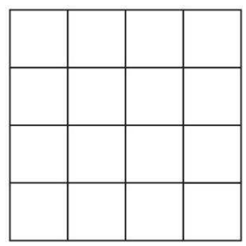
SAMPLE QUESTIONS (MMA): 2019

1. The number of isosceles (but not equilateral) triangles with integer sides and no side exceeding 10 is

   (A) 65 (B) 75 (C) 81 (D) 90.

2. The number of squares in the following figure is



   (A) 25 (B) 26 (C) 29 (D) 30.

3. The number of trailing zeros in $100!$ is

   (A) 21 (B) 23 (C) 24 (D) 25.

4. The number of common terms in the two sequences $\{3, 7, 11, \ldots, 407\}$ and $\{2, 9, 16, \ldots, 709\}$ is

   (A) 13 (B) 14 (C) 15 (D) 16.

5. One needs to choose six real numbers $x_1, x_2, \ldots, x_6$ such that the product of **any** five of them is equal to other number. The number of such choices is

   (A) 3 (B) 33 (C) 63 (D) 93.

6. The volume of the region $S = \{(x, y, z) : |x| + |y| + |z| \leq 1\}$ is

   (A) 1/6 (B) 1/3 (C) 2/3 (D) 4/3.

7. The greatest common divisor of all numbers of the form $p^2 - 1$, where $p \geq 7$ is a prime, is

   (A) 6 (B) 12 (C) 24 (D) 48.

8. Let a and b be two positive integers such that

$$a = k_1 b + r_1 \text{ and } b = k_2 r_1 + r_2,$$

where k_1, k_2, r_1, r_2 are positive integers with $r_2 < r_1 < b$. Then $\gcd(a, b)$ is same as

(A) $\gcd(r_1, r_2)$ (B) $\gcd(k_1, k_2)$ (C) $\gcd(k_1, r_2)$ (D) $\gcd(k_2, r_1)$.

9. If α is a root of $x^2 - x + 1 = 0$, then $\alpha^{2018} + \alpha^{-2018}$ is

(A) -1 (B) 0 (C) 1 (D) 2.

10. A new flag of ISI club is to be designed with 5 vertical strips using some or all of the four colours: green, maroon, red and yellow. In how many ways this can be done so that no two adjacent strips have the same colour ?

(A) 120 (B) 324 (C) 432 (D) 576.

11. The value of λ for which the system of linear equations $2x - y - z = 12$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ has no solution is

(A) 2 (B) -2 (C) 3 (D) -3.

12. The rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 6 & 8 & 10 & 12 \\ 151 & 262 & 373 & 484 \end{bmatrix} \text{ is}$$

(A) 1 (B) 2 (C) 3 (D) 4.

13. If $A = \begin{bmatrix} 2 & i \\ i & 0 \end{bmatrix}$, the trace of A^{10} is

(A) 2 (B) 2(1+i) (C) 0 (D) 2^{10}.

14. Let A be a 3×3 real matrix with all diagonal entries equal to 0. If $1 + i$ is an eigenvalue of A, the determinant of A equals

(A) -4 (B) -2 (C) 2 (D) 4.

15. Let G be a finite group of even order. Then which of the following statements is correct ?

   (A) The number of elements of order 2 in G is even

   (B) The number of elements of order 2 in G is odd

   (C) G has no subgroup of order 2

   (D) None of the above.

16. Consider a large village, where only two newspapers P_1 and P_2 are available to the families. It is known that the proportion of families

   (i) not taking P_1 is 0.48,
   (ii) not taking P_2 is 0.58,
   (iii) taking only P_2 is 0.30.

   The probability that a randomly chosen family from the village takes only P_1 is

   (A) 0.24 (B) 0.28 (C) 0.40 (D) cannot be determined.

17. There are eight coins, seven of which have the same weight and the other one weighs more. In order to find the coin having more weight, a person randomly chooses two coins and puts one coin on each side of a common balance. If these two coins are found to have the same weight, the person then randomly chooses two more coins from the rest and follows the same method as before. The probability that the coin will be identified at the second draw is

   (A) $1/2$ (B) $1/3$ (C) $1/4$ (D) $1/6$.

18. Let $A_1 = (0, 0)$, $A_2 = (1, 0)$, $A_3 = (1, 1)$ and $A_4 = (0, 1)$ be the four vertices of a square. A particle starts from the point A_1 at time 0 and moves either to A_2 or to A_4 with equal probability. Similarly, in each of the subsequent steps, it randomly chooses one of its adjacent vertices and moves there. Let T be the minimum number of steps required to cover all four vertices. The probability $P(T = 4)$ is

   (A) 0 (B) $1/16$ (C) $1/8$ (D) $1/4$.

19. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed with $P(X_i = 1) = P(X_i = -1) = p$ and $P(X_i = 0) = 1 - 2p$ for all $i = 1, 2, \ldots, n$. Define

$$a_n = P\Big(\prod_{i=1}^{n} X_i = 1\Big), b_n = P\Big(\prod_{i=1}^{n} X_i = -1\Big) \text{ and } c_n = P\Big(\prod_{i=1}^{n} X_i = 0\Big).$$

Which of the following is true as n tends to infinity?

(A) $a_n \to 1/3$, $b_n \to 1/3$, $c_n \to 1/3$
(B) $a_n \to p$, $b_n \to p$, $c_n \to 1 - 2p$
(C) $a_n \to 1/2$, $b_n \to 1/2$, $c_n \to 0$
(D) $a_n \to 0$, $b_n \to 0$, $c_n \to 1$.

20. Consider the set of all functions from $\{1, 2, \ldots, m\}$ to $\{1, 2, \ldots, n\}$, where $n > m$. If a function is chosen from this set at random, the probability that it will be strictly increasing is

(A) $\binom{n}{m}/n^m$ (B) $\binom{n}{m}/m^n$ (C) $\binom{m+n-1}{m-1}/n^m$ (D) $\binom{m+n-1}{m}/m^n$.

21. The angle between the tangents drawn from the point $(1, 4)$ to the parabola $y^2 = 4x$ is

(A) $\pi/2$ (B) $\pi/3$ (C) $\pi/4$ (D) $\pi/6$.

22. The x-axis divides the circle $x^2 + y^2 - 6x - 4y + 5 = 0$ into two parts. The area of the smaller part is

(A) $2\pi - 1$ (B) $2(\pi - 1)$ (C) $2\pi - 3$ (D) $2(\pi - 2)$.

23. For $n \geq 1$, let

$$a_n = \frac{1}{2^2} + \frac{2}{3^2} + \cdots + \frac{n}{(n+1)^2} \text{ and } b_n = c_0 + c_1 r + c_2 r^2 + \cdots + c_n r^n,$$

where $|c_k| \leq M$ for all integer k and $|r| < 1$. Then

(A) both $\{a_n\}$ and $\{b_n\}$ are Cauchy sequences
(B) $\{a_n\}$ is a Cauchy sequence, but $\{b_n\}$ is not a Cauchy sequence
(C) $\{a_n\}$ is not a Cauchy sequence, but $\{b_n\}$ is a Cauchy sequence
(D) neither $\{a_n\}$ nor $\{b_n\}$ is a Cauchy sequence.

24. The sum of the infinite series

$$1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \cdots \qquad \text{is}$$

(A) 2 (B) 3 (C) 4 (D) 6.

25. The solution of the differential equation

$$(1 + x^2y^2)ydx + (x^2y^2 - 1)xdy = 0 \qquad \text{is}$$

(A) $xy = \log x - \log y + C$

(B) $xy = \log y - \log x + C$

(C) $x^2y^2 = 2(\log x - \log y) + C$

(D) $x^2y^2 = 2(\log y - \log x) + C$.

26. Let C_i $(i = 0, 1, \ldots, n)$ be the coefficient of x^i in $(1 + x)^n$. Then

$$\frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \cdots + (-1)^n \frac{C_n}{n+2}$$

is equal to

(A) $\frac{1}{n+1}$ (B) $\frac{1}{n+2}$ (C) $\frac{1}{n(n+1)}$ (D) $\frac{1}{(n+1)(n+2)}$.

27. Number of real solutions of the equation $x^7 + 2x^5 + 3x^3 + 4x = 2018$ is

(A) 1 (B) 3 (C) 5 (D) 7.

28. Consider the following functions

$$f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 1, & |x| \le 2 \\ 2, & |x| > 2. \end{cases}$$

Define $h_1(x) = f(g(x))$ and $h_2(x) = g(f(x))$. Which of the following statements is correct?

(A) h_1 and h_2 are continuous everywhere

(B) h_1 is continuous everywhere and h_2 has discontinuity at ± 1

(C) h_2 is continuous everywhere and h_1 has discontinuity at ± 2

(D) h_1 has discontinuity at ± 2 and h_2 has discontinuity at ± 1.

29. Let f be a continuous function with $f(1) = 1$. Define

$$F(t) = \int_t^{t^2} f(x)dx.$$

The value of $F'(1)$ is

(A) -2 (B) -1 (C) 1 (D) 2.

30. Consider the function

$$f(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!}\right) e^{-x},$$

where $n \geq 4$ is a positive integer. Which of the following statements is correct?

(A) f has no local extremum

(B) For every n, f has a local maximum at $x = 0$

(C) f has no local extremum if n is odd and has a local maximum at $x = 0$ when n is even

(D) f has no local extremum if n is even and has a local maximum at $x = 0$ when n is odd.