFFICIENT: If the average speed of the ship was greater than 55 meters per minute, then $r > 55$. Thus, r is definitely greater than 50.

(2): INSUFFICIENT: If the average speed of the ship was less than 60 meters per minute, then $r < 60$. This is not enough information to guarantee that $r > 50$.

4. **(C) 4 additional machines:** Let the work rate of 1 machine be r. Then the work rate of 12 machines is $12r$, and you can set up an RTW chart:

	R	$\times$	T	$=$	W
Original	$12r$		8		$96r$

The shipment work is then $96r$. To figure out how many machines are needed to complete this work in $8 - 2 = 6$ days, set up another row and solve for the unknown rate:

	R	$\times$	T	$=$	W
Original	$12r$		8		$96r$
New	$\boxed{}$		6		$96r$

Therefore, there are $\frac{96r}{6} = 16r$ machines in total, or $16 - 12 = 4$ additional machines.

5. **(C):** You can rephrase the question as follows: What is the ratio of Al's driving distance to the entire distance driven? Alternatively, since the entire distance is the sum of only Al's distance and Barb's distance, you can simply find the ratio of Al's distance to Barb's distance:

(1): INSUFFICIENT. You have no rate information, and so you have no definitive distance relationships:

	R	$\times$	T	$=$	W
Al			$(3/4)t$		
Barb			t		
Total					

(2): INSUFFICIENT. As with Statement (1), you have no definitive distance relationships:

	R	$\times$	T	$=$	W
Al	$(4/5)r$				
Barb	r				
Total					

(1) & (2) TOGETHER: SUFFICIENT. Set up a chart like the one below:

	R	$\times$	T	$=$	W
Al	$(4/5)r$		$(3/4)t$		$(3/5)rt$
Barb	r		t		rt
Total					$(8/5)rt$

Al drove $\dfrac{\frac{3}{5}rt}{\frac{8}{5}rt} = \dfrac{3}{8}$ of the distance. (Alternatively, he drove $\dfrac{3}{5}$ as much as Barb did, meaning that he drove $\dfrac{3}{8}$ of the trip.) Notice that you do not need the absolute time, nor the rate, of either driver's portion of the trip.

6. **(D) 30 seconds:** Initially, Nicky runs 36 meters at a rate of 3 meters per second. Therefore, Nicky runs for 36/3 = 12 seconds before Christina starts running.

Save time on this problem by dealing with the rate at which the distance between Cristina and Nicky changes. Nicky is originally 36 meters ahead of Cristina. If Nicky runs at a rate of 3 meters per second and Cristina runs at a rate of 5 meters per second, then the distance between the two runners is shrinking at a rate of 5 − 3 = 2 meters per second.

You can now figure out how long it will take for Cristina to catch Nicky using a single $RT = D$ equation. The rate at which the distance between the two runners is shrinking is 2 meters per second, and the distance is 36 meters (because that's how far apart Nicky and Cristina are):

	R (meters/sec)	×	T (sec)	=	D (meters)
	2		t		36

$$2t = 36$$
$$t = 18$$

Nicky's total time is 30 seconds: 12 seconds + 18 seconds = 30 seconds.

7. **(D):** First, set up an RTW chart:

	R	×	T	=	W
Mary	$1/m$		m		1
Nancy	$1/n$		n		1

Recall that the question is "Is
$< n$?"

(1): SUFFICIENT. Find out how much time it would take for the task to be performed with both Mary and Nancy working:

	R	×	T	=	W
Mary	$1/m$		m		1
Nancy	$1/n$		n		1
Total	$1/m + 1/n$		t		1

$$\left(\frac{1}{m}+\frac{1}{n}\right)t = 1$$

$$\left(\frac{m+n}{mn}\right)t = 1$$

$$t = \frac{mn}{m+n}$$

Now, set up the inequality described in the statement (that is, twice this time is greater than *m*):

$$2t > m$$

$$2\left(\frac{mn}{m+n}\right) > m$$

$$2mn > mn + m^2 \qquad \text{You can cross multiply by } m+n \text{ because } m+n \text{ is positive.}$$

$$mn > m^2$$

$$n > m \qquad \text{You can divide by } m \text{ because } m \text{ is positive.}$$

Alternatively, you can rearrange the original inequality thus:

$$t > \frac{m}{2}$$

If both Mary and Nancy worked at Mary's rate, then together, they would complete the task in $\frac{m}{2}$ hours. Since the actual time is longer, Nancy must work more slowly than Mary, and thus $n > m$.

(2): SUFFICIENT. You can reuse the computation of *t*, the time needed for the task to be jointly performed:

$$2t < n$$

$$2\left(\frac{mn}{m+n}\right) < n$$

$$2mn < nm + n^2 \qquad \text{Again, you can cross multiply by } m+n \text{ because } m+n \text{ is positive.}$$

$$mn < n^2$$

$$m < n \qquad \text{You can divide by } n \text{ because } n \text{ is positive.}$$

Alternatively, you can rearrange the original inequality thus:

$$t < \frac{n}{2}$$

If both Mary and Nancy worked at Nancy's rate, then together, they would complete the task in $\frac{n}{2}$ hours. Since the actual time is shorter, Mary must work faster than Nancy, and thus $m < n$.

Chapter 4
of
Word Problems

Strategy: Choose Smart Numbers

In This Chapter…

Chapter 4

Strategy: Choose Smart Numbers

Some algebra problems—problems that involve variables—can be turned into arithmetic problems, instead. You're better at arithmetic than algebra (everybody is!), so turning an annoying variable-based problem into one that uses real numbers can be a lifesaver on the GMAT.

Which of the below two problems is easier for you to solve?

How many miles can a car going x miles per hour travel in y hours?

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$ (C) xy

How many miles can a car going 40 miles per hour travel in 3 hours?

(A) $\frac{40}{3}$ (B) $\frac{3}{40}$ (C) 120

You may think that the algebraic version is not difficult at all, but no matter how easy you think it is, it's still easier to work with real numbers.

If you compare the two problems, you'll see that the setup is identical—and this feature is at the heart of how you can turn algebra into arithmetic.

How Do Smart Numbers Work?

Here's how to solve the algebra version of the above problem using smart numbers:

Step 1: Choose smart numbers to replace the variables.

The problem has two variables, x and y. Are the two numbers tied to each other in some way? That is, if you choose a number for one, will that determine the number for the other?

In this case, no. The two variables are completely separate, so you can choose a smart number for each one. Keep these three factors in mind when choosing your smart numbers:

1. If you are picking for more than one variable, pick different numbers for each one. If possible, pick numbers with different characteristics (e.g., one even and one odd).
2. Follow any constraints given in the problem. In this problem, for example, neither x nor y can be negative; it's not possible to drive –3 miles per hour or to travel for –2 hours.
3. Choose numbers that work easily in the problem. They do not have to be realistic in the real world. For example, on this problem, you might say that the car is driving 2 miles per hour for 3 hours. Nobody's going to drive that way in the real world, but it doesn't matter!

On your scrap paper, write:

$$\boxed{\begin{array}{l} x = 2 \\ y = 3 \end{array}}$$

Always write down what you choose for your variables and box it off; you'll be coming back to these numbers repeatedly.

Step 2: Solve the problem using your chosen smart numbers.

Wherever the problem says x, it will now say 2 (since you picked x = 2). Wherever the problem says y, it will now say 3 (since you picked y = 3). Here's the rewritten problem:

How many miles can a car going 2 miles per hour travel in 3 hours?

If you drive 2 miles per hour for 3 hours, then you'll travel 2 × 3 = 6 hours. This is your answer; draw a circle around the answer on your scrap paper, as shown below.

$$6$$

Step 3: Find a match in the answers.

The same rules apply to the answers: replace all of the *x* variables with 2 and all of the *y* variables with 3. Check the answers:

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 6

Only answer (C) equals 6; it is the correct answer.

When and How to Use Smart Numbers

It's crucial to know when you're allowed to use this technique. It's also crucial to know how *you* are going to decide whether to use textbook math or to choose smart numbers; you will typically have time to try just one of the two techniques during your two minutes on the problem.

The choose smart numbers technique can be used any time a problem contains only *unspecified* values. The easiest example of such a problem is one that contains variables throughout; it does not provide real numbers for those variables and it uses those same variables in the answer choices. The problem might also use only percentages or fractions; again, it does not provide real numbers, and the answers consist of percentages or fractions. Whenever a problem has these characteristics, you can choose your own smart numbers to turn the problem into arithmetic.

There is some cost to doing so: it can take extra time compared to the "pure" textbook solution. As a result, the technique is most useful when the problem is a hard one for you. If you find the abstract math involved to be very easy, then you may not want to take the time to transform the problem into arithmetic. As the math gets more complicated, however, the arithmetic form becomes comparatively easier and faster to use.

Try this problem. Solve it twice—once using textbook math and once using smart numbers:

> A train travels at a constant rate. If the train takes 13 minutes to travel *m* kilometers, how long will the train take to travel *n* kilometers?

(A) $\frac{13m}{n}$

(B) $\frac{13n}{m}$

(C) $13mn$

(D) $\frac{n}{13}$

(E) $\frac{m}{13}$

First, how do you know that you can choose smart numbers on this problem? The problem actually does contain a real number: 13.

Think about what's going on. The train might take 13 minutes to travel 1 kilometer or 13 minutes to travel 62 kilometers. It could really be any distance; the problem doesn't depend on the exact number of kilometers. Another clue is the fact that the variables show up again in the answer choices. Put the two pieces together (unspecified amounts for which you could imagine many options and variables in the answer choices) and you know you can use smart numbers.

Step 1: Choose smart numbers to replace the variables.

The number 2 is often a great default number to pick when using the smart numbers technique.

Step 2: Solve the problem using your chosen smart numbers.

In this case, though, 2 is not such a great number. Why?

Sometimes, you have to solve a little bit to figure out what would be a good number to pick. Look at what you'll have to do with that number: the train takes 13 minutes to travel 2 kilometers, so it's going 2 km/13 min. Hmm. Can you think of a number that will make the math less annoying?

Try $m = 26$. If the train travels 26 kilometers in 13 minutes, then it is traveling $\frac{\text{26 kilometers}}{\text{13 minutes}}$ = 2 kilometers per minute.

Next, the problem asks how long the train will take to travel n kilometers. What number would work well for n? Try $n = 4$. If the train travels 4 kilometers and it is traveling 2 km/min, then it takes the train 2 minutes to travel 4 kilometers.

$$\boxed{\begin{array}{c} m = 26 \text{ km} \\ n = 4 \text{ km} \end{array}}$$

$$\text{min} = 2$$

Step 3: Find a match in the answers. Your goal is to find a match for 2. If you can tell that a certain answer will *not* equal 2, you can cross it off without calculating exactly what it does equal.

(A) $\dfrac{13m}{n} = \dfrac{13(26)}{4} =$ too big. Eliminate.

(B) $\dfrac{13n}{m} = \dfrac{13(4)}{26} =$ maybe. Simplify. $\dfrac{13(4)}{26} = \dfrac{4}{2} = 2.$ Match!

(C) $13mn =$ way too big. Eliminate.

(D) $\dfrac{n}{13} = \dfrac{4}{13}$ Eliminate.

(E) $\dfrac{m}{13} = \dfrac{26}{13} = 2.$ Wait a second—this one matches, too!

If your smart numbers aren't smart enough, then you could find yourself with two right answers. (Don't worry, this is rare, but we want you to know what to do if it happens.)

Now what? While the clock is still ticking, either guess between (B) and (E) or, if you have time, try a different set of numbers in the problem.

Afterwards, learn a valuable lesson about how to choose the best smart numbers.

Here's how the problem went:

(choose)
m = 26 km
(calculate)
Therefore, the train is traveling at a rate of 2 km/min.
(choose)
n = 4 km
(calculate)
Therefore, the train goes 4 km in 2 minutes.

Here's the flaw in the choice of numbers: the rate of the train, 2, matches the final answer, also 2. In general, avoid picking numbers that will give you the same numerical answers at different points in the problem; in both of your calculations, the result was 2!

To fix this as efficiently as possible, keep the first part of the calculation as is. For the second part, though, go back and choose something for n that will not again give you 2. For example:

(choose)	m = 26 km
(calculate)	Therefore, the train is traveling at a rate of 2 km/min.
(choose)	n = 6 km
(calculate)	Therefore, the train goes 6 km in 3 minutes.

Now try just those two answer choices again:

(A)

(B) $\frac{13n}{m} = \frac{13(6)}{26} =$ maybe. Simplify. $\frac{13(6)}{26} = \frac{6}{2} = 3$. Match!

(C)

(D)

(E) $\frac{m}{13} = \frac{26}{13} = 2$. Eliminate.

Now, only one answer matches. The correct answer is **(B)**.

Next time you're choosing numbers, you'll know to avoid picking values that return the same numerical outcome as another part of the problem. If you do accidentally find yourself in this situation and you're most of the way through the math, go ahead and try the answers; there may still be only one answer that works. If two answers work and you have the time, then go back, change one of your numbers, and do the math again. If you don't have time, just pick one of the two answers that did work.

It's also a good idea to avoid picking 0 or 1 or a number that was used elsewhere in the problem. For example, it wouldn't be a good idea to choose 13 on this problem.

Let's summarize the process:

Step 0: Recognize that you can choose smart numbers.

The problem talks about some values but doesn't provide real numbers for those values. Rather, it uses variables or only refers to fractions or percentages. The answer choices consist of variable expressions, fractions, or percentages.

Step 1: Choose smart numbers to replace the variables.

Follow all constraints given in the problem. If the problem says that x is odd, pick an odd number for x. If the problem says that $x + y = z$, then note that once you pick for x and y, you have to calculate z. Don't pick your own random number for z!

Avoid choosing 0 or 1. Avoid choosing numbers that show up elsewhere in the problem. If you have to choose more than one number, make sure you choose different numbers.

After considering all of the above, choose numbers that make the problem easier to tackle!

Step 2: Solve the problem using your chosen smart numbers.

Wherever the problem used to have variables or unknowns, it now contains the real numbers that you've chosen. Solve the problem arithmetically and find your target answer.

Step 3: Find a match in the answers.

Plug your smart numbers into the answer choices and look for the choice that matches your target. If, at any point, you can tell that a particular answer will *not* match your target, stop calculating that answer. Cross it off and move to the next answer.

Smart Numbers and Percentages or Fractions

As discussed earlier, percentages or fractions in a problem can also trigger you to use the smart numbers technique.

Try this problem:

> The price of a certain computer is increased by 10%, and then the new price is increased by an additional 5%. The new price is what percentage of the original price?
>
> (A) 120%
> (B) 116.5%
> (C) 115%
> (D) 112.5%
> (E) 110%

How do you know you can use smart numbers? The problem never gives a real price for the computer; the entire problem (including the answers) is in terms of percentages only.

Step 1: Choose smart numbers.
When working with percentages, 100 is a great number to choose. If the problem already uses 100 or 100% somewhere, then you may want to choose a different starting number. In this case, there are no warning signs to avoid 100, so say that the computer's initial price was $100.

Step 2: Solve.
First, the computer's price increased by 10%, so the new price is $100 + $10 = $110. Next, the *new* price is increased by a further 5%, so the price becomes $110 + (0.05)($110) = $110 + $5.50 = $115.50.

This is the beauty of choosing 100 as your starting number. The question asks for the new price as a percentage of the original price. The new price is $115.50 and the original price is $100. “Percentage” literally means “per 100.” In the same way that 80/100 equals 80%, 115.50/100 equals 115.5%. In other words, you can ignore the denominator; your final number already represents the desired percentage.

Step 3: Find a match.

(A) 120%
(B) 115.5% → match
(C) 115%
(D) 112.5%
(E) 110%

The correct answer is **(B)**.

As you can see, this technique is even better when working with percentages (or fractions) because you don't have that final step of replacing variables in the answer choices. When you're done with your calculations, you're done all around!

Let's say that, on the last problem, you weren't able to choose $100 for some reason. Instead, you chose an initial price of $20. How would the problem work? Try it out before continuing to read.

If the initial price is $20, then the first increase of 10% would bring the price to $20 + $2 = $22. The next increase of 5% would bring the price to $22 + (0.05)($22) = $22 + $1.1 = $23.10.

Tip: To calculate 5% quickly, first find 10% of the desired number, then halve the number. For example, to find 5% of 22, first find 10%: 2.2. Next, halve that number: $\frac{2.2}{2}$ = 1.1.

Now what? $23.10 isn't in the answers. Remember that you didn't start with 100! The new price as a percentage of the original is $\frac{23.10}{20}$. How do you turn that into a percentage?

Here's how. First, remember that percentage means "per 100." Manipulate the fraction until you get 100 on the bottom:

$$\frac{23.10}{20} \times \frac{5}{5} = \frac{115.5}{100}$$

The answer is 115.5%.

That last calculation is annoying—you don't want to do it unless you have to. Therefore, if you can pick 100 on a percentage problem, do so.

How to Get Better at Smart Numbers

First, practice the problems at the end of this chapter. Try each problem two times: once using smart numbers and once using the "textbook" method. (Time yourself separately for each attempt.)

When you're done, ask yourself which way you prefer to solve *this* problem and why. Remember that, on the real test, you won't have time to try both methods; you'll have to make a decision and go with it. Learn *how* to make that strategy decision while studying; then, the next time a new problem pops up in front of you that could be solved by choosing smart numbers, you'll be able to make a quick (and good!) decision.

One important note: whenever you learn a new strategy, at first you may find yourself always choosing the textbook approach. You've practiced algebra for years, after all, and you've only been using the smart numbers technique for a short period of time. Keep practicing; you'll get better! Every high-scorer on the Quant section will tell you that choosing smart numbers is invaluable to getting through Quant on time and with a high enough performance to reach a top score.

When NOT to Use Smart Numbers

There are certain scenarios in which a problem contains some of the smart numbers characteristics but not all.

For example, why can't you use smart numbers on this problem from the work backwards strategy chapter?

> Four brothers are splitting a sum of money between them. The first brother receives 50% of the total, the second receives 25% of the total, the third receives 20% of the total, and the fourth receives the remaining $4. How many dollars are the four brothers splitting?

(A) $50
(B) $60
(C) $75
(D) $80
(E) $100

The problem talks about a sum of money but, at first, tells you nothing concrete about this sum of money. Towards the end, though, it does give you one real value: $4. Because the “remaining” percentage has to equal $4 exactly, this problem has just one numerical answer. You can't pick any starting point that you want. (The answer to the above problem is (D), by the way. You can find the solution in Chapter 2, Strategy: Work Backwards.)

Chapter 5
of
Word Problems

Overlapping Sets

In This Chapter…

Chapter 5

Overlapping Sets

Translation problems that involve two or more given sets of data that partially intersect with each other are termed overlapping sets. For example:

> Of 30 integers, 15 are in set A, 22 are in set B, and 8 are in both sets A and B. How many of the integers are in NEITHER set A nor set B?

This problem involves two sets, A and B. The two sets overlap because some of the numbers are in both sets. Thus, these two sets can actually be divided into four categories:

1. Numbers in set A
2. Numbers in set B
3. Numbers in both A and B
4. Numbers in neither A nor B

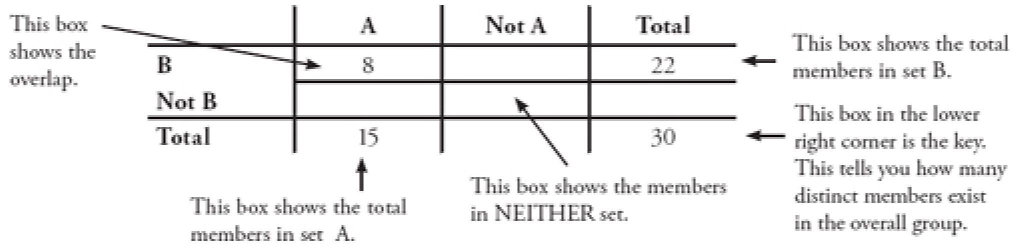
Solving double-set GMAT problems, such as in the example above, involves finding values for these four categories.

The Double-Set Matrix

For GMAT problems involving only *two* categorizations or decisions, the most efficient tool is the **double-set matrix**. Here's how to set one up, using the example from above:

> Of 30 integers, 15 are in set A, 22 are in set B, and 8 are in both set A and B. How many of the integers are in NEITHER set A nor set

B?



Once the information given in the problem has been filled in, as in the chart to the right, complete the chart, using the totals to guide you. Each row and each column sum to a total value.

	A	Not A	Total
B	8	**14**	22
Not B	**7**	**1**	**8**
Total	15	**15**	30

The question asks for the number of integers that are in *neither* set. Look at the chart to find the number of integers that are NOT A and NOT B; the answer is 1.

When you construct a double-set matrix, be careful! As mentioned above, the rows should correspond to the *mutually exclusive options* for one decision: you have A or you don't have A. Likewise, the columns should correspond to the mutually exclusive options for the other decision: you have B or you don't have B. For instance, do not draw the table this way:

INCORRECT

	A	B	Total
Not A			
Not B			
Total			

Note: even if you are accustomed to using Venn diagrams for these problems, you should strongly consider switching to the double-set matrix for problems with only two sets of options. The double-set matrix conveniently displays *all* possible combinations of options, including totals, whereas the Venn diagram only displays a few of them easily.

Overlapping Sets and Percents

Many overlapping-sets problems involve *percents* or *fractions*. The double-set matrix is still effective on these problems, especially if you choose a smart number for the grand total. For problems involving percents, choose a total of 100. For problems involving fractions, choose a common denominator for the total. For example, choose 15 if the problem mentions categories that are $\frac{1}{3}$ and $\frac{2}{5}$ of the total.

> 70% of the guests at Company X's annual holiday party are employees of Company X. 10% of the guests are women who are not employees of Company X. If half the guests at the party are men, what percent of the guests are female employees of Company X?

First, set up your chart. The two groups are Men/Women and Employee/Not Employee. Because the problem uses only percentages, no real numbers, choose 100 for the total number of guests. Then, fill in the other information given in the problem: 70% of the guests are employees, and

10% are women who are not employees. You also know that half the guests are men. (Therefore, you also know that half the guests are women.)

	Men	Women	Total
Employee			70
Not Emp.		10	
Total	50	50	100

What does the question want? Calculate only what you need to answer the question. In this case, the question asks for female employees, so calculate the "Women + Employee" box: 50 – 10 = 40. The percentage of female employees is $\frac{40}{100}$, or 40%.

	Men	Women	TOTAL
Employee	**30**	**40**	70
Not Emp.	**20**	10	**30**
TOTAL	50	50	100

Note that the problem does not require you to complete the matrix with the number of male employees, since you have already answered the question asked in the problem. If you want to check your work, though, you can complete the matrix. (Make sure you have enough time!) The last box you fill in must work both vertically and horizontally.

As with other problems involving smart numbers, you can only assign a number to the total if the problem contains only fractions and/or percents, but no actual *numbers* of items or people. In that case, go ahead and pick a total of 100 (for percent problems) or a common denominator (for fraction problems). If actual quantities appear anywhere in the problem, though, then all the totals are already determined. In that case, you cannot assign numbers, but must solve for them instead.

Overlapping Sets and Algebraic Representation

When solving overlapping sets problems, pay close attention to the wording of the problem. For example, consider the problem below:

> A researcher estimates that 10% of the children in the world are between the ages of 8 and 18 and dislike soccer, and that 50% of the children who like soccer are between the ages of 8 and 18. If 40% of the children in the world are between the ages of 8 and 18, what percentage of children in the world are under age 8 and dislike the game of soccer? (Assume all children are between the ages of 0 and 18.)

It is tempting to fill in the number 50 to represent the percent of children aged 8 to 18 who like soccer. However, this approach is incorrect.

WRONG

	8 to 18	< 8	Total
Like	50		
Dislike	10		
Total			100

Notice that you are told that 50% of the children *who like soccer* are between the ages of 8 and 18. This is different from being told that 50% of the children *in the world* are between the ages of 8 and 18. In this problem, the information you have is a fraction of an unknown number. You do not yet know how many children like soccer. Therefore, you cannot yet write a number for the children aged 8 to 18 who like soccer. Instead, represent the unknown total number of children who like soccer with the variable x. Then, you can represent the number of children aged 8 to 18 who like soccer with the expression $0.5x$.

CORRECT

	8 to 18	< 8	Total
Like	$0.5x$		x
Dislike	10		
Total	40		100

From the relationships in the table, set up an equation to solve for x:

$$0.5x + 10 = 40$$
$$x = 60$$

With this information, you can fill in the rest of the table:

	8 to 18	< 8	Total
Like	$\mathbf{0.5x = 30}$	**30**	$\mathbf{x = 60}$
Dislike	10	**30**	**40**
Total	40	**60**	100

Therefore, 30% of the children are under age 8 and dislike soccer.

Problem Set

1. A is a set of even integers, while B is a set of integers that are multiples of 3. There are 16 integers in set A, 22 integers in set B, and 7 integers in both sets. How many integers are in exactly one of the two sets?

2. Of 28 people in a park, 12 are children and the rest are adults. 8 people have to leave at 3pm; the rest do not. If, after 3pm, there are 6 children still in the park, how many adults are still in the park?

3. 40% of all high school students hate roller coasters; the rest love them. 20% of those students who love roller coasters own chinchillas. What percentage of students love roller coasters but do not own a chinchilla?

Save the below problem set for review, either after you finish this book or after you finish all of the Quant books that you plan to study.

4. Of 30 snakes at the reptile house, 10 have stripes, 21 are poisonous, and 5 have no stripes and are not poisonous. How many of the snakes have stripes AND are poisonous?

5. 10% of all aliens are capable of intelligent thought and have more than 3 arms, and 75% of aliens with 3 arms or less are capable of intelligent thought. If 40% of all aliens are capable of intelligent thought, what percent of aliens have more than 3 arms?

Solutions

1. **24 integers:** Use a Double-Set Matrix to solve this problem. First, fill in the numbers given in the problem: 16 integers in set A (first column total) and 22 integers in set B (first row total). There are 7 integers in both sets (first row, first column). Next, use subtraction to figure out that there are 9 integers in set A but not in set B and 15 integers in set B but not in set A. Finally, add those two numbers: 9 + 15 = 24..

	Set A	NOT Set A	Total
Set B	7	**15**	22
Not Set B	**9**		
Total	16		

2. **14 adults:** Use a double-set matrix to solve this problem. First, fill in the numbers given in the problem: 28 total people in the park, 12 children and the rest (16) adults; 8 leave at 3pm and the rest (20) stay. Then, you are told that there are 6 children left in the park after 3pm. Since you know there are a total of 20 people in the park after 3pm, the remaining 14 people must be adults.

	Children	Adults	Total
Leave at 3			8
Stay	6	**14**	**20**
Total	12	**16**	28

3. **48%:** Since all the numbers in this problem are given in percentages, assign a grand total of 100 students. You know that 40% of all high school students hate roller coasters, so fill in 40 for this total and 60 for the number of students who love roller coasters. You also know that 20% **of those**

students who love roller coasters own chinchillas. It does not say that 20% of all students own chinchillas. Since 60% of students love roller coasters, 20% of 60% own chinchillas. Therefore, fill in 12 for the students who both love roller coasters and own chinchillas. The other 48 roller coaster lovers do not own chinchillas.

	Love RCs	Do Not	Total
Chinchilla	12		
No Chinch.	48		
Total	**60**	40	100

4. **6:** Use a double-set matrix to solve this problem. First, fill in the numbers given in the problem: 30 snakes, 10 with stripes (and therefore 20 without), 21 that are poisonous (and therefore 9 that are not), and 5 that are neither striped nor poisonous. Use subtraction to fill in the rest of the chart. Thus, 6 snakes have stripes and are poisonous.

	Stripes	No Stripes	Total
Poisonous	**6**		21
Not Poison	**4**	5	**9**
Total	10	**20**	30

5. **60%:** Since all the numbers in this problem are given in percentages, assign a grand total of 100 aliens. You know that 10% of all aliens are capable of intelligent thought and have more than 3 arms. You also know that 75% **of aliens with 3 arms or less** are capable of intelligent thought. It does not say that 75% of all aliens are capable of intelligent thought. Therefore, assign the variable x to represent the percentage of aliens with three arms or less. Then, the percentage of aliens with three arms or less who are capable of intelligent thought can be represented by $0.75x$. Since you know that 40% of all aliens are capable of intelligent thought, you can write the equation $10 + 0.75x = 40$, or $0.75x = 30$. Solve for x: $x = 40$. Therefore, 40% of the aliens have three arms or less, and 60% of aliens have more than three arms.

	Thought	No Thought	Total
> 3 arms	10		**60**
≤ 3 arms	$0.75x =$ **30**		$x =$ **40**
Total	40		100

Chapter 6
of
Word Problems

Statistics

In This Chapter…

Chapter 6

Statistics

Averages

The average (or the **arithmetic mean**) of a set is given by the following formula:

$$\text{Average} = \frac{\text{Sum}}{\text{\# of terms}}, \text{ which is abbreviated as } A = \frac{S}{n}$$

The sum, S, refers to the sum of all the terms in the set.

The number, n, refers to the number of terms that are in the set.

The average, A, refers to the average value (arithmetic mean) of the terms in the set.

The language in an average problem will often refer to an “arithmetic mean.” However, occasionally, the concept is implied. “The cost per employee, if equally shared, is $20” means that the *average* cost per employee is $20. Likewise, the “per capita income” is the average income per person in an area.

Here's a commonly used variation of the average formula:

(Average) × (# of terms) = (Sum), or $A \times n = S$

Using the Average Formula

Every GMAT problem dealing with averages can be solved using some form of the average formula. In general, if the average is unknown, the first formula, $A = \frac{S}{n}$, will solve the problem more directly. If the average is known, the second formula, $A \times n = S$, is better.

When you see any GMAT average problem, write down the average formula. Then, fill in any of the three variables (S, n, and A) that are given in the problem:

> The sum of 6 numbers is 90. What is the average term?
>
> $$A = \frac{S}{n}$$
>
> The sum, S, is given as 90. The number of terms, n, is given as 6. By plugging in, you can solve for the average: $\frac{90}{6} = 15$.

Notice that you do *not* need to know each term in the set to find the average!

Sometimes, using the average formula will be more involved. For example:

> If the average of the set $\{2, 5, 5, 7, 8, 9, x\}$ is 6.1, what is the value of x?

Plug the given information into the average formula, and solve for x:

$$A \times n = S \qquad \begin{aligned} (6.1)(7 \text{ terms}) &= 2 + 5 + 5 + 7 + 8 + 9 + x \\ 42.7 &= 36 + x \\ 6.7 &= x \end{aligned}$$

More complex average problems involve setting up two average formulas. For example:

> Sam earned a \$2,000 commission on a big sale, raising his average commission by \$100. If Sam's new average commission is \$900,

how many sales has he made?

To keep track of two average formulas in the same problem, you can set up a table. Sam's new average commission is \$900, and this is \$100 higher than his old average, so his old average was \$800.

Note that the Number and Sum columns add up to give the new cumulative values, but the values in the Average column do *not* add up:

	Average	×	Number	=	Sum
Old Total	800	×	n	=	$800n$
This Sale	2,000	×	1	=	2,000
New Total	900	×	$n + 1$	=	$900(n + 1)$

The right-hand column gives the equation you need:

$$800n + 2{,}000 = 900(n + 1)$$

$$800n + 2{,}000 = 900n + 900$$
$$1{,}100 = 100n$$
$$11 = n$$

Since you are looking for the new number of sales, which is $n + 1$, Sam has made a total of 12 sales.

Median: The Middle Number

Some GMAT problems feature another stats concept: the *median*, or middle value in a list of values place in increasing order. The median is calculated in one of two ways, depending on the number of data points in the set:

1. For sets containing an *odd* number of values, the median is the *unique middle value* when the data are arranged in increasing (or decreasing) order.
2. For sets containing an *even* number of values, the median is the *average (arithmetic mean) of the two middle values* when the data are arranged in increasing (or decreasing) order.

The median of the set {5, 17, 24, 25, 28} is the unique middle number, 24. The median of the set {3, 4, 9, 9} is the mean of the two middle values (4 and 9), or 6.5. Notice that the median of a set containing an *odd* number of values must be a value in the set. However, the median of a set containing an *even* number of values does not have to be in the set—and indeed will not be, unless the two middle values are equal.

Medians of Sets Containing Unknown Values

Unlike the arithmetic mean, the median of a set depends only on the one or two values in the middle of the ordered set. Therefore, you may be able to determine a specific value for the median of a set *even if one or more unknowns are present*.

For instance, consider the unordered set $\{x, 2, 5, 11, 11, 12, 33\}$. No matter whether x is less than 11, equal to 11, or greater than 11, the median of the resulting set will be 11. (Try substituting different values of x to see why the median does not change.)

By contrast, the median of the unordered set $\{x, 2, 5, 11, 12, 12, 33\}$ depends on x. If x is 11 or less, the median is 11. If x is between 11 and 12, the median is x. Finally, if x is 12 or more, the median is 12.

Standard Deviation

The mean and median both give “average” or “representative” values for a set, but they do not tell the whole story. It is possible for two sets to have the same average but to differ widely in how spread out their values are. To describe the spread, or variation, of the data in a set, you use a different measure: the standard deviation (SD).

Standard deviation indicates how far from the average (mean) the data points typically fall. Therefore:

- A small SD indicates that a set is clustered closely around the average (arithmetic mean) value.
- A large SD indicates that the set is spread out widely, with some points appearing far from the mean.

Consider the sets {5, 5, 5, 5}, {2, 4, 6, 8}, and {0, 0, 10, 10}. These sets all have the same mean value of 5. You can see at a glance, though, that the sets are very different, and the differences are reflected in their SDs. The first set has a SD of 0 (no spread at all), the second set has a moderate SD, and the third set has a large SD.

	Set 1	Set 2	Set 3
	{5, 5, 5, 5}	{2, 4, 6, 8}	{0, 0, 10, 10}
Difference from the mean of 5 (in absolute terms)	{0, 0, 0, 0} average spread = 0 SD = 0 An SD of 0 means that all the numbers in the set are equal.	{3, 1, 1, 3} average spread = 2 SD = moderate (technically, SD = $\sqrt{5} \approx 2.24$)	{5, 5, 5, 5} average spread = 5 SD = large (technically, SD = 5) If every absolute difference from the mean is equal, then the SD equals that difference.

You might be asking where the $\sqrt{5}$ comes from in the technical definition of SD for the second set. The good news is that you do not need to know—

the GMAT will not ask you to calculate a specific SD unless a shortcut exists, such as knowing that the SD is 0 if all of the numbers in the set are identical. If you just pay attention to what the *average spread* is doing, you'll be able to answer all GMAT standard deviation problems, which involve either 1) *changes* in the SD when a set is transformed, or 2) *comparisons* of the SDs of two or more sets. Just remember that the more spread out the numbers, the larger the SD.

If you see a problem focusing on changes in the SD, ask yourself whether the changes move the data closer to the mean, farther from the mean, or neither. If you see a problem requiring comparisons, ask yourself which set is more spread out from its mean.

Following are some sample problems to help illustrate SD properties.

1. Which set has the greater standard deviation: {1, 2, 3, 4, 6} or {441, 442, 443, 444, 445}?
2. If each data point in a set is increased by 7, does the set's standard deviation increase, decrease, or remain constant?
3. If each data point in a set is increased by a factor of 7, does the set's standard deviation increase, decrease, or remain constant? (Assume that the set consists of different numbers.)

Answers can be found on the following page.

Answer Key

1. **The first set has the greater SD.** One way to understand this is to observe that the gaps between its numbers are, on average, slightly bigger than the gaps in the second set (because the last two numbers are 2 units apart). Another way to resolve the issue is to observe that the set {441, 442, 443, 444, 445} would have the same standard deviation as {1, 2, 3, 4, 5}. Replacing 5 with 6, which is farther from the mean, will increase the SD of that set.
2. **The SD will not change.** "Increased by 7" means that the number 7 is *added* to each data point in the set. This transformation will not affect any of the gaps between the data points, and thus it will not affect how far the data points are from the mean. If the set were plotted on a number line, this transformation would merely slide the points 7 units to the right, taking all the gaps, and the mean, along with them.
3. **The SD will increase.** "Increased by a *factor* of 7" means that each data point is multiplied by 7. This transformation will make all the gaps between points 7 times as big as they originally were. Thus, each point will fall 7 times as far from the mean. The SD will increase by a factor of 7. Why did the problem specify that the set consists of different numbers? If each data point in the set was the same, then the SD would be 0. Multiplying each data point by 7 would still result in a set of identical numbers, and an identical SD of 0.

Problem Set

1. The average of 11 numbers is 10. When one number is eliminated, the average of the remaining numbers is 9.3. What is the eliminated number?

2. Given the set of numbers {4, 5, 5, 6, 7, 8, 21}, how much higher is the mean than the median?

3. The class mean score on a test was 60, and the standard deviation was 15. If Elena's score was within 2 standard deviations of the mean, what is the lowest score she could have received?

4. Matt gets a $1,000 commission on a big sale. This commission alone raises his average commission by $150. If Matt's new average commission is $400, how many sales has Matt made?

Save the below problem set for review, either after you finish this book or after you finish all of the Quant books that you plan to study.

5. If the average of x and y is 50, and the average of y and z is 80, what is the value of $z - x$?

6. $S = \{1, 2, 5, 7, x\}$

   If x is a positive integer, is the mean of set S greater than 4?

   (1) The median of set S is greater than 2.
   (2) The median of set S is equal to the mean of set S.

7. {9, 12, 15, 18, 21}

   Which of the following pairs of numbers, when added to the set above, will increase the standard deviation of the set?

   I. 14, 16

II. 9, 21
III. 15, 100

(A) II only
(B) III only
(C) I and II
(D) II and III
(E) I, II, and III

Solutions

1. **17:** If the average of 11 numbers is 10, their sum is 11 × 10 = 110. After one number is eliminated, the average is 9.3, so the sum of the 10 remaining numbers is 10 × 9.3 = 93. The number eliminated is the difference between these sums: 110 – 93 = 17.

2. **2:** The mean of the set is the sum of the numbers divided by the number of terms: 56 ÷ 7 = 8. The median is the middle number: 6. Therefore, the mean to the median, 8, is 2 greater than 6.

3. **30:** Elena's score was within 2 standard deviations of the mean. Since one standard deviation is 15, her score is no more than 15 × 2 = 30 points from the mean. The lowest possible score she could have received, then, is 60 – 30, which is equal to 30.

4. **5:** Before the $1,000 commission, Matt's average commission was $250; you can express this algebraically with the equation $S = 250n$.

After the sale, the sum of Matt's commissions increased by $1,000, the number of sales made increased by 1, and his average commission was $400. You can express this algebraically with the equation:

$$S + 1{,}000 = 400(n + 1)$$

$$\begin{aligned} 250n + 1{,}000 &= 400(n + 1) \\ 250n + 1{,}000 &= 400n + 400 \\ 150n &= 600 \\ n &= 4 \end{aligned}$$

Before the big sale, Matt had made 4 sales. Including the big sale, Matt has made 5 sales.

5. **60:** The sum of two numbers is twice their average. Therefore:

$$x + y = 100 \qquad\qquad y + z = 160$$
$$x = 100 - y \qquad\qquad z = 160 - y$$

Substitute these expressions for z and x:

$$z - x = (160 - y) - (100 - y) = 160 - y - 100 + y = 160 - 100 = 60$$

Alternatively, pick smart numbers for x and y. Let $x = 50$ and $y = 50$ (this is an easy way to make their average equal to 50). Since the average of y and z must be 80, you have $z = 110$. Therefore, $z - x = 110 - 50 = 60$.

6. **(B):** Like any other statement or question about the mean of a fixed number of data points, the prompt question can be rephrased to a question about the sum of the numbers in the set. Plug the known values into the equation Sum = Average × Number: is $(1 + 2 + 5 + 7 + x)/5 > 4$?

is $15 + x > 20$?

is $x > 5$?

For reference below, note also that the mean of S is $\frac{1+2+5+7+x}{5} = \frac{15+x}{5} = 3 + \frac{x}{5}$.

(1) INSUFFICIENT: For the median of the set to be greater than 2, x must also be greater than 2. If x were less than or equal to 2, the median would be 2. If x is 3 or 4, then the average of the set will be less than 4. However, if x is greater than or equal to 5, the average of the set will be greater than 4. This statement is insufficient.

(2) SUFFICIENT: This statement is a bit trickier to deal with. You can express the mean as $3 + \frac{x}{5}$, but the median depends on the value of x. However, note that you know the median must be an integer. You know that x must be an integer, so all of the elements in the set are integers. There are an odd number of elements in the set, so one of the elements of the set will be the median.

If the mean equals the median, then you know that $3 + \frac{x}{5}$ must also equal an integer. For that to be the case, you know that x must be a multiple of 5.

Now you can begin testing different values of x:

x	median	mean $\left(3+\frac{x}{5}\right)$
5	5	4
10	5	5

You know $x = 10$, and the average is greater than 4. Statement (2) is sufficient.

7. **(D) II and III:** Fortunately, you do not need to perform any calculations to answer this question. The mean of the set is 15. Take a look at each Roman Numeral:

> I. The numbers 14 and 16 are both very close to the mean (15). Additionally, they are closer to the mean than four of the numbers in the set, and will reduce the spread around the mean. This pair of numbers will reduce the standard deviation of the set.
>
> II. The numbers 9 and 21 are relatively far away from the mean (15). Adding them to the list will increase the spread of the set and increase the standard deviation.
>
> III. While adding the number 15 to the set would actually decrease the standard deviation (because it is the same as the mean of the set), the number 100 is so far away from the mean that it will greatly increase the standard deviation of the set. This pair of numbers will increase the standard deviation.

Chapter 7
of
Word Problems

Weighted Averages

In This Chapter…

Chapter 7

Weighted Averages

The regular formula for averages applies only to sets of data consisting of individual values that are equally weighted—that is, all of the values "count" equally towards the average. Some averages, however, are weighted more heavily towards certain data points.

For example, imagine that your teacher tells you that your mid-term exam will count for 40% of your grade and your final exam will count for 60% of your grade. If you can score a perfect 100 on only one of those components, which one would you want it to be?

Your final exam, of course! It counts more heavily towards your final grade. Next, imagine that you score 100 on your final exam but only 80 on your mid-term exam. What is the weighted average of those two scores?

Any average has to be between the two starting points, in this case 100 and 80. The *regular* average would be 90. Can you tell whether the weighted average is higher or lower than the regular average of 90?

In this case, you do have enough information to tell. The final exam counts for more than 50% of your final score, so the weighted average should be closer to the final exam score of 100 than to 80. The weighted average must be between 90 and 100.

There are two ways to calculate the exact value: *algebraically* or via the *teeter-totter*.

The Algebraic Method

To solve algebraically, set up an equation in which you multiply each exam score by its weight. You scored 100 on your final exam and it has a 60%, or $\frac{3}{5}$, weighting. You scored 80 on your mid-term and it has a 40%, or $\frac{2}{5}$, weighting. Now add them together:

$$100\left(\frac{3}{5}\right)+80\left(\frac{2}{5}\right)=?$$
$$60+32=92$$

The weighted average is 92.

If you prefer just to memorize the formula, go ahead. If, on the other hand, you'd like to know how that formula works, read on—the below explanation will help you to remember how to calculate weighted averages algebraically.

Think of a regular average as one in which each item has exactly equal weight. If there are two items, both are weighted $\frac{1}{2}$. If your teacher weighted the two exams equally, then this would be the calculation:

$$100\left(\frac{1}{2}\right)+80\left(\frac{1}{2}\right)=50+40=90$$

That is, the average is 90, exactly halfway between 80 and 100. The initial equation could be rearranged in this way:

$$100\left(\frac{1}{2}\right)+80\left(\frac{1}{2}\right)=\frac{100+80}{2}$$

Is that starting to look familiar? That's the average formula: find the sum of the two numbers and divide by 2! Technically, regular averages all have these equal weightings, so you can always write the equation in the simplified form: $\frac{\text{sum}}{\text{\# of terms}}$.

Weighted averages, though, must include the individual weightings, so you'll always have a component multiplied by its weighting, and then the next component multiplied by its weighting, and so on.

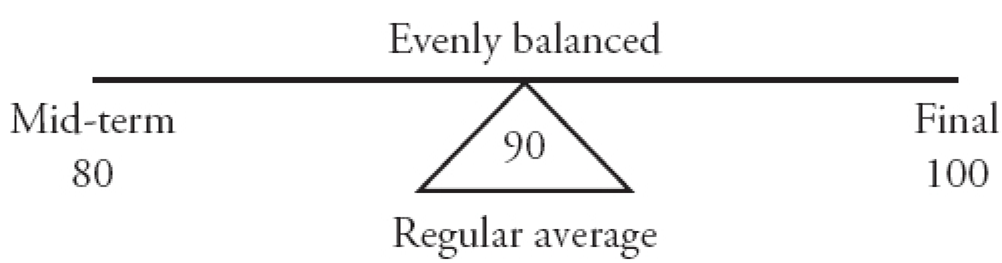
> Weighted Average = (component 1)(weighting 1) + (component 2)(weighting 2)

You can have more than two components, but the GMAT often sticks to just two.

The Teeter-Totter Method

The Teeter-Totter method is very efficient as long as you do understand what a weighted average is and how the concept works in general. If you struggle with the concepts, then you may want to stick with the algebraic method.

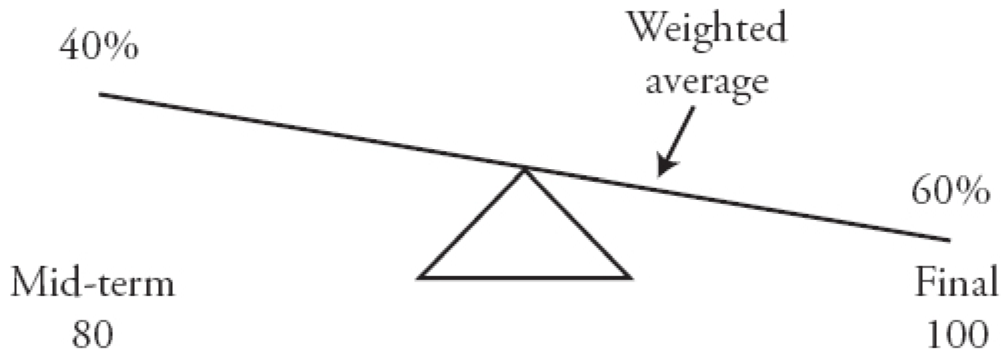
The problem is the same: You scored 100 on your final exam and it has a $\frac{3}{5}$ weighting. You scored 80 on your mid-term and it has a $\frac{2}{5}$ weighting. What is your final grade?



If you did not have a weighted average, then the teeter-totter would be perfectly balanced and you would have a regular average, halfway between

80 and 100.

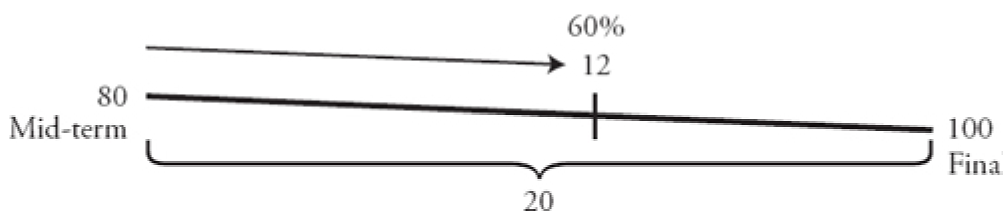
In this case, though, you do have a weighted average:



The weighted average “slips” down towards the heavier end of the teeter-totter, so you know that the weighted average must be between 90 and 100. On some Data Sufficiency questions, this is enough to determine that a statement is sufficient!

If you have to calculate the exact weighted average, here's what you do:

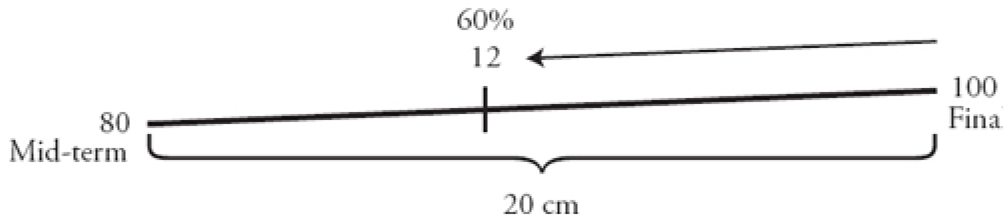
The two “ends” of the teeter-totter are 80 and 100; the difference is 20. The final exam is weighted more heavily, so it is responsible for 60% of that length of 20, and 60% of 20 is equal to 12. Start counting from the higher (lighter) end of the teeter-totter:



Imagine that the final exam weighs down his side of the teeter-totter by 12. Therefore, the average is 80 + 12, which sums to 92.

You don't need to draw out a full teeter-totter, but do draw at least the sloped line. Know which side is heavier. Calculate the “length” of the line and use it to calculate the value of the heavier weighting (in this case, 12); then start counting from the lighter side (80). In this case, you add: 80 + 12 = 92.

If the situation were reversed, and the score of 80 had the 60% weighting, then you would subtract at that last step instead:



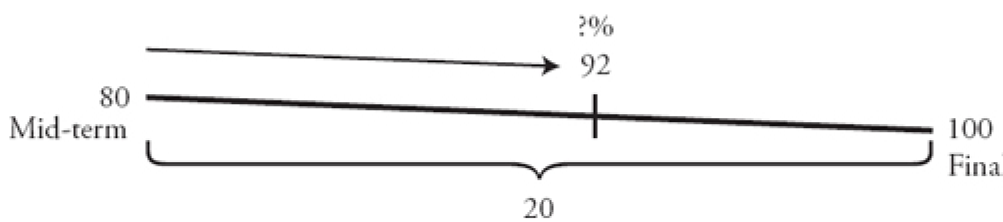
In this case, the mid-term exam is pulling the average 12 units towards him, so start counting from the higher (lighter) end, 100. Subtract from that end: 100 – 12 = 88.

In both cases, do draw the sloped line and place the final number in roughly the appropriate position. That step will tell you whether you should add from the smaller end (80 + 12) or subtract from the larger end (100 – 12).

What if the problem changed the given information? Try this:

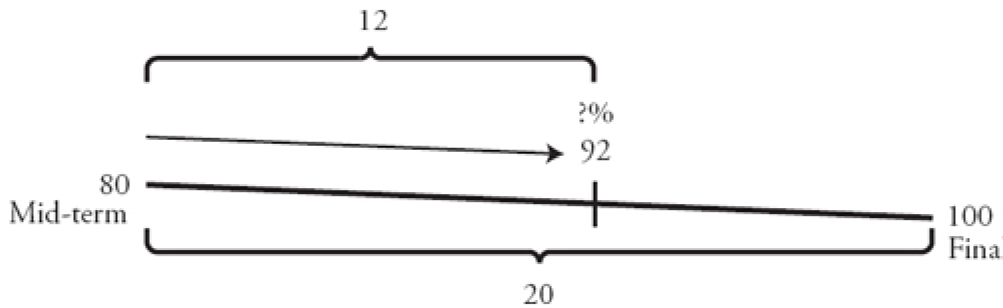
> You score 80 on your mid-term exam and 100 on your final exam. Only these two exams make up your final grade of 92. How heavily did your teacher weight the final exam?

First, draw your teeter-totter:



In this case, you still know the two end points (and the length), but now you're given the final average of 92 and you have to figure out the weighting that results in that average.

Because the weighted average is closer to 100 than to 80, you know that 100 is the heavier weight, so your final exam should be weighted more than 50%. Because the problem asks you to find that weight, find the longer of the two distances, between 80 and 92:



The weighting of the heavier (final exam) side is the fractional part 12 over the total length 20: $\frac{12}{20}=\frac{3}{5}$, which is equal to 60%.

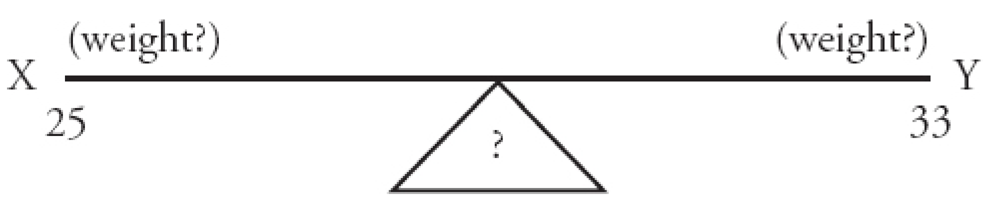
If the problem had asked you to calculate the weighting of the less-heavily weighted value, the mid-term exam, then you would find the shorter of the two distances, 8, and divide by the total distance, 20: $\frac{8}{20}=\frac{2}{5}$, which is equal to 40%.

Try this Data Sufficiency problem:

> The average number of students per class at School X is 25 and the average number of students per class at School Y is 33. Is the average number of students per class for both schools combined less than 29?
>
> (1) There are 12 classes in School X.
> (2) There are more classes in School X than in School Y.

Because this is a Data Sufficiency problem, there's a very good chance that you will not have to complete the calculations. In this case, try the teeter-totter method.

First, the question stem provides this information:



You don't know how to tilt the teeter-totter, because you don't know enough information yet. The question itself, though, implies something very intriguing.

If the two schools are equally weighted, then the “regular” average would be 29. If the average is less than 29, then what would you know?

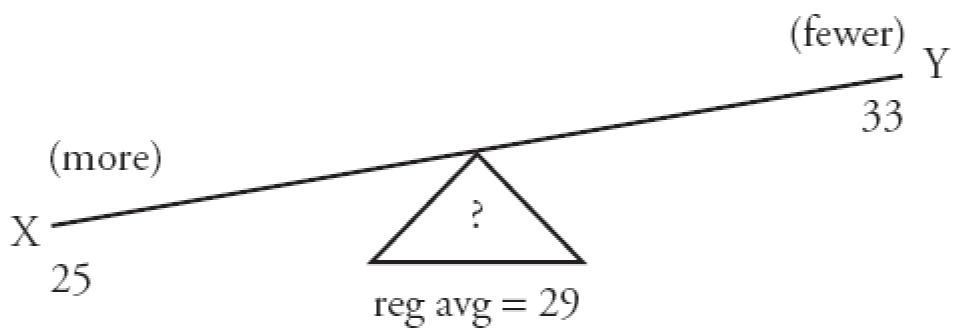
Then, the teeter-totter would have to be tilted down towards School X; this school would be weighted more heavily. Keep this in mind as you examine the statements:

(1) There are 12 classes in School X.

This statement doesn't provide any information about School Y, so it's impossible to tell whether one school is weighted more heavily. Statement (1) is not sufficient.

(2) There are more classes in School X than in School Y.

If there are more classes over at School X, then the weighted average has to tilt down towards this school.



As a result, the weighted average has to be less than the “regular” average of 29.

Statement (2) is sufficient; the correct answer is **(B)**.

On weighted average problems, you can choose whether to use the algebraic method or the teeter-totter method. Try both out on some OG problems and decide which method works better for you.

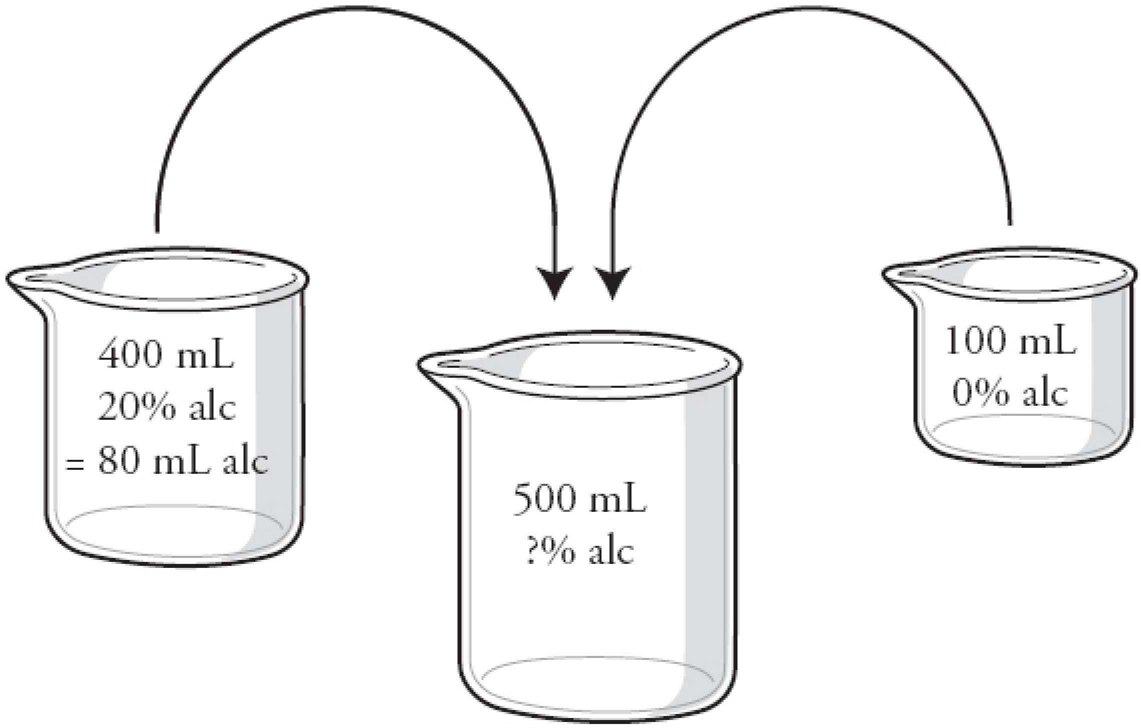
Mixtures, Percents, and Ratios

Percents and ratios can also show up in weighted average problems, particularly in the form of mixtures.

First, try this regular mixtures problem (you don't need to calculate a weighted average for this one):

> A 400 mL solution is 20% alcohol by volume. If 100 mL of water is added, what is the new concentration of alcohol, as a percent of volume?
>
> (A) 5%
> (B) 10%
> (C) 12%
> (D) 12.5%
> (E) 16%

To start, you have two liquid solutions: a 400 mL solution that is 20% alcohol and 80% something else, and a 100 mL solution that is 100% water (and therefore 0% alcohol).



You can actually calculate the mL of alcohol in the 400 mL beaker: 20% of 400 is 80 mL. The 100 mL beaker doesn't contribute any alcohol at all, so the 500 mL beaker contains a total of 80 mL of alcohol. The big beaker, then, is $\frac{80}{500} = \frac{8}{50} = \frac{16}{100} = 16\%$ alcohol. The correct answer is **(E)**.

In this case, only one of the two beakers contributed alcohol to the mixture. What happens when both parts of the problem contribute to the desired mixture?

Try this out:

> Kris-P cereal is 10% sugar by weight, whereas healthier but less delicious Bran-O cereal is 2% sugar by weight. To make a delicious and healthy mixture that is 4% sugar, what should be the ratio of Kris-P cereal to Bran-O cereal, by weight?
>
> (A) 1:2
> (B) 1:3
> (C) 1:4
> (D) 3:1

(E) 4:1

You can use the algebraic method or the teeter-totter—your choice. Both solutions are shown below.

The question asks for a ratio. Note that you don't necessarily need to know the real values of something in order to find a ratio. Call the weight of Kris-P cereal k and the weight of Bran-O cereal b.

To set up an equation:

$$0.1k + 0.02b = 0.04(k + b)$$

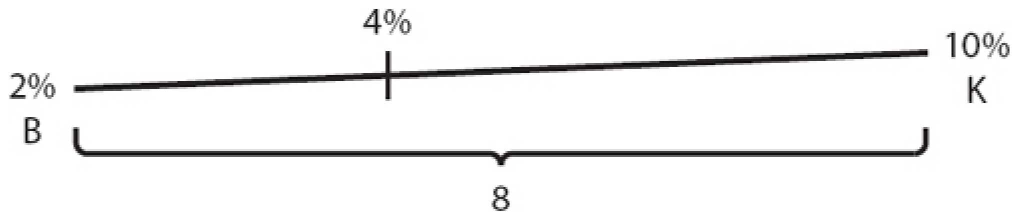
Kris-P is weighted 10% and Bran-O is weighted 2%. The final mixture is weighted 4% and note that the weight of the final mixture is the sum of the two components, k and b.

Because the question asks for the ratio of k to b, manipulate the equation to solve for $\frac{k}{b}$. First, multiply the whole equation by 100 to get rid of the decimals. Then simplify from there:

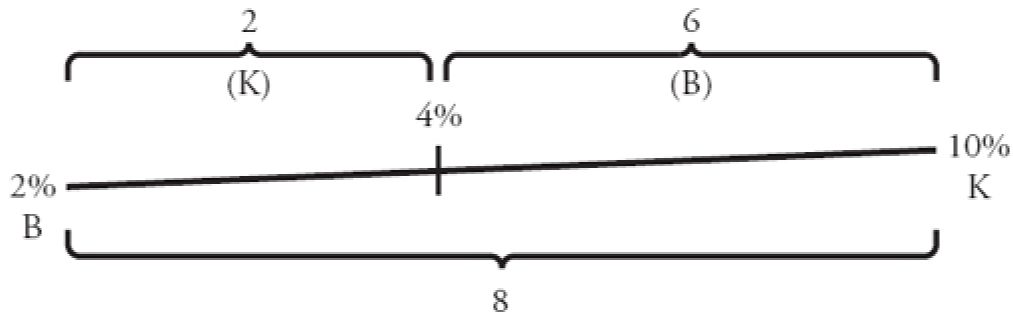
$$\begin{aligned} 10k + 2b &= 4(k + b) \\ 10k + 2b &= 4k + 4b \\ 6k &= 2b \\ \frac{k}{b} &= \frac{2}{6} = \frac{1}{3} \end{aligned}$$

The ratio of Kris-P to Bran-O is 1:3. The correct answer is **(B)**.

To use your teeter-totter, start drawing:



Because 4% is closer to 2%, you know that the Bran-O side is heavier. You also know the “length” is 10 – 2 = 8. Find the two smaller pieces of the line split by the weighted average:



The smaller number, 2, is associated with the less-heavily-weighted end (Kris-P, 10%). The larger number, 6, is associated with the more-heavily-weighted end (Bran-O, 2%). This will always be true: the larger portion always goes with the heavier end of the teeter-totter.

Therefore, the ratio of Kris-P to Bran-O is 2 : 6, or 1 : 3, which is **(B)**.

Again, you can choose whether to use algebra or the teeter-totter; try them both out and see what works best for you.

Problem Set

1. A professional gambler has won 40% of his 25 poker games for the week so far. If, all of a sudden, his luck changes and he begins winning 80% of the time, how many more games must he play to end up winning 60% of all his games for the week?

2. A charitable association sold an average of 66 raffle tickets per member. Among the female members, the average was 70 raffle tickets. The male to female ratio of the association is 1:2. What was the average number of tickets sold by the male members of the association?

3. Tickets to a play cost $10 for children and $25 for adults. If 90 tickets were sold, were more adult tickets sold than children's tickets?

    (1) The average revenue per ticket was $18.

    (2) The revenue from ticket sales exceeded $1,600

Save the below problem set for review, either after you finish this book or after you finish all of the Quant books that you plan to study.

4. A feed store sells two varieties of birdseed: Brand A, which is 40% millet and 60% sunflower, and Brand B, which is 65% millet and 35% safflower. If a customer purchases a mix of the two types of birdseed that is 50% millet, what percent of the mix is Brand A?

5. On a particular exam, the boys in a history class averaged 86 points and the girls in the class averaged 80 points. If the overall class average was 82 points, what was the ratio of boys to girls in the class?

6. A mixture of “lean” ground beef (10% fat) and “super-lean” ground beef (3% fat) has a total fat content of 8%. What is the ratio of “lean” ground beef to “super-lean” ground beef?

Solutions

1. **25 more games:** This is a weighted averages problem. You can set up a table to calculate the number of games the gambler must play to obtain a weighted average win rate of 60%:

Poker Games	First 25 Games	Remaining Games	Total
Wins	$(0.4)25 = 10$	$(0.8)x$	$(0.6)(25 + x)$
Losses			
Total	**25**	x	**25 +** x

Thus, $10 + 0.8x = (0.6)(25 + x)$, $10 + 0.8x = 15 + 0.6x$, $0.2x = 5$, $x = 25$.

2. **58 tickets:** You can answer this question without doing a lot of calculation. Women sold an average of 70 raffle tickets, which is 4 higher than the total average of 66. Each woman is selling, therefore, an average of +4 tickets over the group average. You also know that the ratio of men to women is 1 : 2. There are twice as many women and the total amount of extra tickets that the women sell needs to be canceled out by lower-than-average sales from the men. The positive difference from the average for women multiplied by 2 must cancel with the negative difference from the average for men multiplied by 1. If you call the difference from the average for men *m* then:

$$1 \times m + 2 \times (+4) = 0$$
$$m + 8 = 0$$
$$m = -8$$

The men sold an average of 8 fewer tickets than the total average so subtract: 66 – 8 = 58.

3. **(D).** First things first: how would you recognize that this question is about weighted averages? The two different prices for tickets are like two different data points, and the number of tickets sold will act as the weight.

You can use weighted averages to work through the statements. Statement (1) may seem familiar by now: $18 is closer to $25 than to $10. That means that there must have been more adult tickets sold than children's tickets. Statement 1 is sufficient. Cross off answers (B), (C), and (E) on your grid.

Statement (2) says that the total revenue from ticket sales exceeded $1,750. Where did that number come from?

To figure that out, look at the information in the question stem again. Notice that the question actually told you that 90 tickets were sold. Consider two extreme scenarios:

> 90 children's tickets sold = 90 × $10 = $900 revenue
> 90 adult tickets sold = 90 × $25 = (100 × $25) – (10 × $25) = $2,500 – $250 = $2,250 revenue

If there were an equal number of adult and children's tickets sold, the revenue would be an average of $900 and $2,250, or $1,575. That's the connection to weighted averages. If the revenue is greater than $1,750, it is closer to $2,500 than to $1,000, which means more adult tickets must have been sold. Statement (2) is also sufficient. The correct answer is **(D)**.

4. **60%:** This is a weighted averages problem. You can set up a table to calculate the answer, and assume that you purchased 100 lbs of Brand A:

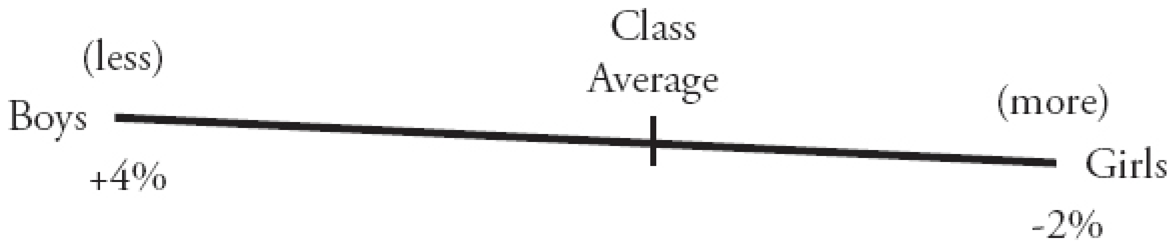
Pounds (lbs)	Brand A	Brand B	Total
Millet	40	$0.65x$	$(0.5)(100 + x)$
Other stuff	60	$0.35x$	$(0.5)(100 + x)$
Total birdseed	100	x	$100 + x$

Thus, $40 + 0.65x = (0.5)(100 + x)$, $40 + 0.65x = 50 + 0.5x$, $0.15x = 10$, $x = \frac{1,000}{15}$.

$$\text{Therefore, Brand A is } \frac{100}{100+\frac{1,000}{15}} = \frac{100}{\frac{1,500}{15}+\frac{1,000}{15}} = \frac{1,500}{2,500} = 60\% \text{ of the total.}$$

5. **1:2:** The boys in the class scored 4 points higher on average than the entire class. Similarly, the girls scored 2 points lower on average than the

class. You can draw a teeter-totter to answer the question. Set up the starting info:



There are more girls because a 2-points difference is smaller than a 4-point difference. What's the actual ratio?

The "length" of the line is 4 + 2 = 6. The girls side "pulls" the average away from boys by 4 points, and so girls are responsible for $\frac{4}{6}$ of the overall length.

But wait! That's a fraction, not a ratio! $\frac{4}{6}$ shows the part-to-whole relationship: 4 out of 6 points in the score spread are attributed to the girls. The boys are responsible for the other 2 out of 6 points in the spread. So the ratio of boys to girls is 2 to 4, or 1:2.

6. **5:2:** The question asks for the ratio of the two types of beef, so you don't need to worry about the actual amount of beef.

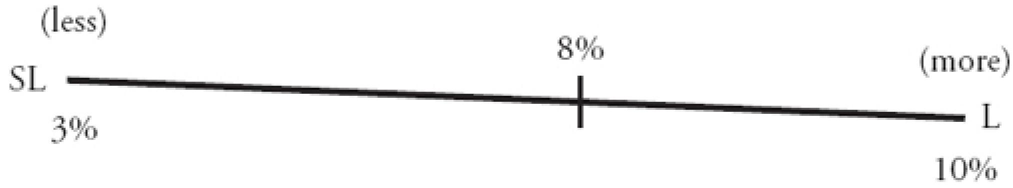
To set this problem up algebraically, first set up an equation, letting L be "lean" beef and S be "super-lean" beef:

$$0.1L + 0.03S = 0.08(L + S)$$

The question asks for the ratio of L to S, or. First, multiply the equation by 100 to get rid of the decimals, then solve for:

$$10L + 3S = 8(L + S)$$
$$10L + 3S = 8L + 8S$$
$$2L = 5S$$

Alternatively, you can draw a teeter-totter to answer the question. Set up the starting information:



There's more lean ground beef, because 8% is closer to 10%. What's the actual ratio?

The “length” of the line is 10 – 3 = 7. The lean side “pulls” the average a total of 8 – 3 = 5 units towards the lean side, so that side is responsible for $\frac{5}{7}$ of the overall average.

But wait! That's a fraction, not a ratio. A fraction is a part-to-whole relationship: 5 parts are lean out of the total 7 parts. The other 2 parts are super-lean. The ratio of lean to super-lean, then, is 5:2.

Chapter 8
of
Word Problems

Consecutive Integers

In This Chapter...

Chapter 8

Consecutive Integers

Consecutive integers are integers that follow one after another from a given starting point, without skipping any integers. For example, 4, 5, 6, and 7 are consecutive integers, but 4, 6, 7, and 9 are not. There are many other types of consecutive patterns. For example:

Consecutive even integers: 8, 10, 12, 14
(8, 10, 14, and 16 is incorrect, as it skips 12)

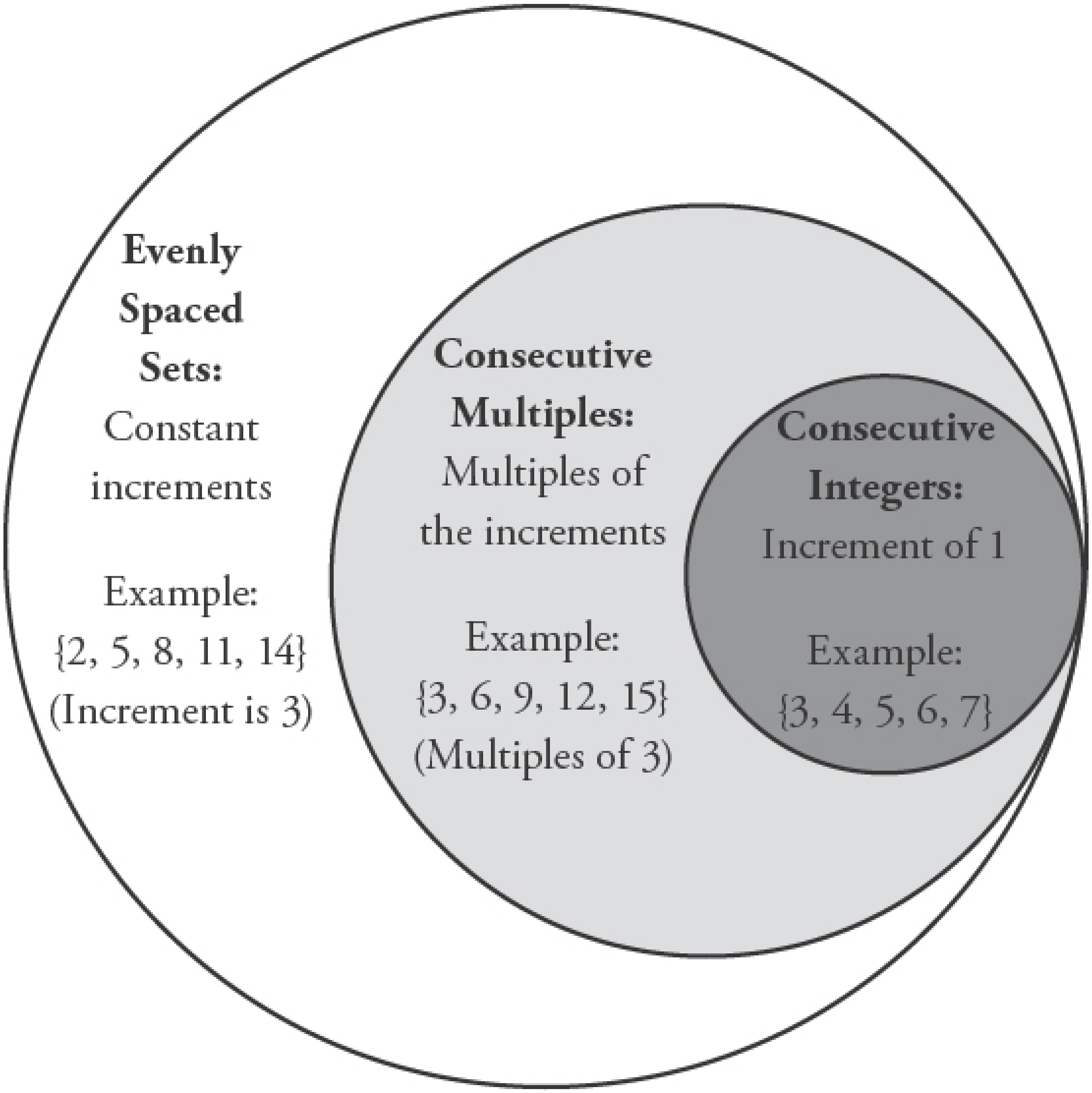
Consecutive primes: 11, 13, 17, 19
(11, 13, 15, and 17 is wrong, as 15 is not prime)

Evenly Spaced Sets

First, think about **evenly spaced sets** the values of the numbers in the set go up or down by the same amount (the **increment**) from one item in the sequence to the next. For instance, the set {4, 7, 10, 13, 16} is evenly spaced because each value increases by 3 over the previous value.

Sets of **consecutive multiples** are special cases of evenly spaced sets: all of the values in the set are multiples of the increment. For example, in the set {12, 16, 20, 24}, the values increase from one to the next by 4, and each element is a multiple of 4. Sets of consecutive multiples must be composed of integers.

Sets of **consecutive integers** are special cases of consecutive multiples: all of the values in the set increase by 1, and all integers are multiples of 1. For example, {12, 13, 14, 15, 16} is a set of consecutive integers.



Counting Integers: Add 1 Before You Are Done

How many integers are there from 6 to 10? Four, right? Wrong! There are actually five integers from 6 to 10. Count them: 6, 7, 8, 9, 10. It is easy to forget that you have to include (or, in GMAT lingo, be inclusive of) extremes. In this case, both extremes (the numbers 6 and 10) must be counted. When you merely subtract ($10 - 6 = 4$), you are forgetting to include the first extreme (6), as it has been subtracted away (along with 5, 4, 3, 2, and 1).

Do you have to methodically count each term in a long consecutive pattern? No. Just remember that if both extremes should be counted, you need to add 1 before you are done. For example:

> How many integers are there from 14 to 765, inclusive?

The formula is **(Last − First + 1):** 765 − 14 + 1 = 752.

This works easily enough if you are dealing with consecutive integers. Sometimes, however, the question will ask about consecutive multiples.

In this case, if you just subtract the largest number from the smallest and add one, you will be overcounting. For example, "All of the even integers between 12 and 24" yields 7 integers: 12, 14, 16, 18, 20, 22, and 24. However, (Last − First + 1) would yield (24 − 12 + 1) = 13, which is too large. How do you amend this? Since the items in the list are going up by increments of 2 (you are counting only the even numbers), you need to divide (Last − First) by 2. Then, add the 1 before you are done:

> (Last − First) ÷ Increment + 1 = (24 − 12) ÷ 2 + 1 = 6 + 1 = 7

For consecutive multiples, the formula is **(Last − First) ÷ Increment + 1**. The bigger the increment, the smaller the result, because there is a larger gap between the numbers you are counting.

Sometimes, it is easier to list the terms of a consecutive pattern and count them, especially if the list is short or if one or both of the extremes are omitted. For example:

> How many multiples of 7 are there between 100 and 150?

Here it may be easiest to list the multiples: 105, 112, 119, 126, 133, 140, 147. Count the number of terms to get the answer: 7. Alternatively, you could find that 105 is the first number, 147 is the last number, and 7 is the increment:

Number of terms = (Last − First) ÷ Increment + 1 = (147 − 105) ÷ 7 + 1 = 6 + 1 = 7

Properties of Evenly Spaced Sets

The following properties apply to all evenly spaced sets:

1. The arithmetic mean (average) and median are equal to each other. For example:

   What is the arithmetic mean of 4, 8, 12, 16, and 20?

   In this example, the median is 12. Since this is an evenly spaced set, the arithmetic mean (average) is also 12.

   What is the arithmetic mean of 4, 8, 12, 16, 20, and 24?

   In this example, the median is the arithmetic mean (average) of the two middle numbers, or the average of 12 and 16. Thus, the median is 14. Since this is an evenly spaced set, the average is also 14.

2. The mean and median of the set are equal to the average of the FIRST and LAST terms. For example:

   What is the arithmetic mean of 4, 8, 12, 16, and 20?

   In this example, the arithmetic mean and median are both equal to (20 + 4) ÷ 2 = 12.

   What is the arithmetic mean of 4, 8, 12, 16, 20, and 24?

   In this example, the arithmetic mean and median are both equal to (24 + 4) ÷ 2 = 14.

   For all evenly spaced sets, the average equals **(First** + **Last)** ÷ **2**.

The Sum of Consecutive Integers

Consider this problem:

> What is the sum of all the integers from 20 to 100, inclusive?

Adding all those integers would take much more time than you have for a GMAT problem. Using the rules for evenly spaced sets, though, you can calculate more easily:

- The formula for the sum of an evenly spaced set is: Sum = Average × Number of Terms
- Average the first and last term to find the average of the set: $100 + 20 = 120$ and $120 \div 2 = 60$.
- Count the number of terms: $100 - 20 = 80$, plus 1 yields 81.
- Multiply the average by the number of terms to find the sum: $60 \times 81 = 4{,}860$.

Note some general facts about sums and averages of consecutive integers:

- The average of an odd number of consecutive integers (1, 2, 3, 4, 5) will always be an integer (3). This is because the "middle number" will be a single integer.
- On the other hand, the average of an even number of consecutive integers (1, 2, 3, 4) will never be an integer (2.5), because there is no true "middle number."

Problem Set

Solve these problems using the rules for consecutive integers.

1. How many terms are there in the set of consecutive integers from -18 to 33, inclusive?

2. What is the sum of all the positive integers up to 100, inclusive?

3. In a sequence of 8 consecutive integers, how much greater is the sum of the last four integers than the sum of the first four integers?

Save the below problem set for review, either after you finish this book or after you finish all of the Quant books that you plan to study.

4. If the sum of the last 3 integers in a set of 6 consecutive integers is 624, what is the sum of the first 3 integers of the set?

5. If the sum of the last 3 integers in a set of 7 consecutive integers is 258, what is the sum of the first 4 integers?

6. The operation ==> is defined by x ==> $y = x + (x + 1) + (x + 2) \ldots + y$. For example, 3 ==> 7 = 3 + 4 + 5 + 6 + 7. What is the value of (100 ==> 150) − (125 ==> 150)?

Solutions

1. **52:** $33 - (-18) = 51$. Then add 1 before you are done: $51 + 1 = 52$.

2. **5,050:** There are 100 integers from 1 to 100, inclusive: $(100 - 1) + 1$. (Remember to add 1 before you are done.) The number exactly in the middle is 50.5. (You can find the middle term by averaging the first and last terms of the set.) Therefore, multiply 100 by 50.5 to find the sum of all the integers in the set: $100 \times 50.5 = 5{,}050$.

3. **16:** Think of the set of eight consecutive integers as follows: n, $(n + 1)$, $(n + 2)$, $(n + 3)$, $(n + 4)$, $(n + 5)$, $(n + 6)$, and $(n + 7)$.

First, find the sum of the first four integers:

$$n + (n + 1) + (n + 2) + (n + 3) = 4n + 6$$

Then, find the sum of the next four integers:

$$(n + 4) + (n + 5) + (n + 6) + (n + 7) = 4n + 22$$

The difference between these two partial sums is:

$$(4n + 22) - (4n + 6) = 22 - 6 = 16$$

Another way you could solve this algebraically is to line up the algebraic expressions for each number so that you can subtract one from the other directly:

$$\begin{array}{ll} \text{Sum of the last four integers.} & (n+4) + (n+5) + (n+6) + (n+7) \\ \text{Less the sum of the first four integers.} & \underline{-\ [n + (n+1) + (n+2) + (n+3)]} \\ & 4 \quad + \quad 4 \quad + \quad 4 \quad + \quad 4 \quad = 16 \end{array}$$

Yet another way to see this outcome is to represent the eight consecutive unknowns with eight lines:

__ __ __ __ ⋮ __ __ __ __

Each of the first four lines can be matched with one of the second four lines, each of which is 4 greater:

__ __ __ __ ⋮ +4 +4 +4 +4

So the sum of the last four numbers is $4 \times 4 = 16$ greater than the sum of the first four.

Finally, you could pick numbers to solve this problem. For example, assume you pick 1, 2, 3, 4, 5, 6, 7, and 8. The sum of the first four numbers is 10. The sum of the last four integers is 26. Again, the difference is $26 - 10 = 16$.

4. **615:** Think of the set of integers as n, $(n + 1)$, $(n + 2)$, $(n + 3)$, $(n + 4)$, and $(n + 5)$. Thus, $(n + 3) + (n + 4) + (n + 5) = 3n + 12 = 624$. Therefore, $n = 204$.

The sum of the first three integers is: $204 + 205 + 206 = 615$.

Alternatively, another way you could solve this algebraically is to line up the algebraic expressions for each number so that you can subtract one from the other directly:

Sum of the last three integers.	$(n + 3) + (n + 4) + (n + 5)$
Less the sum of the first three integers.	$-\ [n \quad + (n + 1) + (n + 2)]$
	$3 + \quad 3 + \quad 3 = 9$

Thus, the sum of the last three numbers is 9 greater than the sum of the first three numbers, so the sum of the first three numbers is $624 - 9 = 615$.

Visually, you can represent the six consecutive unknowns with six lines:

$$\_\_\ \_\_\ \_\_\ \underbrace{\_\_\ \_\_\ \_\_}_{\text{Sum} = 624}$$

$$\text{Average} = 624 \div 3 = 208$$

$$\underbrace{\_\_\ \underline{205}\ \_\_}\ \_\_\ \underline{208}\ \_\_$$

$$\begin{aligned}\text{Sum} &= \text{Average} \times 3\\ &= 205 \times 3 = 615.\end{aligned}$$

5. **330:** Think of the set of integers as n, $(n + 1)$, $(n + 2)$, $(n + 3)$, $(n + 4)$, $(n + 5)$, and $(n + 6)$. $(n + 4) + (n + 5) + (n + 6) = 3n + 15 = 258$. Therefore, $n = 81$. The sum of the first four integers is $81 + 82 + 83 + 84 = 330$.

Alternatively, the sum of the first four integers is $4n + 6$. If $n = 81$, then $4n + 6 = 4(81) + 6 = 330$.

6. **2,800:** This problem contains two components: the sum of all the numbers from 100 to 150, and the sum of all the numbers from 125 to 150. Since you are finding the difference between these components, you are essentially finding just the sum of all the numbers from 100 to 124. You can think of this logically by solving a simpler problem: find the difference (1 ==> 5) − (3 ==> 5).

$$\begin{array}{rr} & 1+2+3+4+5\\ - & 3+4+5\\ \hline & 1+2 \end{array}$$

There are 25 numbers from 100 to 124 ($124 - 100 + 1$). Remember to add 1 before you are done! To find the sum of these numbers, multiply by the average term:

$$\frac{100+124}{2} = 112$$

$$25 \times 112 = 2{,}800$$

Chapter 9
of
Word Problems

Strategy: Draw It Out

In This Chapter…

Chapter 9

Strategy: Draw It Out

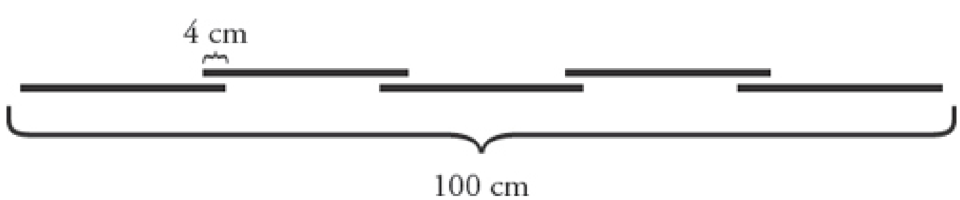
Numerous times throughout this book, you've learned how to draw out a story and find a more “real-world” approach to performing the necessary math. For example, many people find the teeter-totter method for weighted averages easier than the algebraic method.

This chapter contains additional examples of this **draw it out** approach.

Try this problem:

> Five identical pieces of wire are soldered together end-to-end to form one longer wire, with the pieces overlapping by 4 cm at each joint. If the wire thus made is exactly 1 meter long, how long, in centimeters, is each of the identical pieces? (1 meter = 100 cm)
>
> (A) 21.2
> (B) 22
> (C) 23.2
> (D) 24
> (E) 25.4

Draw out what the problem is describing:



That diagram is accurate but it might not exactly match what you'd been assuming in your head. Many people make the mistake of thinking that, because there are five pieces of wire, there are also five spots where the wires join. It turns out that there are only four joints!

The total length is 100 cm plus those extra amounts where the wires overlap. There are four overlaps of 4 cm each, or 16 cm of overlap. The total length shown in the picture, then, is 100 + 16 = 116 centimeters.

Because there are five wires, the length of each one is $\frac{116}{5} = 23.2$ cm. The correct answer is (C).

This problem can also be done algebraically: the relevant equation is $5x - 16 = 100$. Those who don't **draw it out**, though, are more likely to think that there are five joints and write the equation as $5x - 20 = 100$, which leads to answer (D) 24.

How Does Drawing It Out Work?

Essentially, there are multiple ways that you can avoid "textbook" math to get easier answers to many story problems on the GMAT. Whenever you find a problem that could actually be happening to someone in the real world, ask yourself: if I were in this situation right now, how would I try to figure out the answer?

You almost certainly wouldn't start writing equations. Instead, you'd sketch out the situation using a combination of logic, math, and just trying numbers out.

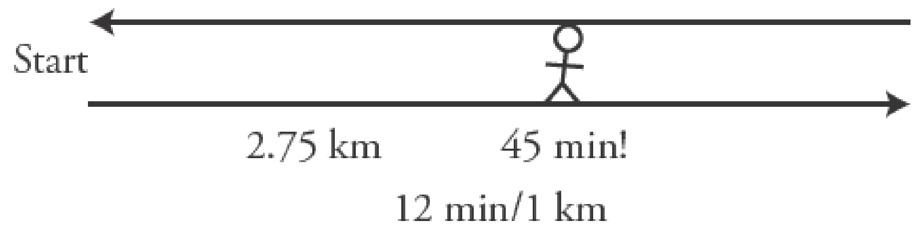
Try drawing out this problem:

> Annika hikes at a constant rate of 12 minutes per kilometer. She has hiked 2.75 kilometers east from the start of a hiking trail when she realizes that she has to be back at the start of the trail in 45 minutes. If Annika continues east, then turns around and retraces her path to reach the start of the trail in exactly 45 minutes, for how many kilometers total did she hike east?
>
> (A) 2.25
> (B) 2.75

(C) 3.25
(D) 3.75
(E) 4.25

This is a pretty nasty problem. There is an algebraic solution; you could also use an *RTD* chart to solve. The best way, though, is to draw it out.

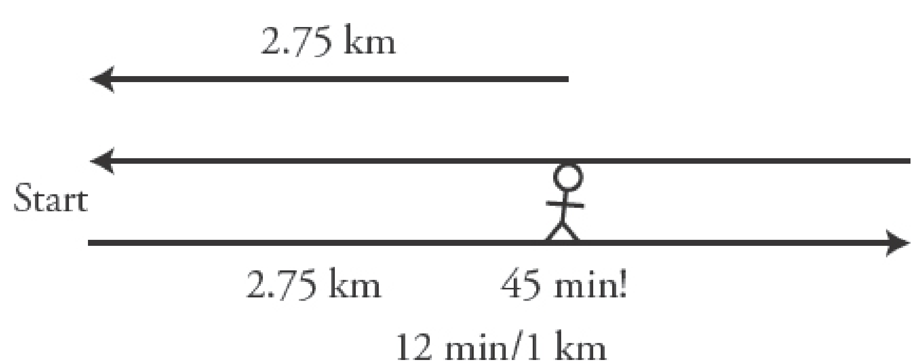
Here's Annika partway down her hiking trail, suddenly realizing that she's got 45 minutes till she needs to get back:



Pretend that isn't Annika at all—now, it's you. Are you going to whip out paper and pencil to start doing some algebra? No way. You're going to use real-world logic to figure out what to do.

What do you want to figure out? The question asks how far you traveled east. You know that part of the distance is 2.75 km, but you don't know how much further east you need to go before turning around. Glance at the answers. Hey, the answer can't be (A) and it can only be (B) if you have to turn around right now. Hmm.

First, if you didn't go a step further, how far would you have to go to get back?



You're going to need 2.75 kilometers to get back. How long is that going to take? You're going 1 kilometer every 12 minutes. Let's see, that would be 3 kilometers in (count it out) 12, 24, 36 minutes. How can you find the time for 2.75 kilometers?

If you hike 1 kilometer in 12 minutes, then you can hike a quarter of that distance in a quarter of the time: 0.25 kilometers in 3 minutes. Subtract from 3 kilometers in 36 minutes: 36 min − 3 min = 33 min.

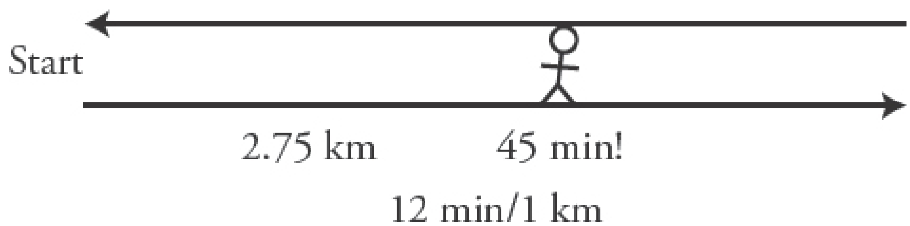
That takes care of 33 of your remaining 45 minutes. You have 12 minutes left.

How convenient! You already know that you can hike 1 kilometer in 12 minutes. You need to hike west *and* cover that same distance back, so you're going to continue for half a kilometer and then turn around.

Now, total: you'll hike 2.75 + 0.5 = 3.25 kilometers west before you turn around. The correct answer is (D).

Here's another way to draw it out:

Go back to the beginning.



Step back from the problem for a second—forget that you want 2.75 km plus some unknown distance. Look at your diagram. You're asking yourself how far you travel east before you turn around and exactly retrace your steps; that is, you want to know the distance of *half* of your trip. If you can calculate the total distance, you'll be done!

You know that you travel 2.75 kilometers to start. Then, you travel another 45 minutes at 12 minutes per kilometer. You can count it out again or do the straight math—12 × 4 = 48—so it takes 48 minutes to go 4 kilometers. How can you find the distance for 45 minutes?

If you hike 1 kilometer in 12 minutes, then you hike 0.25 kilometers in 3 minutes. Subtract: you hike 4 − 0.25 = 3.75 kilometers in 48 − 3 = 45 minutes.

Therefore, you travel 2.75 + 3.75 = 6.5 kilometers total. Half of that, 3.25 kilometers, is spent hiking east. The correct answer is **(D)**.

There are usually multiple ways to draw out the problem to get to the answer, so you aren't stuck trying to figure out the main textbook math method. Put yourself in the situation and ask yourself how you would go about this in the real world. You'd almost never start writing a bunch of equations; rather, you'd do “back of the envelope” arithmetic and estimation to get to the answer (or close enough!).

Rate and work problems, in particular, often lend themselves well to the draw it out method. Whenever you run across a problem that you think might work, try drawing out the problem to gain practice. At first, you may feel a bit slow, but you'll gain efficiency and accuracy with practice!

Write Out the Scenarios

Here's another variation on the draw it out method:

> During a week-long sale at a car dealership, the most number of cars sold on any one day was 12. If at least 2 cars were sold each day, was the average daily number of cars sold during that week more than 6?
>
> (1) During that week, the second smallest number of cars sold on any one day was 4.
> (2) During that week, the median number of cars sold was 10.

You're the manager of the car dealership and the owner has asked you to figure this out. How would you do so?

Start drawing out the scenarios. You know that the “highest” day was 12, but you don't know which day of the week that was. So how can you draw this out?

Glance at the statements. Neither statement provides information about a specific day of the week, either. Rather, they provide information about the least number of sales and the median number of sales.

The use of median is interesting. Maybe you should try organizing the number of sales from smallest to largest?

Draw out seven slots (one for each day) and add the information given in the question stem:

$\underline{\geq 2}$ ___ ___ ___ ___ ___ $\underline{12}$

The question asks whether the average number of daily sales for the week is more than 6. Because this is a yes/no DS question, test each statement to see whether it can give you both a "Yes, the average is more than 6" answer and a "No, the average is not more than 6" answer. If so, then you'll know the statement is insufficient.

> (1) During that week, the second smallest number of cars sold on any one day was 4.

Draw out a version of the scenario that includes statement (1):

$$\underline{\geq 2}\quad \underline{4}\quad \underline{}\quad \underline{}\quad \underline{}\quad \underline{}\quad \underline{12}$$

Can you find a way to make the average less than 6? Keep the first day at 2 and make the other days as small as possible:

$$\underline{\geq 2}\quad \underline{4}\quad \underline{4}\quad \underline{4}\quad \underline{4}\quad \underline{4}\quad \underline{12}$$

The sum of the numbers is 34. The average is $\frac{34}{7}$, which is a little less than 5.

Can you also make the average greater than 6? Try this:

$$\underline{3}\quad \underline{4}\quad \underline{12}\quad \underline{12}\quad \underline{12}\quad \underline{12}\quad \underline{12}$$

You may be able to eyeball that and tell it will be greater than 6. If not, calculate: the sum is 66, so the average is just less than 10.

Statement (1) is not sufficient because the average might be greater than or less than 6.

Cross off answers (A) and (D) and move to the statement (2).

> (2) During that week, the median number of cars sold was 10.

Again, draw out the scenario (using only the second statement this time!).

$$\underline{\geq 2}\quad \underline{}\quad \underline{}\quad \underline{10}\quad \underline{}\quad \underline{}\quad \underline{12}$$

Can you make the average less than 6? The three lowest days could each be 2. Then, the next three days could each be 10.

$$\underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{12}$$

The sum is 6 + 30 + 12 = 48. The average is $\frac{48}{7}$ just less than 7, but bigger than 6. The numbers cannot be made any smaller—you have to have a minimum of 2 a day. Once you hit the median of 10 in the middle slot, you have to have something greater than or equal to the median for the remaining slots to the right.

The smallest possible average is still bigger than 6, so this statement is sufficient to answer the question. The correct answer is (B).

If a problem talks about a set of numbers but doesn't give you the value of all of those numbers, try drawing out slots to represent each number in the set and stepping through the allowed possibilities. Make sure to test the extreme cases, depending upon what parameters the problem allows.

If a problem includes information about the median, you will probably want to order the numbers from least to greatest.

Maximizing and Minimizing

In other cases, a story problem might ask you to find the minimum or maximum possible value of something.

For example:

> There are enough available spaces on a school team to select at most $\frac{1}{3}$ of 50 students trying out for the team. What is the greatest number of students that could be rejected while still filling all available spaces for the team?
>
> (A) 32
> (B) 33
> (C) 34
> (D) 35
> (E) 36

You're asked to maximize the number of rejected students. You first have to fill all available spaces on the team, though. If at most $\frac{1}{3}$ of the students can be selected, then at most $\frac{50}{3}$, or $16\frac{2}{3}$, students can be selected. It's impossible to select $\frac{2}{3}$ of a person, of course, so the maximum possible is actually 16.

The maximum number of rejected students, then, is 50 − 16 = 34. The correct answer is **(C)**.

This problem has a hidden *integer constraint*: you have to assume that the numbers can only be integers, since you can't split a person. Many Word Problems have similar hidden constraints. Notice also that you have to be careful to round in the right direction—not up, but down. If the maximum number of available spaces is $16\frac{2}{3}$, then you cannot select 17 students. The maximum is 16.

Try another. How would you draw this problem out?

> Orange Computers is breaking up its conference attendees into groups. Each group must have exactly 1 person from Division A, 2 people from Division B, and 3 people from Division C. There are 20 people from Division A, 30 people from Division B, and 40 people from Division C at the conference. What is the smallest number of people who will not be able to be assigned to a group?
>
> (A) 12
> (B) 5
> (C) 2
> (D) 1
> (E) 0

You're in charge of the conference and you have to figure this out. First, jot down the given information on your scrap paper. Then, think about how you would figure this out in the real world.

Div.	Total People	Per Group
A	20	1

B	30	2
C	40	3

Try out some scenarios. If you start with Division A, you can make 20 groups.

Wait! Then you'd need 40 people from B and you have only 30. Hmm.

Okay, if you start with Division B, you can make 15 groups. Oh, but now you don't have enough people in C. Division C is the limiting factor—start there.

From Division C, you can make 13 groups of 3, using a total of 39 people. One person is left over.

Do you have enough Division B people? You need 13 × 2 = 26. Okay, there are 4 Division B people left over.

You'll also take 13 people from Division A, leaving 7 left over.

If you have 13 groups, then there are 1 + 4 + 7 = 12 people without a group. The correct answer is **(A)**.

Some max/min problems will be more like the first one, where the path of the math is fairly straightforward, but you have to make decisions along the way about maximizing or minimizing other pieces in order to get to your desired answer.

In others, the starting point won't be so obvious. As with the second problem, you'll test a couple of cases until you find the limiting factor, and then you'll follow the math from there.

In both cases, make sure to pay attention to any constraints, especially those not explicitly stated. People and cars and rabbits cannot be split into fractional parts.

When in Doubt, Draw It Out

Drawing out a problem, doing "back of the envelope" rough calculations, and using logic to get through the math are fantastic ways to get through some especially annoying story problems—and not just when you're in doubt! As you get better at working in this way, you'll find that these methods are very effective even when you do know how to do the "textbook" version of the math.

Your steps are straightforward: put yourself in the problem. Pretend that you have to figure this out in the real world. Then ask yourself what you would do in order to find the answer—even if just to estimate and narrow down the answers.

Problem Set

1. A bookshelf holds both paperback and hardcover books. The ratio of paperback books to hardcover books is 22 to 3. How many paperback books are on the shelf?

   (1) The number of books on the shelf is between 202 and 247, inclusive.

   (2) If 18 paperback books were removed from the shelf and replaced with 18 hardcover books, the resulting ratio of paperback books to hardcover books on the shelf would be 4 to 1.

2. a, b, and c are integers in the set $\{a, b, c, 51, 85, 72\}$. Is the median of the set greater than 70?

   (1) $b > c > 69$

   (2) $a < c < 71$

3. Velma has exactly one week to learn all 71 Japanese hiragana characters. If she can learn at most a dozen of them on any one day and will only have time to learn four of them on Friday, what is the least number of hiragana that Velma will have to learn on Saturday?

4. An eccentric casino owner decides that his casino should only use chips in \$5 and \$7 denominations. Which of the following amounts cannot be paid out using these chips?

   (A) \$31 (B) \$29 (C) \$26 (D) \$23 (E) \$21

Solutions

1. **(D):** The question stem states that the ratio of paperback books to hardcover books is 22 to 3. What does this tell you? Try writing out scenarios:

# of Paperbacks	# of Hardcovers	Total # of Books
22	3	25
44	6	50
66	9	75

Although the total number of books changes, it is always a multiple of 25. This makes sense because the number of paperbacks will be in multiples of 22, the number of hardbacks will be in multiples of 3, and 22 + 3 sums to 25.

(1) SUFFICIENT: There is only one multiple of 25 between 202 and 247, so the total number of books must be 225. You could stop here, because only one possible value for the total implies only one possible value for the number of paperback books.

(2) SUFFICIENT: Hmm. Since this statement gives very specific information, writing out scenarios seems tricky since it would be hard to find a scenario that fits. It's more direct to set up and solve algebraically.

Let p be the number of paperbacks and h be the number of hardbacks. From the question set, $\frac{p}{h} = \frac{22}{3}$. From this statement, you get: $\left(\frac{p-18}{h+18}\right) = \frac{4}{1}$. This gives you two equations and two unknowns, so it is possible to solve for p, and you could stop here.

However, for the sake of completeness, the calculation follows:

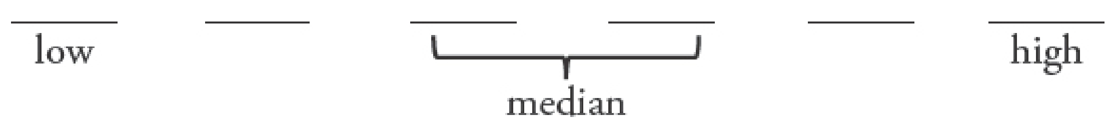
Cross-multiply both equations: $3p = 22h$, or $h = \frac{3p}{22}$, and $(p - 18) = 4(h + 18)$.

Substitute for p into the statement equation:

$$(p - 18) = 4((3p/22) + 18)$$
$$p - 18 = 6p/11 + 72$$
$$p - 6p/11 = 90$$
$$11p/11 - 6p/11 = 90$$
$$5p/11 = 90$$
$$p = (90)(11)/5$$
$$p = (18)(11) = 198$$

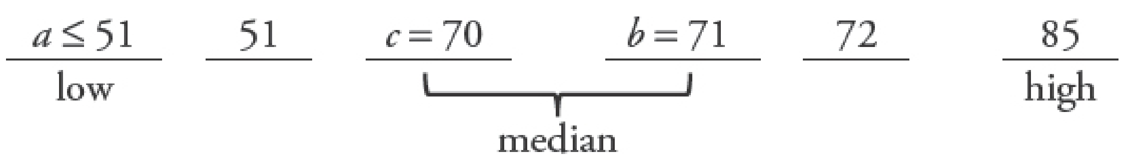
The correct answer is **(D)**.

2. **(A):** Draw out six spaces and imagine they contain values ordered from low to high. What would the median be? Since there are an even number of numbers, the median of a set of six integers is the average of the two middle terms (the 3rd and 4th) when the terms are placed in order from low to high.

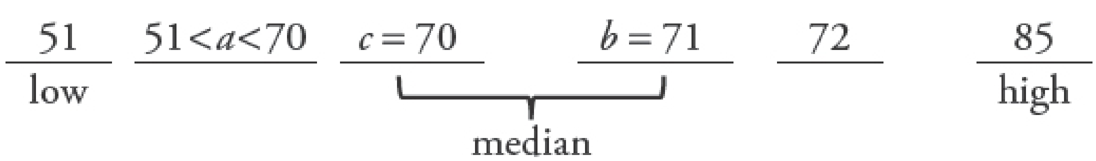


(1) SUFFICIENT: Look at the minimum case. If c is an integer greater than 69, the smallest c can be is 70. By similar logic, the smallest b could be is 71. In this case, the set is {51, 70, 71, 72, 85, a}. The only unknown is the value of a:

If $a \leq 51$, the ordered set is {a, 51, 70, 71, 72, 85}; median = (70 + 71)/2 = 70.5.



If 51 < $a \leq 70$, the ordered set is {51, a, 70, 71, 72, 85}; median = (70 + 71)/2 = 70.5.



If $a = 71$, the ordered set is {51, 70, a, 71, 72, 85}; median = (71 + 71)/2 = 71.

$$\underset{\text{low}}{\underline{\;51\;}} \quad \underline{c=70} \quad \underbrace{\underline{a=71} \qquad \underline{b=71}}_{\text{median}} \quad \underline{\;72\;} \quad \underset{\text{high}}{\underline{\;85\;}}$$

If a = 72, the ordered set is {51, 70, 71, 72, a, 85}; median = (71 + 72)/2 = 71.5.

$$\underset{\text{low}}{\underline{\;51\;}} \quad \underline{c=70} \quad \underbrace{\underline{b=71} \qquad \underline{\;72\;}}_{\text{median}} \quad \underline{a=72} \quad \underset{\text{high}}{\underline{\;85\;}}$$

If a > 72, the order of the first four terms doesn't change from the line above, so the median is 71.5.

In all cases, the median is greater than 70, so the answer is a definite “yes.”

Furthermore, if c and b are larger than the minimum case you tested, say c = 71 and b = 72 or c = 100 and b = 150, a quick check reveals that there is no a value that would make the median less than or equal to 70.

(2) INSUFFICIENT: Look at the maximum case. If c is an integer less than 71, the greatest c can be is 70. By similar logic, the greatest a could be is 69. In this case, the set is {b, 51, 69, 70, 72, 85}. The only unknown is the value of b:

If $b \leq 51$, the ordered set is {b, 51, 69, 70, 72, 85}; median = (69 + 70)/2 = 69.5.

$$\underset{\text{low}}{\underline{b \leq 51}} \quad \underline{\;51\;} \quad \underbrace{\underline{a=69} \qquad \underline{c=70}}_{\text{median}} \quad \underline{\;72\;} \quad \underset{\text{high}}{\underline{\;85\;}}$$

If b = 70, the ordered set is {51, 69, 70, 70, 72, 85}; median = (70 + 70)/2 = 70.

$$\underset{\text{low}}{\underline{\;51\;}} \quad \underline{a=69} \quad \underbrace{\underline{b=70} \qquad \underline{c=70}}_{\text{median}} \quad \underline{\;72\;} \quad \underset{\text{high}}{\underline{\;85\;}}$$

If b = 71, the ordered set is {51, 69, 70, 71, 72, 85}; median = (70 + 71)/2 = 70.5.

$$\underset{\text{low}}{51}\qquad a=69\qquad \underbrace{c=70\qquad b=71}_{\text{median}}\qquad 72\qquad \underset{\text{high}}{85}$$

In some cases, the median is greater than 70, but in other cases, it isn't. The answer is “maybe.”

Thus, the correct answer is **(A)**.

3. **7:** Draw it out! Draw seven slots and label them for the days of the week. The problem states that Velma will learn 4 hiragana on Friday and at most 12 on any other day. Finally, you want the least possible number that she will need to learn on Saturday:

≤12	≤12	≤12	≤12	≤12	4	least?
Sun	M	Tu	W	Th	F	Sat

To minimize the number of hiragana that Velma will have to learn on Saturday, consider the extreme case in which she learns *as many* hiragana *as possible* on the other days. If Velma learns the maximum of 12 hiragana on the other five days (besides Saturday), then she will have $67 - 5(12) = 7$ left for Saturday:

12	12	12	12	12	4	7
Sun	M	Tu	W	Th	F	Sat

4. **(D):** The payouts will have to be in the sum of some integer number of \$5 chips and some integer number of \$7 chips. Which of the answer choices cannot be the sum? One efficient way to eliminate choices is first to cross off any multiples of 7 and/or 5, which eliminates (E). Now, any other possible sums must have at least one 5 and one 7 in them. So you can subtract off 5's one at a time until you reach a multiple of 7. (It is easier to subtract 5's than 7's, because our number system is base-10.) Thus:

> Answer choice (A): $31 - 5 = 26$; $26 - 5 = 21$, a multiple of 7; this eliminates (A). (In other words, $31 = 3 \times 7 + 2 \times 5$.)
>
> Answer choice (B): $29 - 5 = 24$; $24 - 5 = 19$; $19 - 5 = 14$, a multiple of 7; this eliminates (B).
>
> Answer choice (C): $26 - 5 = 21$, a multiple of 7; this eliminates (C).

So the answer must be **(D)**, 23. You check by successively subtracting 5 and looking for multiples of 7: $23 - 5 = 18$, not a multiple of 7; $18 - 5 = 13$, also not a multiple of 7; $13 - 5 = 8$, not a multiple of 7; and no smaller result will be a multiple of 7 either.

Chapter 10
of
Word Problems

Extra Overlapping Sets and Consecutive Integers

In This Chapter…

Chapter 10

Extra Overlapping Sets and Consecutive Integers

Two Sets, Three Choices: Still Double-Set Matrix

Very rarely, you might need to consider more than two options for one or both of the dimensions of your chart. As long as each set of distinct options is complete and has no overlaps, you can extend the chart.

For instance, if respondents can answer "Yes," "No," or "Maybe" to a survey question, and the question specifies whether the respondents are male or female, then you might set up the following matrix:

	Yes	No	Maybe	Total
Female				
Male				
Total				

The set of three answer choices is complete (there are no other options). Also, the choices do not overlap (no respondent can give more than one response). So this extended chart is fine.

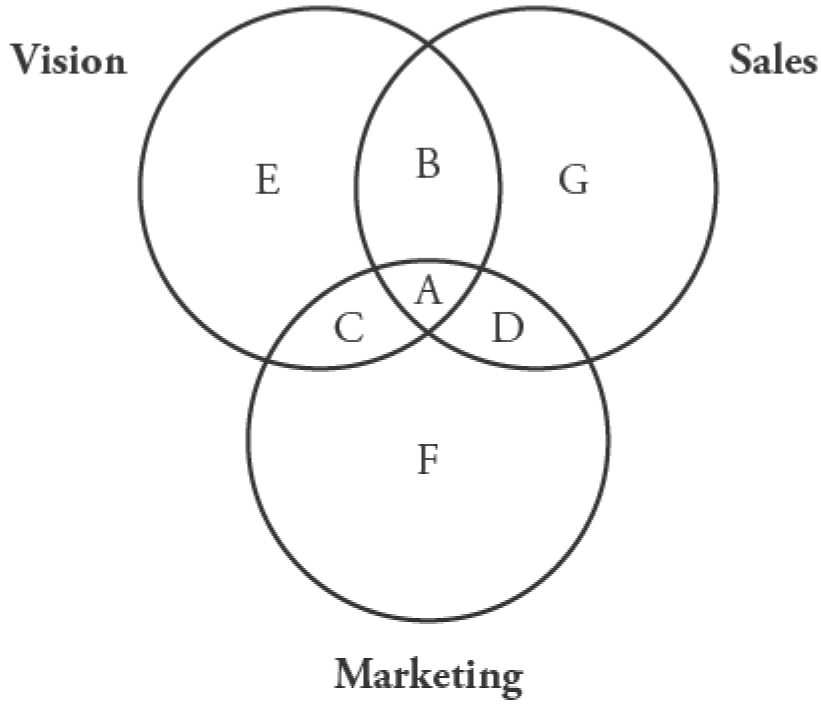
You rarely need to do real computation, but setting up an extended chart such as this can be helpful on certain Data Sufficiency problems, so that you can see what information is or is not sufficient to answer the given question.

Three-Set Problems: Venn Diagrams

Problems that involve three overlapping sets can be solved by using a Venn diagram. The three overlapping sets are usually three teams or clubs, and each person is either *on* or *not on* any given team or club. That is, there are only two choices for any club: member or not. For example:

> Workers are grouped by their areas of expertise and are placed on at least one team. There are 20 workers on the Marketing team, 30 on the Sales team, and 40 on the Vision team. Five workers are on both the Marketing and Sales teams, 6 workers are on both the Sales and Vision teams, 9 workers are on both the Marketing and Vision teams, and 4 workers are on all three teams. How many workers are there in total?

In order to solve this problem, use a Venn diagram instead of a double-set matrix.



Begin your Venn diagram by drawing three overlapping circles.

Notice that there are seven different sections in a Venn diagram. There is one innermost section **(A)** where all three circles overlap. This contains individuals who are on all three teams.

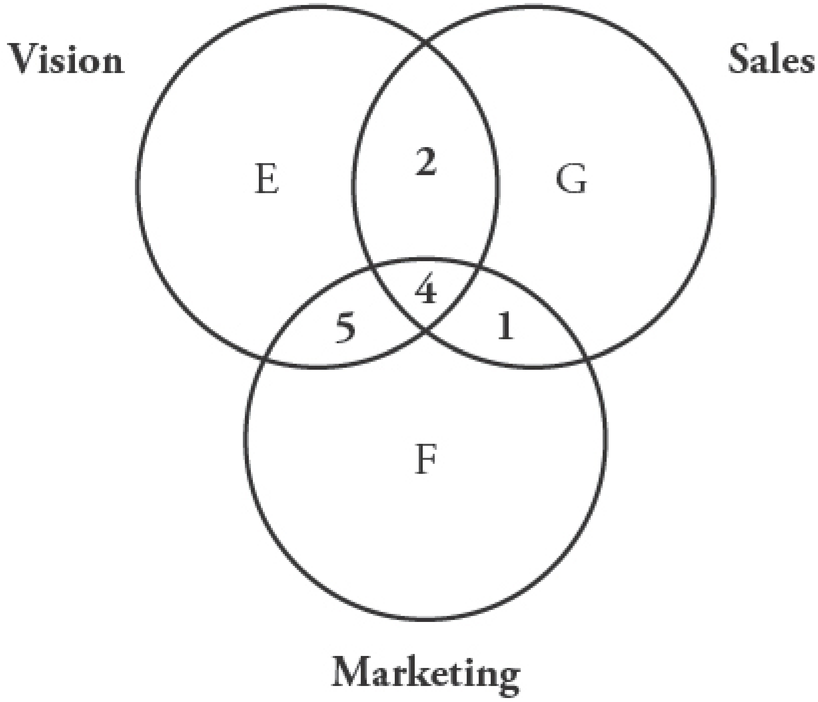
There are three sections **(B, C, and D)** where two circles overlap. These contain individuals who are on two teams. There are three non-overlapping sections **(E, F, and G)** that contain individuals who are on only one team.

Venn diagrams are easier to work with if you remember one simple rule: work from the inside out.

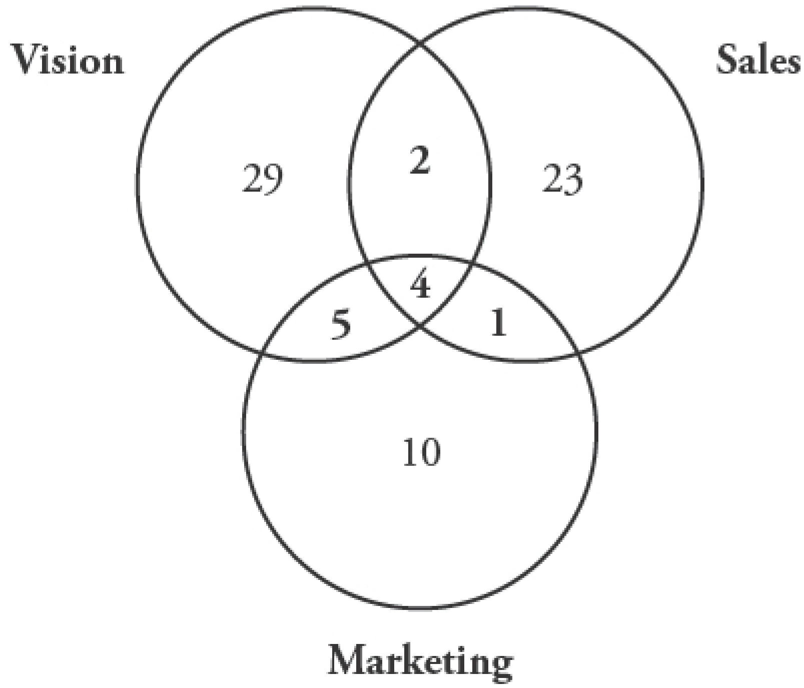
That is, it is easiest to begin by filling in a number in the innermost section **(A)**. Then, fill in numbers in the middle sections **(B, C, and D)**. Fill in the outermost sections **(E, F, and G)** last.

First: *Workers on all three teams.* Fill in the innermost circle. This is given in the problem as 4.

Second: *Workers on two teams.* Here you must remember to subtract those workers who are on all three teams. For example, the problem says that there are 5 workers on the Marketing and Sales teams. However, this includes the 4 workers who are on all three teams.



Therefore, in order to determine the number of workers who are on the Marketing and Sales teams exclusively, subtract the 4 workers who are on all three teams. You are left with $5 - 4 = 1$. The number of workers on the Marketing and Vision teams exclusively is $9 - 4 = 5$. The number of workers on the Sales and Vision teams exclusively is $6 - 4 = 2$.



Third: *Workers on one team only.* Here you must remember to subtract those workers who are on two teams and those workers who are on three teams. For example, the problem says that there are 20 workers on the Marketing team. But this includes the 1 worker who is on the Marketing and Sales teams, the 5 workers who are on the Marketing and Vision teams, and the 4 workers who are on all three teams. Subtract all of these workers to find that there are $20 - 1 - 5 - 4 = 10$ people who are on the Marketing team exclusively. There are $30 - 1 - 2 - 4 = 23$ people on the Sales team exclusively. There are $40 - 2 - 5 - 4 = 29$ people on the Vision team exclusively.

In order to determine the total, add all seven numbers together: $29 + 5 + 4 + 2 + 1 + 23 + 10$. Thus, there are 74 total workers.

Products of Consecutive Integers and Divisibility

Can you come up with a series of three consecutive integers in which none of the integers is a multiple of 3? Go ahead, try it! You will find that any set of three consecutive integers must contain one multiple of 3. The result is that the product of any set of three consecutive integers is divisible by 3, as shown below:

$$1 \times 2 \times 3 = 6 \qquad 4 \times 5 \times 6 = 120$$
$$2 \times 3 \times 4 = 24 \qquad 5 \times 6 \times 7 = 210$$
$$3 \times 4 \times 5 = 60 \qquad 6 \times 7 \times 8 = 336$$

According to the Factor Foundation Rule, every number is divisible by all the factors of its factors. If there is always a multiple of 3 in a set of three consecutive integers, the product of three consecutive integers will always be divisible by 3. Additionally, there will always be at least one multiple of 2 (an even number) in any set of three consecutive integers. Therefore, the product of three consecutive integers will also be divisible by 2. Thus, the product of three consecutive integers will always be divisible by 3!: $3 \times 2 \times 1 = 6$.

The same logic applies to a set of four consecutive integers, five consecutive integers, and any other number of consecutive integers. For instance, the product of any set of 4 consecutive integers will be divisible by 4!—$4 \times 3 \times 2 \times 1 = 24$—since that set will always contain one multiple of 4, at least one multiple of 3, and another even number (a multiple of 2).

This rule applies to any number of consecutive integers: The product of k consecutive integers is always divisible by k factorial ($k!$).

Sums of Consecutive Integers and Divisibility

Find the sum of any five consecutive integers:

$4 + 5 + 6 + 7 + 8 = 30$	Notice that both sums are multiples of 5.
$13 + 14 + 15 + 16 + 17 = 75$	In other words, both sums are divisible by 5.

You can generalize this observation. For any set of consecutive integers with an ODD number of items, the sum of all the integers is ALWAYS a multiple of the number of items. This is true because the sum equals the average multiplied by the number of items. The average of {13, 14, 15, 16, 17} is 15, so $15 \times 5 = 13 + 14 + 15 + 16 + 17$.

Find the sum of any four consecutive integers:

$1 + 2 + 3 + 4 = 10$	Notice that NEITHER sum is a multiple of 4.
$8 + 9 + 10 + 11 = 38$	In other words, both sums are NOT divisible by 4.

For any set of consecutive integers with an EVEN number of items, the sum of all the items is NEVER a multiple of the number of items. This is true because the sum equals the average multiplied by the number of items. For an even number of integers, the average is never an integer, so the sum is never a multiple of the number of items. The average of {8, 9, 10, 11} is 9.5, so $9.5 \times 4 = 8 + 9 + 10 + 11$. That is, $8 + 9 + 10 + 11$ is NOT a multiple of 4.

Consider this Data Sufficiency problem:

Is k^2 odd?

(1) $k - 1$ is divisible by 2.
(2) The sum of k consecutive integers is divisible by k.

Statement (1) tells you that $k - 1$ is even. Therefore, k is odd, so k^2 will be odd. SUFFICIENT.

Statement (2) tells you that the sum of k consecutive integers is divisible by k. Therefore, this sum divided by k is an integer. Moreover, the sum of k consecutive integers divided by k is the average (arithmetic mean) of that set of k integers. As a result, statement (2) is telling you that the average of the k consecutive integers is itself an integer:

$$\frac{(\text{Sum of } k \text{ integers})}{k} = (\text{Average of } k \text{ integers}) = \text{Integer}$$

If the average of this set of consecutive integers is an integer, then k must be odd. SUFFICIENT.

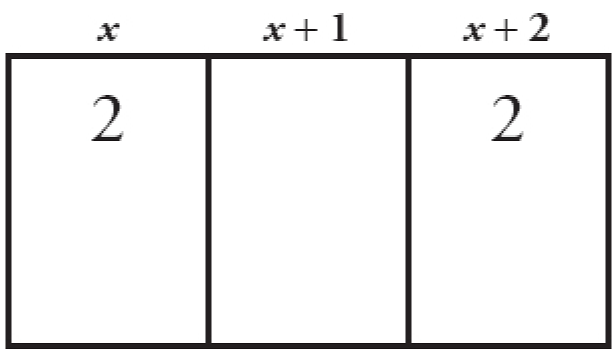
The correct answer is **(D)**.

Consecutive Integers and Divisibility

You can use prime boxes to keep track of factors of consecutive integers. (Refer to the Divisibility and Primes chapter of the *Number Properties* Strategy Guide for more information on prime boxes.) Consider the following problem:

If x is an even integer, is $x(x + 1)(x + 2)$ divisible by 4?

You know $x(x + 1)(x + 2)$ is the product of three consecutive integers, because x is an integer. If there is one even integer in a series of consecutive integers, the product of the series is divisible by 2. If there are two even integers in a series of consecutive integers, the product of the series is divisible by 4. Set up prime boxes:



If x is even, then $x + 2$ is even, so 2 is a factor of $x(x + 1)(x + 2)$ twice. Therefore, the product $2 \times 2 = 4$ is a factor of the product of the series. The answer to the question then is “yes.”

Scheduling

Scheduling problems, which require you to determine possible schedules satisfying a variety of constraints, can usually be tackled by careful consideration of **extreme possibilities**, usually the earliest and latest possible time slots for the events to be scheduled. Consider the following problem:

> How many days after the purchase of Product X does its standard warranty expire? (1997 is not a leap year.)
>
> (1) When Mark purchased Product X in January 1997, the warranty did not expire until March 1997.
>
> (2) When Santos purchased Product X in May 1997, the warranty expired in May 1997.

Rephrase the two statements in terms of extreme possibilities:

> (1) Shortest possible warranty period: Jan. 31 to Mar. 1 (29 days later) Longest possible warranty period: Jan. 1 to Mar. 31 (89 days later) Note that 1997 was not a leap year.
>
> (2) Shortest possible warranty period: May 1 to May 2, or similar (1 day later) Longest possible warranty period: May 1 to May 31 (30 days later)

Even taking both statements together, there are still two possibilities—29 days and 30 days—so both statements together are still insufficient.

Note that, had the given year been a leap year, the two statements together would have become sufficient! Moral of the story: *Read the problem very, very carefully.*

Problem Set

1. If r, s, and t are consecutive positive multiples of 3, is rst divisible by 27, 54, or both?

2. Is the sum of the integers from 54 to 153, inclusive, divisible by 100?

3. When it is 2:01pm Sunday afternoon in Nullepart, it is Monday in Eimissaan. When it is 1:00pm Wednesday in Eimissaan, it is also Wednesday in Nullepart. When it is noon Friday in Nullepart, what is the possible range of times in Eimissaan?

4. Is the average of n consecutive integers equal to 1?

   (1) n is even.

   (2) If S is the sum of the n consecutive integers, then $0 < S < n$.

5. Students are in clubs as follows: Science–20, Drama–30, and Band–12. No student is in all three clubs, but 8 are in both Science and Drama, 6 are in both Science and Band, and 4 are in Drama and Band. How many different students are in at least one of the three clubs?

6. If x, y, and z are consecutive integers, is $x + y + z$ divisible by 3?

7. A list kept at Town Hall contains the town's average daily temperature in Fahrenheit, rounded to the nearest integer. A particular completed month has either 30 or 31 days. How many days does the month have?

   (1) The median temperature is 73.5.

   (2) The sum of the average daily temperatures is divisible by 3.

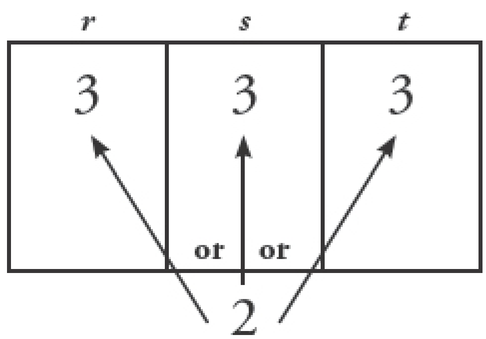
8. The 38 movies in the video store fall into the following three categories: 10 action, 20 drama, and 18 comedy. However, some movies are classified under more than one category: 5 are both action and drama, 3

are both action and comedy, and 4 are both drama and comedy. How many action–drama–comedy movies are there?

9. Of 60 children, 30 are happy, 10 are sad, and 20 are neither happy nor sad. There are 20 boys and 40 girls. If there are 6 happy boys and 4 sad girls, how many boys are neither happy nor sad?

Solutions

1. **Both:** Because *r*, *s*, and *t* are all multiples of 3, the product *rst* must have THREE 3's as factors. Additionally, at least one of the integers must be even, so the product will have a 2 as a factor, because every other multiple of 3 is even (for example, 3, **6**, 9, **12**, etc.). 27 = 3 × 3 × 3 can be constructed from the known prime factors and is therefore a factor of the product *rst*. 54 = 2 × 3 × 3 × 3 can also be constructed from the known prime factors and therefore is also factor of the product *rst*.



2. **No:** There are 100 integers from 54 to 153, inclusive. For any even number of consecutive integers, the sum of all the integers is *never* a multiple of the number of integers. Thus, the sum of the integers from 54 to 153 will not be divisible by 100.

3. **Anywhere from 10pm Friday to 1am Saturday:** The first statement tells you that the time in Eimissaan is at least 10 hours ahead of the time in Nullepart; given this information, the second statement tells you that the time in Eimissaan is at most 13 hours ahead of the time in Nullepart. (The second statement *by itself* could allow Nullepart time to be ahead of Eimissaan time, but that situation is already precluded by the first statement.) Therefore, the time in Eimissaan is between 10 and 13 hours ahead of the time in Nullepart.

4. **(D):** (1) SUFFICIENT: Statement (1) states that there is an even number of consecutive integers. This statement tells you nothing about the actual

values of the integers, but the average of an even number of consecutive integers will never be an integer. Therefore, the average of the *n* consecutive integers cannot equal 1.

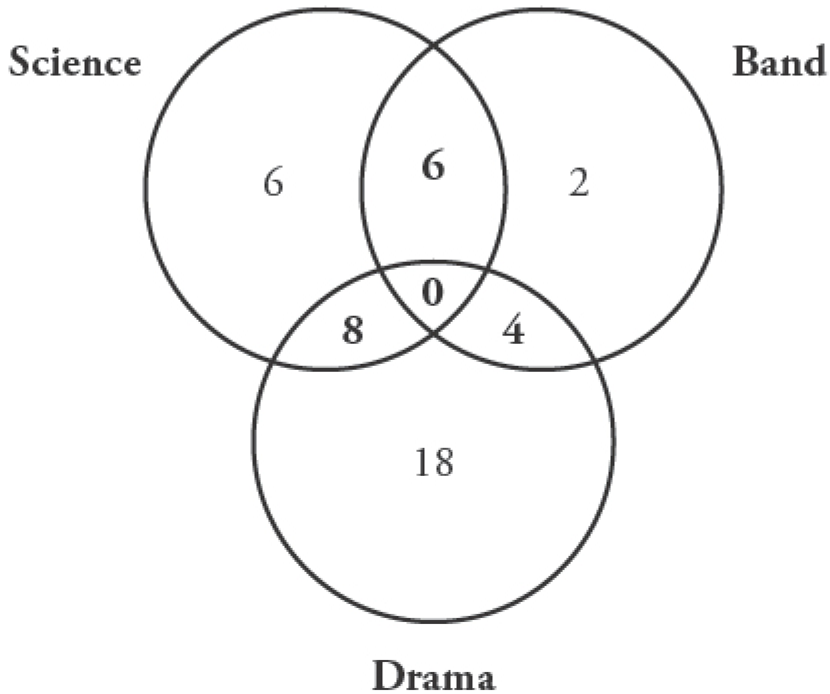
(2) SUFFICIENT: You know that the sum of the *n* consecutive integers is positive, but smaller than *n*. Perhaps the most straightforward way to interpret this statement is to express it in terms of the average of the *n* numbers, rather than the sum. Using the formula Average = Sum ÷ Number, you can reinterpret the statement by dividing the compound inequality by *n*:

$$0 < S < n \qquad \frac{0}{n} < \frac{S}{n} < \frac{n}{n} \qquad 0 < \frac{S}{n} < 1$$

This tells you that the average integer in set *S* is larger than 0 but less than 1. Therefore, the average number in the set does NOT equal 1, so the statement is sufficient. The correct answer is (D).

As a footnote, this situation can happen ONLY when there is an even number of integers, and when the "middle numbers" in the set are 0 and 1. For example, the set of consecutive integers {0, 1} has a median number of 0.5. Similarly, the set of consecutive integers {-3, -2, -1, 0, 1, 2, 3, 4} has a median number of 0.5.

5. **44 different students:** There are three overlapping sets here. Therefore, use a Venn diagram to solve the problem. First, fill in the numbers given in the problem, working from the inside out: no students in all three clubs, 8 in Science and Drama, 6 in Science and Band, and 4 in Drama and Band. Then, use the totals for each club to determine how many students are in only one club. For example, you know that there are 30 students in the Drama club. So far, you have placed 12 students in the circle that represents the Drama club (8 who are in Science and Drama, and 4 who are in Band and Drama). Therefore, 30 − 12 = 18, the number of students who are in only the Drama Club. Use this process to determine the number of students in just the Science and Band clubs as well. To find the number of students in at least one of the clubs, sum all the numbers in the diagram:



6 + 18 + 2 + 6 + 8 + 4 = 44.

6. **Yes:** For any odd number of consecutive integers, the sum of those integers is divisible by the number of integers. There are three consecutive integers (x, y, and z), so the rule applies in this case.

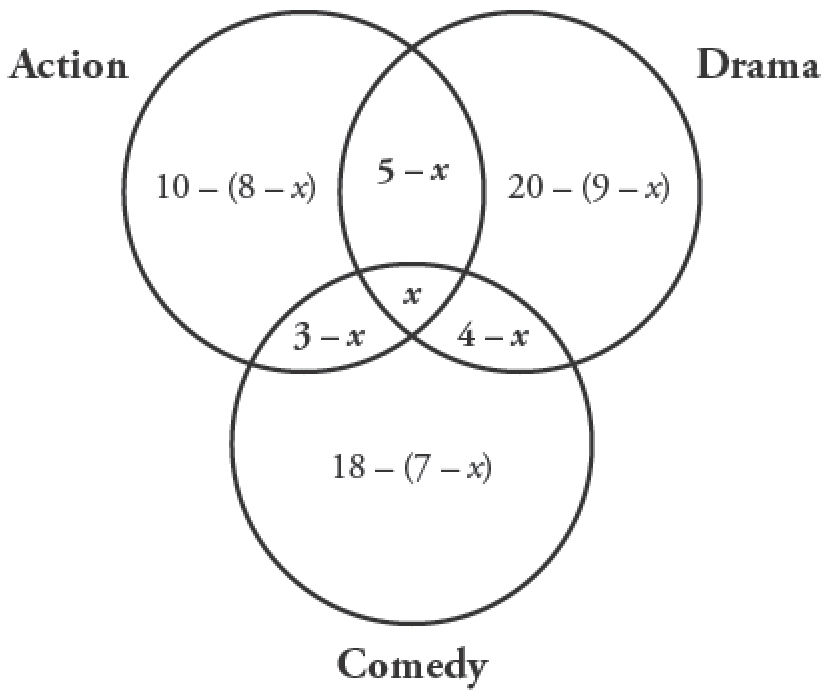
7. **(A):** This question is really about evens and odds. A list of values contains either 30 or 31 elements. Does the list have an even or an odd number of elements?

(1) SUFFICIENT: Since every item in the list is an integer, the only way for the median to be a non-integer is if there is an even number of items in the list (and therefore no middle term—in this case, the median is calculated as the average of the two middle terms). Therefore, the month must have an even number of days, so it must contain 30 days.

(2) INSUFFICIENT: The sum of either 30 or 31 values can be divisible by 3. Since there are no constraints on what the temperatures might be, it is perfectly possible to have a list of 30 values or a list of 31 values that add up to a multiple of 3. For example, if the temperature every day was 60 degrees, the sum of the temperatures would be divisible by 3 no matter how many days the month contained.

Therefore, the correct answer is **(A)**.

8. **2:** There are three overlapping sets here; therefore, use a Venn diagram to solve the problem. First, fill in the numbers given in the problem, working from the inside out. Assign the variable x to represent the number of action–drama–comedy movies. Then, create variable expressions, using the totals given in the problem, to represent the number of movies in each of the other categories. You know that there is a total of 38 movies; therefore, you can write the following equation to represent the total number of movies in the store:



$$
\begin{array}{lr}
 & 10 - 8 + x \\
 & 20 - 9 + x \\
 & 18 - 7 + x \\
 & 5 - x \\
 & 4 - x \\
 & 3 - x \\
+ & x \\
\hline
 & 36 + x = 38 \\
 & x = \ \ 2
\end{array}
$$

If you are unsure of the algebraic solution, you can also guess a number for *x* and fill in the rest of the diagram until the total number of movies reaches 38.

9. **8 boys:** Use a double-set matrix to solve this problem, with the “mood” set divided into three categories instead of only two. First, fill in the numbers given in the problem: of 60 children, 30 are happy, 10 are sad, and 20 are neither happy nor sad; 20 are boys and 40 are girls. You also know there are 6 happy boys and 4 sad girls. Therefore, by subtraction, there are 6 sad boys and there are 8 boys who are neither happy nor sad.

	Happy	Sad	Neither	Total
Boys	6	**6**	**8**	20
Girls		4		40
Total	30	10	20	60

Appendix A
of
Word Problems

Data Sufficiency

In This Chapter…

Appendix A

Data Sufficiency

Data Sufficiency (DS) problems are a cross between math and logic. Imagine that your boss just walked into your office and dumped a bunch of papers on your desk, saying, “Hey, our client wants to know whether they should raise the price on this product. Can you answer that question from this data? If so, which pieces do we need to prove the case?” What would you do?

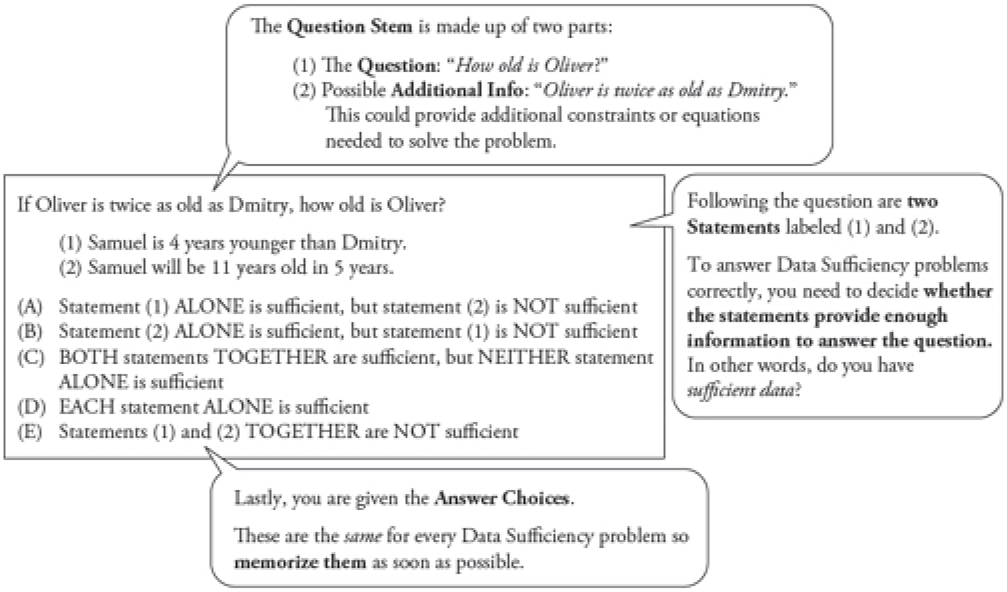
The client has asked a specific question: should the company raise the price? You have to decide which pieces of information will allow you to answer that question—or, possibly, that you don't have enough information to answer the question at all.

This kind of logical reasoning is exactly what you use when you answer DS questions.

How Data Sufficiency Works

If you already feel comfortable with the basics of Data Sufficiency, you may want to move quickly through this particular section of the chapter—but you are encouraged to read it. There are a few insights that you may find useful.

Every DS problem has the same basic form:



The question stem contains the question you need to answer. The two statements provide *given* information, information that is true. DS questions look strange but you can think of them as deconstructed Problem Solving (PS) questions. Compare the DS-format problem above to the PS-format problem below:

> Samuel is 4 years younger than Dmitry, and Samuel will be 11 years old in 5 years. If Oliver is twice as old as Dmitry, how old is Oliver?"

The two questions contain exactly the same information; that information is just presented in a different order. The PS question stem contains all of the givens as well as the question. The DS problem moves some of the givens down to statement (1) and statement (2).

As with regular PS problems, the given information in the DS statements is always true. In addition, the two statements won't contradict each other. In the same way that a PS question wouldn't tell you that $x > 0$ *and* $x < 0$, the two DS statements won't do that either.

In the PS format, you would go ahead and calculate Oliver's age. The DS format works a bit differently. Here is the full problem, including the answer choices:

> If Oliver is twice as old as Dmitry, how old is Oliver?

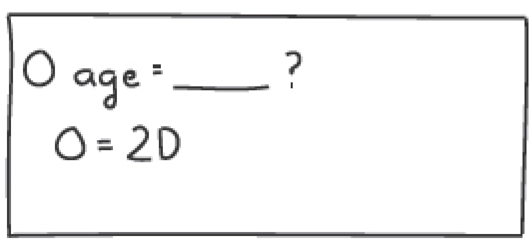
(1) Samuel is 4 years younger than Dmitry.
(2) Samuel will be 11 years old in 5 years.

(A) Statement (1) ALONE is sufficient, but statement (2) is NOT sufficient.
(B) Statement (2) ALONE is sufficient, but statement (1) is NOT sufficient.
(C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient.
(E) Statements (1) and (2) TOGETHER are NOT sufficient.

Despite all appearances, the question is not actually asking you to calculate Oliver's age. Rather, it's asking *which pieces of information* would allow you to calculate Oliver's age.

You may already be able solve this one on your own, but you'll see much harder problems on the test, so your first task is to learn how to work through DS questions in a systematic, consistent way.

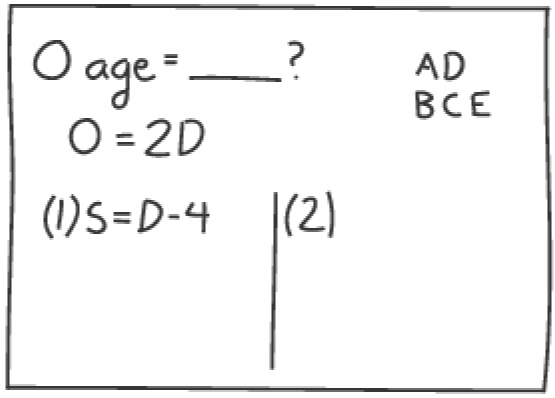
As you think the problem through, jot down information from the question stem:



Hmm. If they tell you Dmitry's age, then you can find Oliver's age. Remember that!

Take a look at the first statement. Also, write down the $\frac{AD}{BCE}$ answer grid (you'll learn why as you work through the problem):

(1) Samuel is 4 years younger than Dmitry.



The first statement doesn't allow you to figure out anyone's real age. Statement (1), then, is *not sufficient*. Now you can cross off the top row of answers, (A) and (D).

Why? Here's the text for answers (A) and (D):

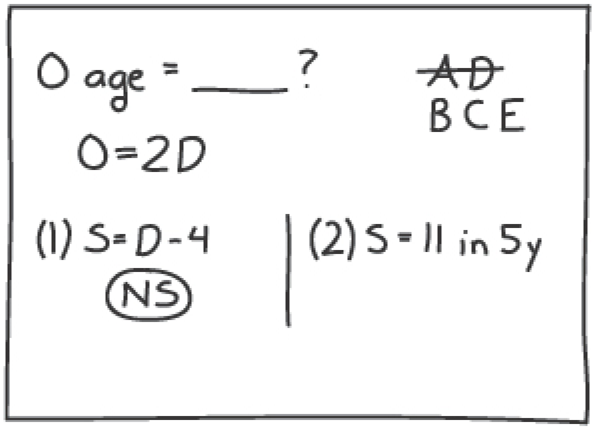
(A) Statement (1) ALONE is sufficient, but statement (2) is NOT sufficient.
(D) EACH statement ALONE is sufficient.

Both answers indicate that statement (1) is sufficient to answer the question. Because statement (1) is *not* sufficient to find Oliver's age, both (A) and (D) are wrong.

The answer choices will always appear in the order shown for the above problem, so any time you decide that statement (1) is not sufficient, you will always cross off answers (A) and (D). That's why your answer grid groups these two answers together.

Next, consider statement (2), but remember one tricky thing: forget what statement (1) told you. Because of the way DS is constructed, you must evaluate the two statements separately before you look at them together:

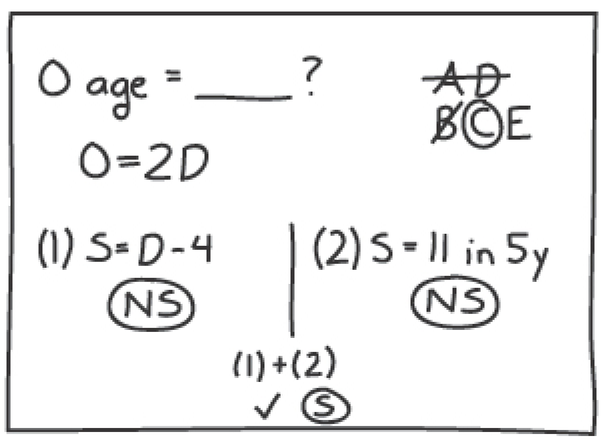
(2) Samuel will be 11 years old in 5 years.



It's useful to write the two statements side-by-side, as shown above, to help remember that statement (2) is separate from statement (1) and has to be considered by itself first.

Statement (2) does indicate how old Sam is now, but says nothing about Oliver or Dmitry. (Remember, you're looking *only* at statement (2) now.) By itself, statement (2) is not sufficient, so cross off answer (B).

Now that you've evaluated each statement by itself, take a look at the two statements together. Statement (2) provides Sam's age, and statement (1) allows you to calculate Dmitry's age if you know Sam's age. Finally, the question stem allows you to calculate Oliver's age if you know Dmitry's age:



As soon as you can tell that you *can* solve, put down a check mark or write an S with a circle around it (or both!). Don't actually calculate Oliver's age;

the GMAT doesn't give you any extra time to calculate a number that you don't need.

The correct answer is **(C)**.

The Answer Choices

The five Data Sufficiency answer choices will always be exactly the same (and presented in the same order), so memorize them before you go into the test.

Here are the five answers written in an easier way to understand:

(A) Statement (1) does allow you to answer the question, but statement (2) does not.

(B) Statement (2) does allow you to answer the question, but statement (1) does not.

(C) Neither statement works on its own, but you can use them *together* to answer the question.

(D) Statement (1) works by itself *and* statement (2) works by itself.

(E) Nothing works. Even if you use both statements together, you still can't answer the question.

Answer (C) specifically says that neither statement works on its own. For this reason, you are required to look at each statement by itself *and decide that neither one works* before you are allowed to evaluate the two statements together.

Here's an easier way to remember the five answer choices; we call this the “twelve-ten” mnemonic (memory aid):

1	only statement 1
2	only statement 2
T	together
E	either one
N	neither/nothing

Within the next week, memorize the DS answers. If you do a certain number of practice DS problems in that time frame, you'll likely memorize the answers without conscious effort—and you'll solidify the DS lessons you're learning right now.

Starting with Statement (2)

If statement (1) looks hard, start with statement (2) instead. Your process will be the same, except you'll make one change in your answer grid.

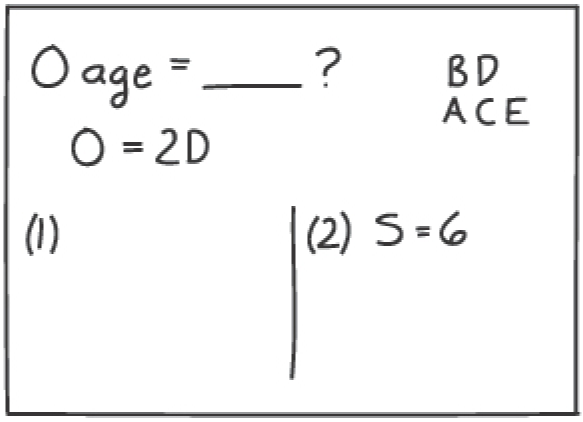
Try this problem:

> If Oliver is twice as old as Dmitry, how old is Oliver?
>
> (1) Two years ago, Dmitry was twice as old as Samuel.
> (2) Samuel is 6 years old.

(From now on, the answer choices won't be shown. Start memorizing!)

Statement (1) is definitely more complicated than statement (2), so start with statement (2) instead. Change your answer grid to $\begin{matrix}\text{BD}\\\text{ACE}\end{matrix}$. (You'll learn why in a minute.)

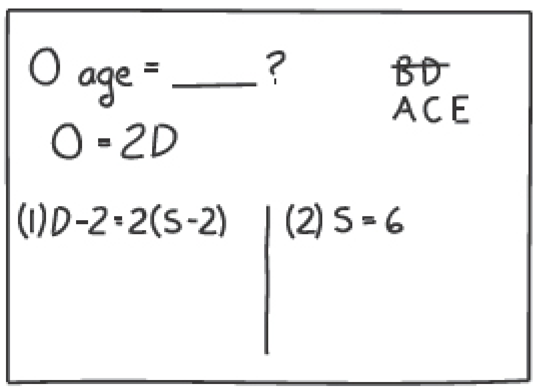
> (2) Samuel is 6 years old.



Statement (2) is not sufficient to determine Oliver's age, so cross off the answers that say statement (2) is sufficient: (B) and (D). Once again, you can cross off the entire top row; when starting with statement (2), you always will keep or eliminate these two choices at the same time.

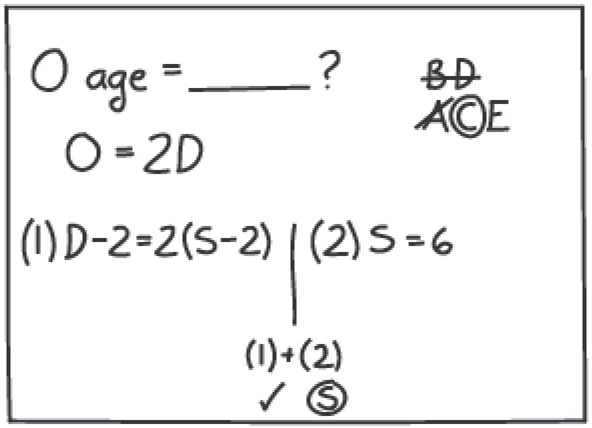
Now assess statement (1):

(1) Two years ago, Dmitry was twice as old as Samuel.



Forget all about statement (2); only statement (1) exists. By itself, is the statement sufficient?

Nope! Too many variables. Cross off (A), the first of the remaining answers in the bottom row, and assess the two statements together:



You can plug Samuel's age (from the second statement) into the formula from statement (1) to find Dmitry's age, and then use Dmitry's age to find Oliver's age. Together, the statements are sufficient.

The correct answer is **(C)**.

The two answer grids work in the same way, regardless of which one you use. As long as you use the AD/BCE grid when starting with statement (1), or the BD/ACE grid when starting with statement (2), you will always:

- cross off the *top* row if the first statement you try is *not* sufficient;
- cross off the *bottom* row if the first statement you try *is* sufficient; and
- assess the remaining row of answers one answer at a time.

Finally, remember that you must assess the statements separately before you can try them together—and you'll only try them together if neither one is sufficient on its own. You will only consider the two together if you have already crossed off answers (A), (B), and (D).

Value vs. Yes/No Questions

Data Sufficiency questions come in two “flavors”: Value or Yes/No.

On Value questions, it is necessary to find a single value in order to answer the question. If you can't find any value or you can find two or more values, then the information is not sufficient.

Consider this statement:

(1) Oliver's age is a multiple of 4.

Oliver could be 4 or 8 or 12 or any age that is a multiple of 4. Because it's impossible to determine one particular value for Oliver's age, the statement is not sufficient

What if the question changed?

Is Oliver's age an even number?

(1) Oliver's age is a multiple of 4.
(2) Oliver is between 19 and 23 years old.

This question is a Yes/No question. There are three possible answers to a Yes/No question:

1. Always Yes: Sufficient!
2. Always No: Sufficient!
3. Maybe (or Sometimes Yes, Sometimes No): Not Sufficient

It may surprise you that Always No is sufficient to answer the question. Imagine that you ask a friend to go to the movies with you. If she says, "No, I'm sorry, I can't," then you did receive an answer to your question (even though the answer is negative). You know she can't go to the movies with you.

Apply this reasoning to the Oliver question. Is statement 1 sufficient to answer the question *Is Oliver's age an even number*?

(1) Oliver's age is a multiple of 4.

If Oliver's age is a multiple of 4, then Yes, he must be an even number of years old. The information isn't enough to tell how old Oliver actually is—he could be 4, 8, 12, or any multiple of 4 years old. Still, the information is sufficient to answer the specific question asked.

Because the statement tried first is sufficient, cross off the bottom row of answers, (B), (C), and (E).

Next, check statement (2):

(2) Oliver is between 19 and 23 years old.

Oliver could be 20, in which case his age is even. He could also be 21, in which case his age is odd. The answer here is Sometimes Yes, Sometimes No, so the information is not sufficient to answer the question.

The correct answer is **(A)**: the first statement is sufficient but the second is not.

The DS Process

This section summarizes everything you've learned in one consistent DS process. You can use this on every DS problem on the test.

Step 1: Determine whether the question is Value or Yes/No.

Value: The question asks for the value of an unknown (e.g., What is x?).

A statement is **Sufficient** when it provides **1 possible value**.

A statement is **Not Sufficient** when it provides **more than 1 possible value** (or none at all).

Yes/No: The question asks whether a given piece of information is true (e.g., Is x even?).

Most of the time, these will be in the form of Yes/No questions.

A statement is **Sufficient** when the answer is **Always Yes** or **Always No**.

A statement is **Not Sufficient** when the answer is **Maybe** or **Sometimes Yes, Sometimes No**.

Step 2: Separate given information from the question itself.

If the question stem contains given information—that is, any information other than the question itself—then write down that information separately from the question itself. This is true information that you must consider or use when answering the question.

Step 3: Rephrase the question.

Most of the time, you will not write down the entire question stem exactly as it appears on screen. At the least, you'll simplify what is written on screen. For example, if the question stem asks, "What is the value of *x*?" then you might write down something like $x =$ ________ ?

For more complicated question stems, you may have more work to do. Ideally, before you go to the statements, you will be able to articulate a fairly clear and straightforward question. In the earlier example, $x =$ ________ ? is clear and straightforward.

What if this is the question?

If $xyz \neq 0$, is $\frac{3x}{2} + y + 2z = \frac{7x}{2} + y$?

(1) $y = 3$ and $x = 2$

(2) $z = -x$

Do you need to know the individual values of *x*, *y*, and *z* in order to answer the question? Would knowing the value of a combination of the variables, such as $x + y + z$, work? It's tough to tell.

In order to figure this out, ***rephrase*** the question stem, which is a fancy way of saying: simplify the information a lot. Take the time to do this before you address the statements; you'll make your job much easier!

If you're given an equation, the first task is to put the "like" variables together. Also, when working with the question stem, make sure to carry the question mark through your work:

$$y - y + 2z = \frac{7x}{2} - \frac{3x}{2}?$$

That's interesting: the two *y* variables cancel out. Keep simplifying:

$$2z = \frac{4x}{2}?$$

$$2z = 2x?$$

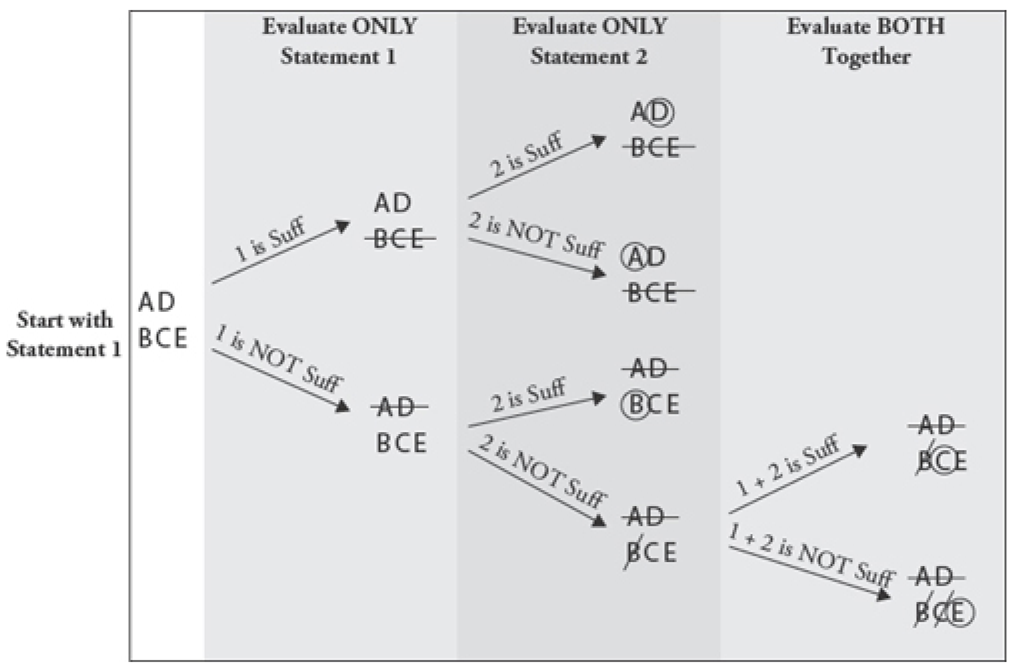
$$z = x?$$

That whole crazy equation is really asking a much simpler question: is $z = x$?

It might seem silly to keep writing that question mark at the end of each line, but don't skip that step or you'll be opening yourself up to a careless error. By the time you get to the end, you don't want to forget that this is still a *question*, not a statement or given.

Step 4: Use the Answer Grid to Evaluate the Statements

If you start with statement 1, then write the AD/BCE grid on your scrap paper.



Here is the rephrased problem:

If $xyz \neq 0$, is $z = x$?

(1) $y = 3$ and $x = 2$

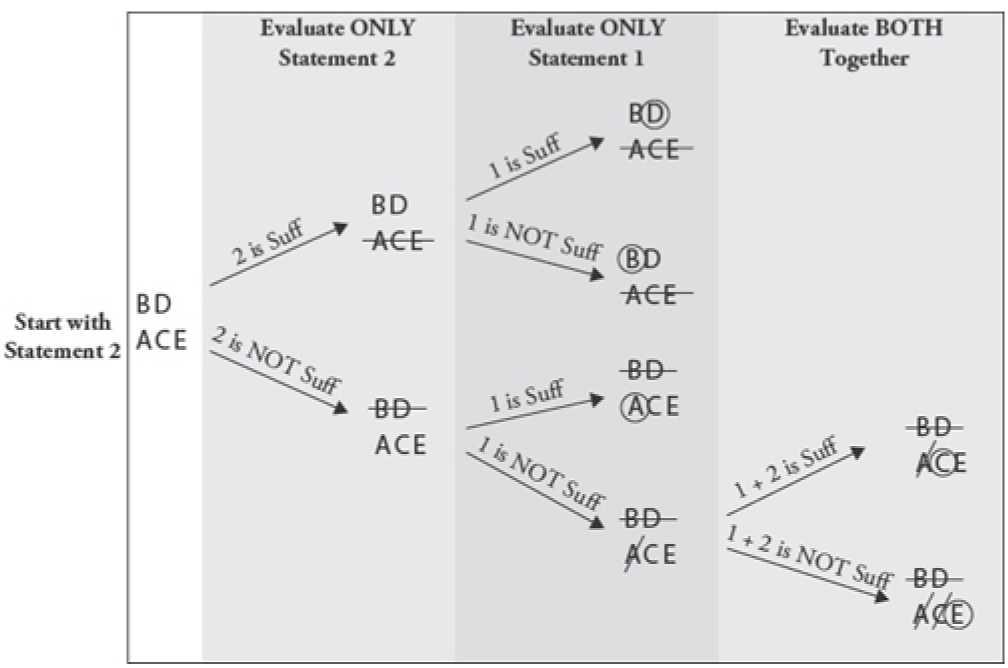
(2) $z = -x$

Statement (1) is useless by itself because it says nothing about z. Cross off the top row of answers: $\dfrac{\cancel{AD}}{BCE}$

Statement (2) turns out to be very useful. None of the variables is 0, so if $z = -x$, then those two numbers cannot be equal to each other. This statement is sufficient to answer the question: no, z does not equal x. You can circle B on your grid: $\dfrac{\cancel{AD}}{BCE}$

The correct answer is **(B)**.

If you decide to start with statement (2), your process is almost identical, but you'll use the BD/ACE grid instead. For example:



First, evaluate statement (1) by itself and, if you've crossed off answers (A), (B), and (D), then evaluate the two statements together.

Whether you use AD/BCE or BD/ACE, remember to

- cross off the *top* row if the first statement you try is *not* sufficient, and
- cross off the *bottom* row if the first statement you try *is* sufficient.

Pop Quiz! Test Your Skills

Have you learned the DS process? If not, go back through the chapter and work through the sample problems again. Try writing out each step yourself.

If so, prove it! Give yourself up to four minutes total to try the following two problems:

1. Are there more engineers than salespeople working at SoHo Corp?

   (1) SoHo Corp employs $\frac{2}{3}$ as many clerical staff as engineers and salespeople combined.

   (2) If 3 more engineers were employed by SoHo Corp and the number of salespeople remained the same, then the number of engineers would be double the number of salespeople employed by the company.

2. At SoHo Corp, what is the ratio of managers to non-managers?

   (1) If there were 3 more managers and the number of salespeople remained the same, then the ratio of managers to non-managers would double.

   (2) There are 4 times as many non-managers as managers at SoHo Corp.

How did it go? Are you very confident in your answers? Somewhat confident? Not at all confident?

Before you check your answers, go back over your work, using the DS process discussed in this chapter as your guide. Where can you improve? Did you write down (and use!) your answer grid? Did you look at each statement separately before looking at them together (if necessary)? Did you mix up any of the steps of the process? How neat is the work on your scrap paper? You may want to rewrite your work before you review the answers.

Pop Quiz Answer Key

1. Engineers vs. Salespeople

Step 1: Is this a Value or Yes/No question?

1. Are there more engineers than salespeople working at SoHo Corp?

This is a Yes/No question.

Steps 2 and 3: What is given and what is the question? Rephrase the question.

The question stem doesn't contain any given information. In this case, the question is already about as simplified as it can get: are there more engineers than salespeople?

Step 4: Evaluate the statements.

If you start with the first statement, use the AD/BCE answer grid.

(1) SoHo Corp employs $\frac{2}{3}$ as many clerical staff as engineers and salespeople combined.

If you add up the engineers and salespeople, then there are fewer people on the clerical staff…but this indicates nothing about the relative number of engineers and salespeople. This statement is not sufficient. Cross off (A) and (D), the top row, of your answer grid.

(2) If 3 more engineers were employed by SoHo Corp and the number of salespeople remained the same, then the number of engineers would be double the number of salespeople employed by the company.

This one sounds promising. If you add only 3 engineers, then you'll have twice as many engineers as salespeople. Surely, that means there are more engineers than salespeople?

Don't jump to any conclusions. Test some possible numbers; think about fairly extreme scenarios. What if you start with just 1 engineer? When you add 3, you'll have 4 engineers. If there are 4 engineers, then there are half as many, or 2, salespeople. In other words, you start with 1 engineer and 2 salespeople, so there are more salespeople. Interesting.

According to this one case, the answer to the Yes/No question *Are there more engineers than salespeople?* is no.

Can you find a yes answer? Try a larger set of numbers. If you start with 11 engineers and add 3, then you would have 14 total. The number of salespeople would have to be 7. In this case, then, there are more engineers to start than salespeople, so the answer to the question *Are there more engineers than salespeople?* is yes.

Because you can find both yes and no answers, statement (2) is not sufficient. Cross off answer (B).

Now, try the two statements together. How does the information about the clerical staff combine with statement (2)?

Whenever you're trying some numbers and you have to examine the two statements together, see whether you can reuse the numbers that you tried earlier.

If you start with 1 engineer, you'll have 2 salespeople, for a total of 3. In this case, you'd have 2 clerical staff, and the answer to the original question is no.

If you start with 11 engineers, you'll have 7 salespeople, for a total of 18. In this case, you'd have 12 clerical staff, and the answer to the original question is yes.

The correct answer is **(E)**. The information is not sufficient even when both statements are used together.

2. Managers vs. Non-Managers

Step 1: Is this a Value or a Yes/No question?

2. At SoHo Corp, what is the ratio of managers to non-managers?

This is a Value question. You need to find one specific ratio—or know that you can find one specific ratio—in order to answer the question.

Steps 2 and 3: What is given and what is the question? Rephrase the question.

Find the ratio of managers to non-managers, or M : N.

Step 4: Evaluate the statements.

If you start with the second statement, use the BD/ACE answer grid. (Note: this is always your choice; the solution with the BD/ACE grid shown is just for practice.)

(2) There are 4 times as many non-managers as managers at SoHo Corp.

If there is 1 manager, there are 4 non-managers. If there are 2 managers, there are 8 non-managers. If there are 3 managers, there are 12 non-managers.

What does that mean? In each case, the ratio of managers to non-managers is the same, 1 : 4. Even though you don't know how many managers and non-managers there are, you do know the ratio. (For more on ratios, see the Ratios chapter of the *Fractions, Decimals, & Percents GMAT Strategy Guide*.)

This statement is sufficient; cross (A), (C), and (E), the bottom row, off of the grid.

(1) If there were 3 more managers and the number of salespeople remained the same, then the ratio of managers to non-managers would double.

First, what does it mean to *double* a ratio? If the starting ratio were 2 : 3, then doubling that ratio would give you 4 : 3. The first number in the ratio doubles relative to the second number.

Test some cases. If you start with 1 manager, then 3 more would bring the total number of managers to 4. The *manager* part of the ratio just quadrupled (1 to 4), not doubled, so this number is not a valid starting point. Discard this case.

If you have to add 3 and want that number to double, then you need to start with 3 managers. When you add 3 more, that portion of the ratio doubles from 3 to 6. The other portion, the non-managers, remains the same.

Notice anything? The statement says nothing about the relative number of non-managers. The starting ratio could be 3 : 2 or 3 : 4 or 3 : 14, for all you know. In each case, doubling the number of managers would double the ratio (to 6 : 2, or 6 : 4, or 6 : 14). You can't figure out the specific ratio from this statement.

The correct answer is **(B)**: statement (2) is sufficient, but statement (1) is not.

Proving Insufficiency

The Pop Quiz solutions used the Testing Cases strategy: testing real numbers to help determine whether a statement is sufficient. You can do this whenever the problem allows for the possibility of multiple numbers or cases.

When you're doing this, your goal is to try to prove the statement insufficient. For example:

> If x and y are positive integers, is the sum of x and y between 20 and 26, inclusive?
>
> (1) $x - y = 6$

Test your first case. You're allowed to pick any numbers for x and y that make statement 1 true *and* that follow any constraints given in the question stem. In this case, that means the two numbers have to be positive integers and that $x - y$ has to equal 6.

Case #1: 20 − 14 = 6. These numbers make statement 1 true and follow the constraint in the question stem, so these are legal numbers to pick. Now, try to answer the Yes/No question: 20 + 14 = 34, so no, the sum is not between 20 and 26, inclusive.

You now have a *no* answer. Can you think of another set of numbers that will give you the opposite, a *yes* answer?

Case #2: 15 − 9 = 6. In this case, the sum is 24, so the answer to the Yes/No question is yes, the sum is between 20 and 26, inclusive.

Because you have found both a yes and a no answer, the statement is not sufficient.

Here's a summary of the process:

1. Notice that you can test cases. You can do this when the problem allows for multiple possible values.
2. Pick numbers that make the statement true and that follow any givens in the question stem. If you realize that you picked numbers that make the statement false or contradict givens in the question stem, *discard* those numbers and start over.
3. Your first case will give you one answer: a yes or a no on a Yes/No problem, or a numerical value on a value problem.
4. Try to find a second case that gives you a *different* answer. On a Yes/No problem, you'll be looking for the opposite of what you found for the first case. For a Value problem, you'll be looking for a different numerical answer. (Don't forget that whatever you pick still has to make the statement true and follow the givens in the question stem!)

The usefulness of trying to prove insufficiency is revealed as soon as you find two different answers. You're done! That statement is not sufficient, so you can cross off an answer or answers and move to the next step.

What if you keep finding the same answer? Try this:

> If x and y are positive integers, is the product of x and y between 20 and 26, inclusive?

(1) x is a multiple of 17.

Case #1: Test $x = 17$. Since y must be a positive integer, try the smallest possible value first: $y = 1$. In this case, the product is 17, which is not between 20 and 26 inclusive. The answer to the question is *no*; can you find the opposite answer?

Case #2: If you make $x = 34$, then xy will be too big, so keep $x = 17$. The next smallest possible value for y is 2. In this case, the product is 34, which is also not between 20 and 26 inclusive. The answer is again no.

Can you think of a case where you will get a *yes* answer? No! The smallest possible product is 17, and the next smallest possible product is 34. Any additional values of x and y you try will be equal to or larger than 34.

You've just proved the statement sufficient because it is impossible to find a yes answer. Testing Cases can help you to figure out the “theory” answer, or the mathematical reasoning that proves the statement is sufficient.

This won't always work so cleanly. Sometimes, you'll keep getting all no answers or all yes answers but you won't be able to figure out the theory behind it all. If you test three or four different cases, and you're actively seeking out the opposite answer but never find it, then go ahead and assume that the statement is sufficient, even if you're not completely sure why.

Do make sure that you're trying numbers with different characteristics. Try both even and odd. Try a prime number. Try zero or a negative or a fraction. (You can only try numbers that are allowed by the problem, of course. In the case of the above problems, you were only allowed to try positive integers.)

Here's how Testing Cases would work on a Value problem:

If x and y are prime numbers, what is the product of x and y?

(1) The product is even.

Case #1: $x = 2$ and $y = 3$. Both numbers are prime numbers and their product is even, so these are legal numbers to try. In this case, the product is 6. Can you choose numbers that will give a different product?

Case #2: $x = 2$ and $y = 5$. Both numbers are prime numbers and their product is even, so these are legal numbers to try. In this case, the product is 10.

The statement is not sufficient because there are at least two different values for the product of x and y.

In short, when you're evaluating DS statements, go into them with an “I'm going to try to prove you insufficient!” mindset.

- If you do find two different answers (yes and no, or two different numbers), then immediately declare that statement not sufficient.
- If, after several tries, you keep finding the same answer despite trying different kinds of numbers, see whether you can articulate why; that statement may be sufficient after all. Even if you can't say why, go ahead and assume that the statement is sufficient.

Now you're ready to test your Data Sufficiency skills. As you work through the chapters in this book, test your progress using some of the *Official Guide* problem set lists found online, in your Manhattan GMAT Student Center. Start with lower-numbered problems first, in order to practice the process, and work your way up to more and more difficult problems.