1. The digit in the unit place of the number 7^{78} is

(A) 1 (B) 3 (C) 7 (D) 9.

2. If P is an integer from 1 to 50, what is the probability that $P(P+1)$ is divisible by 4?

(A) 0.25 (B) 0.50 (C) 0.48 (D) none of these.

3. If the co-efficient of p^{th}, $(p+1)^{th}$ and $(p+2)^{th}$ terms in the expansion of $(1+x)^n$ are in Arithmetic Progression (A.P.), then which one of the following is true?

(A) $n^2 + 4(4p+1) + 4p^2 - 2 = 0$
(B) $n^2 + 4(4p+1) + 4p^2 + 2 = 0$
(C) $(n-2p)^2 = n+2$
(D) $(n+2p)^2 = n+2$.

4. The number of terms with integral coefficients in the expansion of $(17^{\frac{1}{3}} + 19^{\frac{1}{2}}x)^{600}$ is

(A) 99 (B) 100 (C) 101 (D) 102.

5. Let A be the set of all prime numbers, B be the set of all even prime numbers, and C be the set of all odd prime numbers.

Consider the following three statements in this regard:

(I) $A = B \cup C$.
(II) B is a singleton set.
(III) $A = C \cup \{2\}$.

Then which one of the following holds?

(A) None of the above statements is true.
(B) Exactly one of the above statements is true.
(C) Exactly two of the above statements are true.
(D) All the above three statements are true.

6. A die is thrown thrice. If the first throw is a 4 then the probability of getting 15 as the sum of three throws is

   (A) $\frac{1}{108}$ (B) $\frac{1}{6}$ (C) $\frac{1}{18}$ (D) none of these.

7. You are given three sets A, B, C in such a way that (i) the set $B \cap C$ consists of 8 elements, (ii) the set $A \cap B$ consists of 7 elements, and (iii) the set $C \cap A$ consists of 7 elements. The minimum number of elements in the set $A \cup B \cup C$ is

   (A) 8 (B) 14 (C) 15 (D) 22.

8. A Pizza Shop offers 6 different toppings, and they do not take an order without any topping. I can afford to have one pizza with a maximum of 3 toppings. In how many ways can I order my pizza?

   (A) 20 (B) 35 (C) 41 (D) 21.

9. Let $f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^{2018}}{2018}$. Then $f'(1)$ is equal to

   (A) 0 (B) 2017 (C) 2018 (D) 2019.

10. Let $f'(x) = 4x^3 - 3x^2 + 2x + k$, $f(0) = 1$ and $f(1) = 4$. Then $f(x)$ is equal to

    (A) $4x^4 - 3x^3 + 2x^2 + x + 1$ (B) $x^4 - x^3 + x^2 + 2x + 1$
    (C) $x^4 - x^3 + x^2 + 2(x + 1)$ (D) none of these.

11. The sum of 99^{th} power of all the roots of $x^7 - 1 = 0$ is equal to

    (A) 1 (B) 2 (C) -1 (D) 0.

12. Let $A = \{10, 11, 12, 13, \ldots, 99\}$. How many pairs of numbers x and y are possible so that $x + y \geq 100$ and x and y belong to A?

    (A) 2405 (B) 2455 (C) 1200 (D) 1230.

13. In a certain town, 20% families own a car, 90% own a phone, 5% own neither a car nor a phone and $30,000$ families own both a car and a phone.

Consider the following statements in this regard:

(i) 10% families own both a car and a phone.
(ii) 95% families own either a car or a phone.
(iii) $2,00,000$ families live in the town.

Then which one of the following is true?

(A) (i) & (ii) are correct and (iii) is wrong.
(B) (i) & (iii) are correct and (ii) is wrong.
(C) (ii) & (iii) are correct and (i) is wrong.
(D) (i), (ii) & (iii) are correct.

14. In a room there are 8 men, numbered $1, 2, \ldots, 8$. These men have to be divided into 4 teams in such a way that (i) every team has exactly 2 members, and (ii) there are no common members between any two teams. For example, $\{(1,2),(3,4),(5,6),(7,8)\}$ $\{(1,5),(2,7),(3,8),(4,6)\}$ are two such 4-team combinations. The total number of such 4-team combinations is

(A) $\frac{8!}{2^4}$ (B) $\frac{8!}{2^4(4!)}$ (C) $\frac{8!}{4!}$ (D) $\frac{8!}{(4!)^2}$.

15. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is

(A) 6 (B) 9 (C) 12 (D) 18.

16. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & a & b \\ 1 & 0 & 1 \end{pmatrix}$. Then A^{-1} does not exist if (a, b) is equal to

(A) $(1, -1)$ (B) $(1, 0)$ (C) $(-1, -1)$ (D) $(0, 1)$.

17. The value of ${}^{13}C_3 + {}^{13}C_5 + {}^{13}C_7 + \cdots + {}^{13}C_{13}$ is

(A) 4096 (B) 4083 (C) $2^{13} - 1$ (D) $2^{12} - 1$.

18. If $x + y = \pi$, the expression $\cot \frac{x}{2} + \cot \frac{y}{2}$ can be written as

(A) $2 \text{ cosec } x$ (B) $\text{cosec } x + \text{cosec } y$

(C) $2 \sin x$ (D) $\sin x + \sin y$.

19. The area of the region formed by line segments joining the points of intersection of the circle $x^2 + y^2 - 10x - 6y + 9 = 0$ with the two axes in succession in a definite order (clockwise or anticlockwise) is

(A) 16 (B) 9 (C) 3 (D) 12.

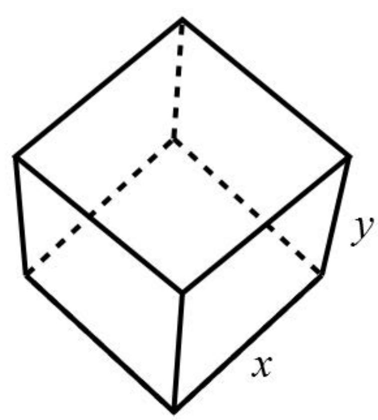
20. The value of $\tan\left(\sin^{-1}(\frac{3}{5}) + \cot^{-1}(\frac{3}{2})\right)$ is

(A) $\frac{1}{18}$ (B) $\frac{11}{6}$ (C) $\frac{13}{6}$ (D) $\frac{17}{6}$.

21. A box with a square base of length x and height y has an open top and its volume is 32 cubic centimetres, as shown in the figure below. The values of x and y that minimize the surface area of the box are

(A) $x = 4$ cm $\&\ y = 2$ cm (B) $x = 3$ cm $\&\ y = \frac{32}{9}$ cm

(C) $x = 2$ cm $\&\ y = 8$ cm (D) none of these.



The box with base length x and side length y.

22. Let the sides opposite to the angles A, B, C in a triangle ABC be represented by a, b, c respectively. If $(c+a+b)(a+b-c) = ab$, then the angle C is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$.

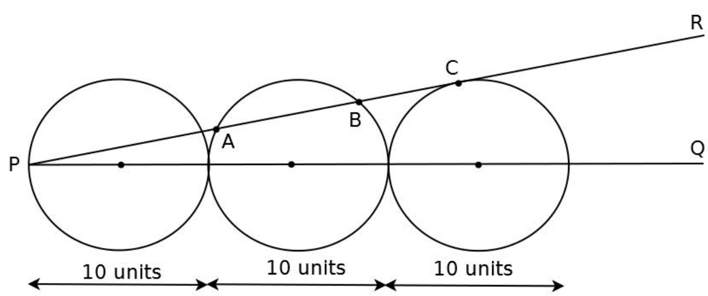
23. Let A be the point of intersection of the lines $3x - y = 1$ and $y = 1$. Let B be the point of reflection of the point A with respect to the y-axis. Then the equation of the straight line through B that produces a right angled triangle ABC with $\angle ABC = 90°$, and C lies on the line $3x - y = 1$, is

(A) $3x - 3y = 2$ (B) $2x + 3 = 0$
(C) $3x + 2 = 0$ (D) $3y - 2 = 0$.

24. Let $[x]$ denote the largest integer less than or equal to x. The number of points in the open interval $(1, 3)$ in which the function $f(x) = a^{[x^2]}, a > 1$ is not differentiable, is

(A) 0 (B) 3 (C) 5 (D) 7.

25. There are three circles of equal diameter (10 units each) as shown in the figure below. The straight line PQ passes through the centres of all the three circles. The straight line PR is a tangent to the third circle at C and cuts the second circle at the points A and B, as shown in the figure.



Then the length of the line segment AB is

(A) 6 units (B) 7 units (C) 8 units (D) 9 units.

26. The area of the region bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x-axis in the first quadrant is

(A) 9 (B) $\frac{27}{4}$ (C) 36 (D) 18.

27. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is

(A) 2 (B) 1

(C) ∞ (D) not a convergent series.

28. Let $f(x) = e^{-\left(\frac{1}{x^2-3x+2}\right)}$; $x \in \mathbb{R}$ & $x \notin \{1, 2\}$. Let $a = \lim_{x \to 1^+} f(x)$ and $b = \lim_{x \to 1^-} f(x)$. Then

(A) $a = \infty, b = 0$ (B) $a = 0, b = \infty$

(C) $a = 0, b = 0$ (D) $a = \infty, b = \infty$.

29. Let $f(x) = (x-1)(x-2)(x-3)g(x)$; $x \in \mathbb{R}$ where g is a twice differentiable function. Then

(A) there exists $y \in (1, 3)$ such that $f''(y) = 0$.
(B) there exists $y \in (1, 2)$ such that $f''(y) = 0$.
(C) there exists $y \in (2, 3)$ such that $f''(y) = 0$.
(D) none of the above is true.

30. Let $0.01^x + 0.25^x = 0.7$. Then

(A) $x \geq 1$ (B) $0 < x < 1$

(C) $x \leq 0$ (D) no such real number x is possible.

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