1. If $f(x) = xe^{x(1-x)}, x \in \mathbb{R}$, then $f(x)$ is

   (a) Decreasing on $(-\frac{1}{2}, 1]$
   (b) Decreasing on $(\frac{1}{2}, 1)$
   (c) Decreasing on $(\frac{1}{2}, 1]$
   (d) None of the above

2. $\int \left(\frac{1}{x}\tan\left(\frac{1}{x}\right)\right)^2 dx$ is equal to

   (a) $x - \tan(x) + C$
   (b) $\frac{1}{x} + \tan(\frac{1}{x}) + C$
   (c) $\frac{1}{x} - \tan(\frac{1}{x}) + C$
   (d) None of the above

3. If $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$, then the maximum value of xy for all $x \geq 0, y > 0$, is

   (a) 2500
   (b) 3000
   (c) 1200
   (d) None of the above

4. If $f(x) = ax + b$ and $f^{-1}(x) = bx + a$, with $a, b, x \in \mathbb{R}$, then what is the value of $a + b$?

   (a) -2
   (b) -1
   (c) 0
   (d) 1

5. If, $\ln(a + c), \ln(c - a), \ln(a - 2b + c)$ are in Arithmetic Progression, then

   (a) a, b, c are in Arithmetic Progression
   (b) a, b, c are in Geometric Progression
   (c) a, b, c are in Harmonic Progression
   (d) None of the above

6. Let $x > 0$ and $\log_2 x + \log_2 \sqrt{x} + \log_2 \sqrt[4]{x} + \log_2 \sqrt[8]{x} + \ldots = 4$. Then x is equal to

   (a) 2
   (b) 3
   (c) 4
   (d) None of the above

7. A fair coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then the value of n is

   (a) 14
   (b) 16
   (c) 24
   (d) None of the above

8. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $I + 2A + 3A^2 + \ldots..$ is equal to

   (a) $\begin{bmatrix} 4 & 1 \\ -4 & 0 \end{bmatrix}$
   (b) $\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix}$
   (c) $\begin{bmatrix} 5 & 2 \\ -8 & -3 \end{bmatrix}$
   (d) $\begin{bmatrix} 5 & 2 \\ -3 & -8 \end{bmatrix}$

9. $\lim_{x \to \frac{\pi}{4}} [\tan(x)]^{\tan(2x)}$ is equal to

   (a) 1
   (b) e
   (c) -1
   (d) None of the above

10. The sum of all squared numbers between 50 and 500 is
    (a) 3704
    (b) 3655
    (c) 4233
    (d) None of the above

11. Coefficient of x^{99} in the expansion of $(x+1)(x+3)(x+5).....(x+199)$ is equal to
    (a) 10250
    (b) 10000
    (c) 10500
    (d) None of the above

12. If $N = n!$, where n is a natural number with $n > 2$, then

$$\lim_{N\to\infty} [\log_2 N]^{-1} + [\log_3 N]^{-1} + [\log_4 N]^{-1} + ... + [\log_n N]^{-1}$$

    is,
    (a) 1
    (b) 2
    (c) 3
    (d) None of the above

13. The final score in a recreational soccer game between Team A and Team B, is 6 goals for A to 3 goals for B. How many possibilities exist for the score at the end of first half?
    (a) 20
    (b) 24
    (c) 28
    (d) None of the above

14. Integers a, b, c and d, not necessarily distinct, are chosen independently and at random from 0 to 2007 (both inclusive). What is the probability that $ad - bc$ is even?

   (a) $\frac{3}{8}$
   (b) $\frac{7}{16}$
   (c) $\frac{9}{16}$
   (d) $\frac{5}{8}$

15. If the function f satisfies the relation $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{N}$. Further if $f(1) = 2$ and $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, then value of a, (where $a \in \mathbb{N}$), is equal to

   (a) 3
   (b) 1
   (c) 2
   (d) 4

16. If for any real number y, $[y]$ is the greatest integer less than or equal to y, then the value of the integral $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2\sin x]dx$, is

   (a) $-\pi$
   (b) 0
   (c) $-\frac{\pi}{2}$
   (d) None of the above

17. The value of real number x for which the matrix $\begin{bmatrix} 6 & 4 & 2x \\ 8 & 2 & -2 \\ 0 & 6 & 8 \end{bmatrix}$ has no inverse is

   (a) $\frac{12}{11}$
   (b) $\frac{11}{12}$
   (c) 1
   (d) 0

18. The sum of the infinite series

$$\frac{5}{13} + \frac{55}{13^2} + \frac{555}{13^3} + \dots$$

is equal to

(a) $\frac{31}{18}$
(b) $\frac{65}{32}$
(c) $\frac{65}{36}$
(d) $\frac{75}{36}$

19. Water is being poured at the rate of 2 cubic metres per second into a cone which has semi vertical angle of 45°. The rate at which perimeter of water surface changes when the height of water in the cone is 2 metres is

(a) 2 metres per second
(b) 1 metre per second
(c) 3 metres per second
(d) 4 metres per second

20. The number of real solutions for $x^2 + 5|x| + 6 = 0$ is

(a) 0
(b) 2
(c) 3
(d) 4

21. $\lim_{x\to\infty} \left(1 - \frac{4}{x-1}\right)^{3x-1}$ is

(a) e^{-12}
(b) e^{12}
(c) e^{-4}
(d) e^{-3}

22. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$, then which of the following is true?

   (a) $I < \frac{2}{3}$ and $J > 2$
   (b) $I > \frac{2}{3}$ and $J < 2$
   (c) $I > \frac{2}{3}$ and $J > 2$
   (d) $I < \frac{2}{3}$ and $J < 2$

23. The sum of the infinite series

$$1 + \frac{1}{4 \times 2!} + \frac{1}{16 \times 4!} + \frac{1}{64 \times 6!} + \dots$$

   is

   (a) $\frac{e-1}{2\sqrt{e}}$
   (b) $\frac{e+1}{2\sqrt{e}}$
   (c) $\frac{e-1}{\sqrt{e}}$
   (d) $\frac{e+1}{\sqrt{e}}$

24. A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Jack picks a random three - second time interval to watch the light. What is the probability that the color changes while he is watching?

   (a) $\frac{1}{3}$
   (b) $\frac{1}{7}$
   (c) $\frac{1}{10}$
   (d) None of the above

25. The term independent of x in the binomial expansion of

$$\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}} \right)^{10}$$

   is

   (a) 4
   (b) 120
   (c) 210
   (d) 310

26. $\lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{\frac{1}{n}}$ is equal to

(a) e

(b) $\frac{1}{e}$

(c) $\frac{\pi}{4}$

(d) $\frac{4}{\pi}$

27. Largest possible area of a right angled triangle having hypotenuse of length 4cm is equal to

(a) 3 sq cm

(b) 4 sq cm

(c) 5 sq cm

(d) None of the above

28. Let f be a one-to-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false: $f(x) = 1, f(y) \neq 1$ and $f(z) \neq 2$. Then $f^{-1}(1)$ is equal to

(a) x

(b) y

(c) z

(d) None of the above

29. If $f(x) = ae^{2x} + be^x + cx$ satisfies the conditions $f(0) = 1, f'(\log 2) = 31$ and $\int_0^{\log 4}(f(x) - cx)dx = \frac{39}{2}$, then

(a) $a = 5, b = 6, c = 3$

(b) $a = 5, b = -6, c = 3$

(c) $a = -5, b = 6, c = 3$

(d) None of the above

30. If x_1, x_2, x_3 and x_4 are the roots of the equation

$$x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x \cos\beta - \sin\beta = 0$$

then $\tan^{-1}(x_1) + \tan^{-1}(x_2) - \tan^{-1}(x_3) - \tan^{-1}(x_4)$ is equal to

(a) β

(b) $\frac{\pi}{2} - \beta$

(c) $\pi - \beta$

(d) None of the above