Sample Questions 2019
Test Code PCB (Short Answer Type)

- **The questions are divided into *two* groups, GROUP A and GROUP B.**
- **GROUP B consists of the following *FIVE* sections:**
  (i) Computer Science, (ii) Engineering and Technology, (iii) Mathematics, (iv) Physics, (v) Statistics.
- **GROUP A is compulsory for all. Candidates will have to answer from *only one* section in GROUP B.**

Group A

A1. Consider an $n \times n$ matrix $A = I_n - \alpha\alpha^T$, where I_n is the $n \times n$ identity matrix and α is an $n \times 1$ column vector such that $\alpha^T\alpha = 1$. Show that $A^2 = A$.

[6]

A2. Let there be a pile of 2018 chips in the center of a table. Suppose there are two players who could alternately remove one, two or three chips from the pile. At least one chip must be removed, but no more than three chips can be removed in a single move. The player that removes the last chip wins. Does the first player (the player who starts playing the game) have a winning strategy in this game, that is, whatever moves his opponent makes, he can always make his moves in a certain way ensuring his win? Justify your answer.

[12]

A3. Let n, r, and s be positive integers, each greater than 2. Prove that $n^r - 1$ divides $n^s - 1$ if and only if r divides s.

[10]

A4. Let A and B be two non-empty finite subsets of $\mathbb{Z}$, the set of all integers. Define $A + B = \{a + b : \ a \in A, \ b \in B\}$. Prove that $|A + B| \geq |A| + |B| - 1$, where $|S|$ denotes the cardinality of a finite set S.

[12]

Group B

Section I : Computer Science

C1. Consider an array of length n consisting only of positive and negative integers. Design an algorithm to rearrange the array so that all the negative integers appear before all the positive integers, using $O(n)$ time and only constant amount of extra space.

[10]

C2. You can climb up a staircase of n stairs by taking steps of one or two stairs at a time.

(a) Formulate a recurrence relation for counting a_n, the number of distinct ways in which you can climb up the staircase.

(b) Mention the boundary conditions for your recurrence relation.

(c) Find a closed form expression for a_n by solving your recurrence.

$[3 + 2 + 5 = 10]$

C3. An n-variable Boolean function $f : \{0,1\}^n \to \{0,1\}$ is called symmetric if its value depends only on the number of 1's in the input. Let σ_n denote the number of such functions.

(a) Calculate the value of σ_4.

(b) Derive an expression for σ_n in terms of n.

$[3 + 7 = 10]$

C4. Let the valid moves along a staircase be U (one step *up*) and D (one step *down*). For example, the string $s = UUDU$ represents the sequence of moves as two steps up, then one step down, and then again one step up. Suppose a person is initially at the base of the staircase. A string denoting a sequence of steps that takes the person below the base is invalid. For example, the sequence $UUDDDU$ is invalid. Let L be the language defined by the set of valid strings which represent scenarios in which the person never returns to the base of the staircase after the final step.

(a) **Show that L is not regular.**

(b) **Write a context free grammar for accepting L.**

[$5 + 5 = 10$]

C5. Consider a max-heap of n distinct integers, $n \geq 4$, stored in an array $\mathcal{A}[1 \ldots n]$. The *second minimum* of $\mathcal{A}$ is the integer that is less than all integers in $\mathcal{A}$ except the minimum of $\mathcal{A}$. Find all possible array indices of $\mathcal{A}$ in which the second minimum can occur. Justify your answer.

[10]

C6. The following function computes an array `SPF`, where, for any integer `1 < i < 1000`, `SPF[i]` is the *smallest prime factor* of `i`. For example, `SPF[6]` is 2, and `SPF[11]` is 11.

There are five missing parts in the following code, commented as `/* Blank */`. For each of them, copy the entire line with the comment and fill the blank appropriately in your answer sheet.

```
int SPF[1000];

void findSPF() {
  SPF[1] = 1;

  // Initializing SPF of every number to be itself
  for (int i = 2; i < 1000; i++) {
    _____; /* Blank 1 */
  }

  // SPF of every even number is 2
  for (int i = 4; i < 1000; i += 2) {
        SPF[i] = _____; /* Blank 2 */
  }

  // For odd numbers, updating the SPFs of their multiples
  for (int i = _____; i * i < 1000; i++) { /* Blank 3 */
    if (SPF[i] == i) { // No smaller factor of i found yet
      for (int j = _____; j < 1000; j+= i) { /* Blank 4 */
        if (SPF[j] == j) {
           SPF[j] = _____; /* Blank 5 */
```

```
                }
            }
        }
    }
}
```

[$5 \times 2 = 10$]

C7. **A context switch from a process P_{old} to a process P_{new} consists of the following steps:**

- **Step I: saving the context of P_{old};**
- **Step II: running the scheduling algorithm to pick P_{new};**
- **Step III: restoring the saved context of P_{new}.**

Suppose Steps I and III together take T_0 units of time. The scheduling algorithm takes nT_1 units of time, where n is the number of ready-to-run processes. The scheduling policy is round-robin with a time slice of 10ms. Compute the CPU utilization for the following scenario: k processes become ready at almost the same instant in the order $P_1, P_2, \ldots, P_k$; each process requires exactly one CPU burst of 20ms and no I/O burst.

[10]

C8. **Consider a 5-stage instruction pipeline. The stages and the corresponding stage delays are given below.**

Instruction	Stage delay
Fetch instruction (FI)	3 ns
Decode instruction (DI)	4 ns
Fetch operand (FO)	7 ns
Execute instruction (EI)	10 ns
Write result (WR)	7 ns

Assume that there is no delay between two consecutive stages. Consider a processor with a branch prediction mechanism by which it is always able to correctly predict the direction of the branch at the FI stage itself, without executing the branch instruction. A program consisting of a sequence of 10 instructions I1, I2, ..., I10, is executed in the pipeline, where the 5^{th} instruction (I5) is the only branch instruction and its branch target is the 8^{th} instruction (I8).

(a) Draw the pipeline diagram over time showing how the instructions I1, I2, . . . , I10 flow through the pipeline stages in this processor.

(b) Calculate the time (in ns) needed to execute the program.

$[5 + 5 = 10]$

C9. **The data link layer uses a fixed-size sliding window protocol, where the window size for the connection is equal to *twice* the bandwidth-delay product of the network path. Consider the following three scenarios, in each of which only the given parameter changes as specified (no other parameters change). For each scenario, explain whether the throughput (not utilization) of the connection increases, decreases, remains the same, or cannot be determined:**

(a) the packet loss rate L decreases to $L/3$;

(b) the minimum value of the round trip time R increases to $1.8R$;

(c) the window size W decreases to $W/3$.

$[3 + 4 + 3 = 10]$

C10. **Consider two $n \times 1$ vectors u and v, stored as tables `U(ind,val)` and `V(ind,val)` with the same schema. A row (i, u_i) of table `U` specifies that the i-th element of vector u has value u_i (similarly for v, respectively). Only the non-zero entries of the vectors are stored in the corresponding tables. For example, if the vector u equals $(0, 1, 3, 0, 2, 0)$, then it is represented in table `U` as:**

ind	val
2	1
3	3
5	2

Write a relational algebra expression or an SQL query to compute the sum u + v of the two vectors u and v. Explain your solution.

[10]

Section II : Engineering and Technology

E1. A small block starts sliding down an inclined plane which forms an angle of θ with the horizontal. The coefficient of friction μ depends on the distance x covered by the block as $\mu = ax$, where a is a constant. Find (a) the distance covered by the block till it stops, and (b) it's maximum velocity over the distance traversed.

$[5 + 5 = 10]$

E2. A thin and rigid horizontal rod, fixed at both ends, passes through a small hole made in a uniform disc of radius r. The disc remains suspended vertically from the rod and oscillates with small angular amplitude about the rod. Derive expressions for:

(a) the distance of the hole from the center of the disc for which the time period of such small oscillations is minimum;

(b) the corresponding minimum time period.

$[7 + 3 = 10]$

E3. An engine operates at 75% of the efficiency of a Carnot engine operating between two temperatures T_1 and T_2 ($T_1 > T_2$). This engine has a power output of 100 W and discharges heat at the rate of 300 J/s into the low-temperature reservoir with $T_2 = 27°$C. Compute the temperature T_1 of the high-temperature reservoir.

[10]

E4. A piston can move slowly inside a horizontal cylinder closed at both ends. Initially the piston divides the space inside the cylinder into two equal parts each of volume 10 liters and containing an ideal gas at the same pressure 10 kPa. What amount of work has to be done by an external agent in order to isothermally increase the volume of one part of the gas to 9 times the other part, by slowly moving the piston? Assume that $\ln(5/3) = 0.511$.

[10]

E5. Two strings are said to be *anagrams* if one string can be formed by rearranging the letters of the other, such as spot formed from post; peek formed from keep. The following function checks whether two strings are anagram or not. The function returns true if two given strings s_1 and s_2 are anagrams, and returns false if they are not anagrams. There are five missing parts in the following code, commented as `//Blank`. For each of them, copy the entire line with the comment and fill the blank appropriately in your answer sheet.

```
Boolean CheckAnagram (String s1, String s2)
{
    int LetterArray[26]; // an integer array starts from
                         // index position 0 (ZERO)
    int Length1, Length2;
    Boolean flag = true;

    Length1 = strlen(s1); //strlen(s) returns the length
                        // of a string.
    Length2 = strlen(s2);

    if(Length1 != Length2) // != is the relational operator
                           // for "not equal to" relation.
        return _________; //Blank 1

    s1.toUpper(s1); // converts a string to its uppercase
    s2.toUpper(s2);
    for(i = 0; i < 26; i++) LetterArray[i]=0;
    for(i = 0; i <  __________; i++) //Blank 2
    {
        ++LetterArray[s1[i]-65];//ASCII value of 'A' is 65
                            //and ++ denotes unary
                            // increment operator
        --LetterArray[_________]; //Blank 3
                              //-- denotes the unary
                              // decrement operator
    }
    for(i = 0; i < 26; i++)
        if(__________!=0) //Blank 4
            { flag=___________; //Blank 5
              break;
            }
    return flag;
```

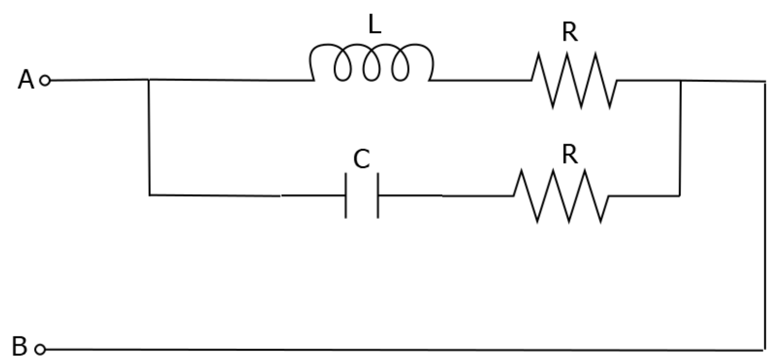
```
}
```

$[5 \times 2 = 10]$

E6. **A proton beam passes without deviation through a region of space where there are uniform, transverse, and mutually perpendicular electric and magnetic fields with field strengths $E = 120$ kV/m and $B = 50 \times 10^{-3}$ T respectively. The beam strikes a grounded target. Find the force imparted by the beam on the target if the beam current is equal to 0.8 mA. Assume the charge to mass ratio of a proton to be 10^8 coulombs/kg.**

[10]

E7. **In the circuit shown below, determine the resonant frequency and the resonant impedance between terminals A and B, when $R = 100\ k\Omega$, $C = 10\ \mu F$, and $L = 100\ mH$.**
Find the condition in terms of R, C and L, under which the impedance between A and B at resonance equals R.



$[(3 + 4) + 3 = 10]$

E8. **A short shunt $230\,$V D.C. motor runs with a speed of 1200 rpm at no load. The motor has 4 poles and 2000 conductors connected in 8 parallel paths. The magnetic flux per pole is 11×10^{-3} weber. The resistances equivalent to shunt field and armature winding are $115\,\Omega$ and $1\,\Omega$ respectively, while the voltage drop at each brush is 1 V.**

(a) **At no load, calculate**
   (i) **back e.m.f. generated across the armature,**
   (ii) **current drawn from the 230 V supply.**

(b) **At full load the motor draws a current of 32 A. Calculate**

   (i) **back e.m.f. generated across the armature,**

(ii) **kilowatt equivalent of the power delivered to the load.**

$[(2 + 2) + (3 + 3) = 10]$

E9. For the n-p-n transistor circuit C_1 shown below, assume that $V_{BE} < 0.1$V in the cut-off region and is in the range 0.7 V – 0.8 V when in saturation, while $V_{CE} = 0.2$V in the saturation region. Assume the DC current gain $\beta = 20$, and for each diode in the circuits C_1 and C_2, the forward direction drop is 0.7 V.

(a) **Determine the output voltage v_o of the circuit C_1 in the cases when**

(i) $v_{i_1} = 0.2$ **V**, $v_{i_2} = v_{i_3} = 5$ **V.**

(ii) $v_{i_1} = v_{i_2} = v_{i_3} = 5$ **V.**

(b) **Next, consider that the output of C_1 is connected to the input v_i of each of the k copies of circuit C_2. For case (ii) above, determine the maximum value of k such that the output voltage of C_1, *i.e.*, v_o remains unchanged.**

[(2+5)+3=10]

E10. Using J-K flip-flops, design a 4-bit Möbius counter which begins with all the output bits as 0's. Starting from the left, these bits toggle to 1, one after another, till all the output bits are 1's. Then these bits become 0, one after another, from the left till all are 0's.

(a) Provide the state transition table.

(b) Derive the excitation functions (in the minimized form) of the flip-flops.

(c) Draw the corresponding circuit.

$[3 + 4 + 3 = 10]$

Section III : Mathematics

M1. **(a)** **Show that for every $\theta \in (0, \pi/2)$, there exists a unique real x_θ such that**

$$(\sin\theta)^{x_\theta} + (\cos\theta)^{x_\theta} = \frac{3}{2}.$$

(b) **Consider the following improper integral**

$$\int_0^4 \frac{x}{|x^2 - 4|^{1/3}}\, \mathrm{d}x.$$

Justify whether this improper integral converges.

[$8 + 7 = 15$]

M2. **Let a_n be a decreasing sequence such that $\sum_{n=1}^{\infty} a_n$ is convergent. Show that the sequence na_n goes to zero as $n \to \infty$.**

[15]

M3. **Let $f : (-1, 1) \to \mathbb{R}$ be twice differentiable such that $f(1/n) = 0$ for all $n \in \mathbb{N}$. Show that (i) $f'(0) = 0$, and (ii) $f''(0) = 0$.**

[$7 + 8 = 15$]

M4. **(a)** **Let A and B be two invertible $n \times n$ real matrices. Show that**

$$\det(xA + (1 - x)B) = 0$$

has finitely many real solutions for x.

(b) **Suppose W is a subspace of $\mathbb{R}^n$ of dimension d. Show that**

$$|\{(x_1, \ldots, x_n) \in W : x_i \in \{0, 1\}\}| \leq 2^d.$$

[$10 + 5 = 15$]

M5. Suppose the collection $\{A_1, \ldots, A_k\}$ forms a group under matrix multiplication, where each A_i is an $n \times n$ real matrix. Let $A = \sum_{i=1}^{k} A_i$.

(a) Show that $A^2 = kA$.

(b) If the trace of A is zero, then show that A is the zero matrix.

$[5 + 10 = 15]$

M6. (a) Let G be a non-cyclic group of order 57. Prove that there is a unique Sylow-19 subgroup of G. Hence or otherwise, determine the number of elements in G of order 3.

(b) For a ring D, $a \in D$ and $n \in \mathbb{N}$, we define $n \cdot a$ as $a + \cdots + a$ (n times). Suppose D is an integral domain such that $n \cdot a = 0$ for some nonzero $a \in D$ and some positive integer n. Show that D has non-zero characteristic.

$[(6 + 5) + 4 = 15]$

M7. (a) Let

$$H_k(n) = \sum_{1 \leq i_1 < i_2 < \cdots < i_k \leq n} i_1 i_2 \cdots i_k.$$

Show that $H_k(n) = nH_{k-1}(n-1) + H_k(n-1)$.

(b) Let G be a simple graph with $n \geq 2$ vertices and m edges. Prove that if $m > \binom{n-1}{2}$, then G is connected.

$[7 + 8 = 15]$

Section IV : Physics

P1. **Consider a particle of mass m moving in a plane $\{r, \theta\}$ under a central force**

$$F(r) = -\frac{k}{r^2} + \frac{k^{'}}{r^3}.$$

where $k > 0$ and $k^{'} < 0$.

(a) **Write the equation of motion for the particle using the Lagrangian technique.**

(b) **Show that the orbital angular momentum l is conserved for the system.**

(c) **If $l^2 > -mk^{'}$, show that the trajectory of the particle is given by**

$$r = \left[A \cos\left(\sqrt{1 + \frac{mk'}{l^2}}\ \theta \right) + \frac{mk}{l^2 + mk'} \right]^{-1}.$$

$[5 + 2 + 8 = 15]$

P2. (a) **An atomic clock is a classic example of time dilation for atomic systems. Its working principle is based on the fact that an atom makes a transition between two of its internal energy states, and the frequency change due to this transition can be measured in a spectroscope. Consider an atom of mass M undergoing this transition. Using the average kinetic energy of thermal motion of the atom, derive an expression for the observed change in frequency due to this transition at temperature T. Assume the Boltzmann constant as k and the speed of light in vacuum as c.**

(b) **In an inertial frame, two events have the space-time coordinates $\{x_1, y, z, t_1\}$ and $\{x_2, y, z, t_2\}$ respectively. Let $x_2 - x_1 = 3c\ (t_2 - t_1)$, where c represents the velocity of light in vacuum. Consider another inertial frame which moves along the x-axis with velocity u with respect to the first one. Find the value of u for which the events are simultaneous in the second frame.**

$[10 + 5 = 15]$

P3. (a) **Consider a particle of mass m inside a one-dimensional box of length L. At $t = 0$, let the state of the particle be given by the following normalized wave function:**

$$\psi(x,0) = \begin{cases} A\sin(\frac{4\pi x}{L})\cos(\frac{3\pi x}{L}) & \textbf{if } 0 \le x \le L \\ 0 & \textbf{otherwise} \end{cases}$$

(i) **Derive an expression for A.**

(ii) **What is the average momentum of the system at $t = 0$?**

(iii) **Find the wave function at time $t = T$.**

(b) **Consider a simple harmonic oscillator of mass m in one dimension with the following Hamiltonian:**

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2,$$

$\hat{p}$ and $\hat{x}$ being the momentum and position operators, respectively and ω be the frequency. Show that

$$\left\langle n \left| \frac{1}{2}m\omega^2\hat{x}^2 \right| n \right\rangle = \left\langle n \left| \frac{\hat{p}^2}{2m} \right| n \right\rangle,$$

where $|n\rangle$ is any eigenstate of the Hamiltonian. You can use the creation and annihilation operators to solve the problem.

$[(2 + 3 + 3) + 7 = 15]$

P4. (a) **N particles are distributed among three states with energy levels $E = 0$, $E = kT$ and $E = 2kT$ according to the Maxwell-Boltzmann distribution. If the total equilibrium energy is 2000kT, calculate the value of N.**

(b) (i) **Find the radius of the Fermi sphere (k_F) of free electrons in a monovalent metal with an FCC structure in which the volume of the unit cell is a^3.**

(ii) **In a band structure calculation, the energy dispersion relation for electrons is found to be**

$$E(k) = E_0(\cos k_x a + \cos k_y a + \cos k_z a)$$

where E_0 is a constant and a is the lattice spacing. Determine the effective mass at the boundary of the first Brillouin Zone.

$[7 + (4 + 4) = 15]$

P5. (a) **A charged particle moves in an arbitrary magnetic field $\overrightarrow{B}$ and a time-independent electric field $\overrightarrow{E} = -\bigtriangledown\phi$, where ϕ is the scalar potential. Using Lorentz force law, show that the energy of the particle is constant.**

(b) $100\,\mu$**C of charge is uniformly distributed on the slant surface of a right circular cone having semi-vertical angle 30^0 and height 30 cm. The cone is uniformly rotated about its axis at an angular speed of $\frac{300\sqrt{3}}{\pi}$ rpm. Compute the magnetic dipole moment associated with the cone.**

$[7 + 8 = 15]$

P6. For the transistor configuration shown below:

(a) **Find the Q-point (I_c, V_{CE}).**

(b) **Find the values of the current through the resistances marked as R_c and R_L. For the BJT, assume that $\beta_{dc} = 200$ and $I_c \approx I_E$.**

[(4+3)+(4+4)=15]

P7. (a) **Consider the circuit below with ideal op-amps. Find v_0 and i_0.**

(b) **Draw the diagram of a circuit that counts the number of ones in a 7-bit input using only full-adders. Use as few full-adders as possible. Recall that a full adder receives two input bits along with an extra carry bit, and outputs two bits, namely, the sum and the carry bit.**

$[5 + 10 = 15]$

Section V : Statistics

S1. Let $X_1, X_2, \ldots, X_{2n+1}$ be independent and identically distributed with probability density function

$$f_\theta(x) = \begin{cases} 1 + \cos(2\pi(x-\theta)) & \textbf{if } \theta - 1/2 \leq x \leq \theta + 1/2 \\ 0 & \textbf{otherwise} \end{cases}$$

Define $X_{min} = \min\{X_1, X_2, \ldots, X_{2n+1}\}$, $X_{max} = \max\{X_1, X_2, \ldots, X_{2n+1}\}$ and $\widetilde{X} =$ median$\{X_1, X_2, \ldots, X_{2n+1}\}$.

(a) Show that $\theta - X_{min}$ and $X_{max} - \theta$ have the same distribution.

(b) Prove that $\widetilde{X}$ is an unbiased estimator of θ.

(c) For $\theta = 1$, find the value of α that maximizes the probability $P(\alpha \leq X_1 \leq \alpha + 0.2)$.

[5 + 5 + 5 = 15]

S2. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed as $U(0, \theta)$.

(a) Find the maximum likelihood estimator of θ.

(b) Consider two estimators $T_1 = c_1\bar{X}$ and $T_2 = c_2\widehat{\theta}$, where $\bar{X} = \sum_{i=1}^n X_i/n$ and $\widehat{\theta}$ is the maximum likelihood estimator of θ. Find c_1 and c_2 such that T_1 and T_2 are unbiased for θ.

(c) Which of the two estimators above will you prefer? Justify your answer.

[3 + 6 + 6 = 15]

S3. Let X be a random variable taking values $1, 2, 3, \ldots$. If $E(X)$ is finite, show that

$$\sum_{k\geq 1} 2^{k-1} P(X \geq 2^k) \leq E(X) \leq \sum_{k \geq 0} 2^k P(X \geq 2^k).$$

[15]

S4. Suppose that X follows a beta distribution with probability density function

$$f_\theta(x) = \frac{1}{\textbf{Beta}(\theta+1, 11-\theta)} x^\theta (1-x)^{10-\theta}; \quad 0 \leq x \leq 1; \quad 0 \leq \theta \leq 10.$$

(a) **Describe how you will construct a most powerful test of size α based on a single observation for testing the null hypothesis $H_0 : \theta = 5$ against the alternative hypothesis $H_1 : \theta = 6$.**

(b) **Show that the power of this test is larger than α.**

(c) **Is this a uniformly most powerful test for $H_0 : \theta = 5$ against $H_1 : \theta > 5$? Justify your answer.**

$[5 + 5 + 5 = 15]$

S5. (a) **Suppose that we want to use the method of least squares to fit a regression line of the form $y = \alpha + \beta x$, to the data set consisting of n observations (x_i, y_i), $i = 1, 2, \ldots, n$, where all x_i's are distinct. For $1 \leq i < j \leq n$, let β_{ij} be the slope of the line passing through (x_i, y_i) and (x_j, y_j). Show that the least squares estimate of the regression coefficient can be expressed as an weighted average of the β_{ij}s.**

(b) **Consider a data set $(x_1, y_1), (x_2, y_2), \ldots, (x_{20}, y_{20})$, where $x_i = a$ for all $1 \leq i \leq 10$ and $x_i = b$ for all $10 < i \leq 20$ ($a \neq b$). Two regression functions**

$$y = \alpha_0 + \alpha_1 x \quad \textbf{and} \quad y = \beta_0 + \beta_1 x^3$$

are fitted to this data set using the method of least squares. Which of these two models will lead to smaller residual sum of squares? Justify your answer.

$[8 + 7 = 15]$

S6. From a set of N elements, a non-empty subset is chosen at random such that all non-empty subsets are equally likely to be selected. Let X be the number of elements in the chosen subset. Show that

(a) $E(X) = N/[2 - (\frac{1}{2})^{N-1}]$.

(b) **Var**$(X) = [N2^{2N-2} - N(N+1)2^{N-2}]/(2^N - 1)^2$.

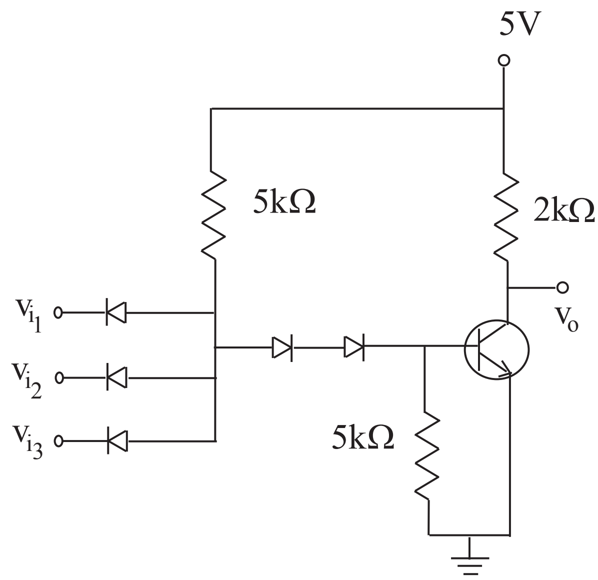
$[6 + 9 = 15]$

S7. In a game, three players A, B and C roll a perfect dice. The players play in the following sequence $A, B, C, A, B, C, A, \ldots$. Find the probability of the following events.

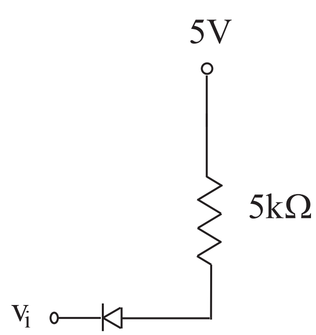
(a) **A is the first to throw a 'six', B the second and C the third.**

(b) **The first 'six' is thrown by A, the second 'six' by B and the third 'six' by C.**

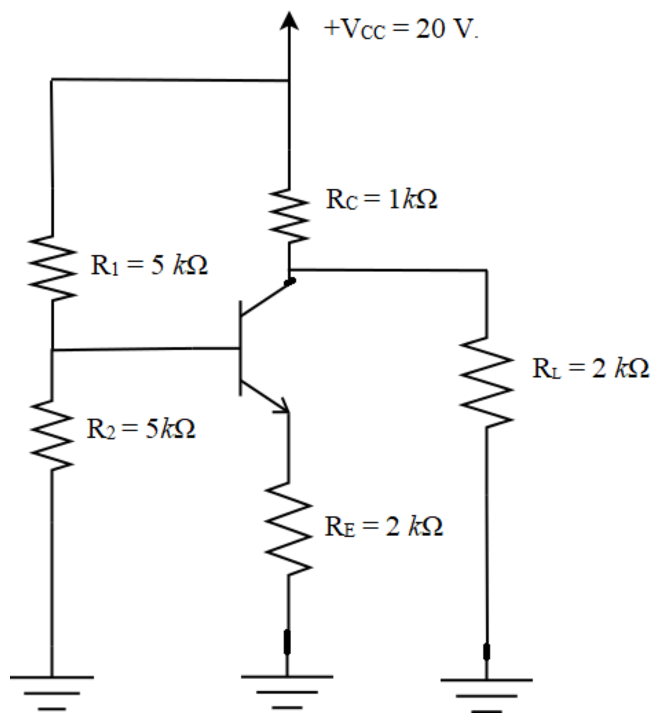
$[7 + 8 = 15]$

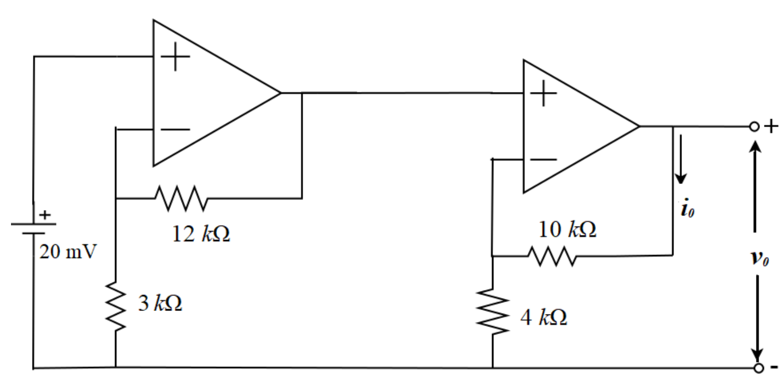


Circuit C_1



Circuit C_2


+Vcc = 20 V.
Rc = 1kΩ
R1 = 5 kΩ
RL = 2 kΩ
R2 = 5kΩ
RE = 2 kΩ


20 mV
12 kΩ
3 kΩ
10 kΩ
4 kΩ
i0
v0