1.

If $F(x) = \int_e^x \log t \, dt$ for all positive x, then $F'(x) =$

(A) x

(B) $\frac{1}{x}$

(C) $\log x$

(D) $x \log x$

(E) $x \log x - 1$

2.

If $F(1) = 2$ and $F(n) = F(n-1) + \frac{1}{2}$ for all integers $n > 1$, then $F(101) =$

(A) 49
(B) 50
(C) 51
(D) 52
(E) 53

3.

If $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is invertible under matrix multiplication, then its inverse is

(A) $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

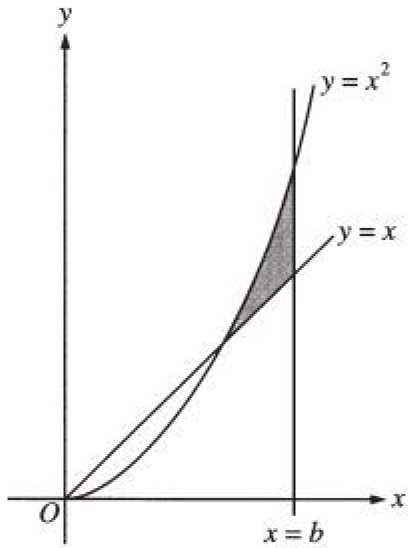
(B) $\frac{1}{a^2 + b^2}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(C) $\frac{1}{a^2 + b^2}\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(D) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(E) $\frac{1}{a^2 - b^2}\begin{pmatrix} -b & a \\ a & b \end{pmatrix}$

4.



If $b > 0$ and if $\int_0^b x\,dx = \int_0^b x^2\,dx$, then the area of the shaded region in the figure above is

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

5.

Consider the following sequence of instructions.

1. Set $k = 999$, $i = 1$, and $p = 0$.
2. If $k > i$, then go to step 3; otherwise go to step 5.
3. Replace i with $2i$ and replace p with $p + 1$.
4. Go to step 2.
5. Print p.

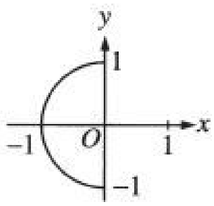
If these instructions are followed, what number will be printed at step 5 ?

(A) 1
(B) 2
(C) 10
(D) 512
(E) 999

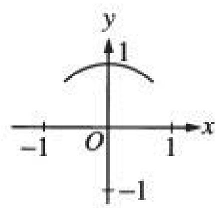
6.

Which of the following indicates the graph of $\left\{(\sin t, \cos t)\colon -\frac{\pi}{2} \le t \le 0\right\}$ in the xy-plane?

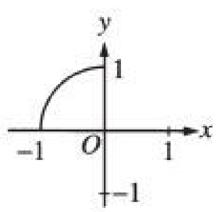
(A)



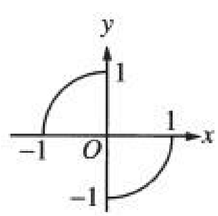
(D)



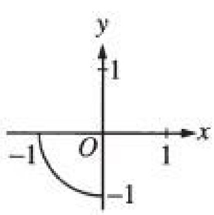
(B)



(E)



(C)



7.

$$\int_0^1 \frac{x}{1 + x^2}\, dx =$$

(A) 1

(B) $\frac{\pi}{4}$

(C) $\tan^{-1} \frac{\sqrt{2}}{2}$

(D) $\log 2$

(E) $\log \sqrt{2}$

8.

If S is a nonempty finite set with k elements, then the number of one-to-one functions from S onto S is

(A) $k!$
(B) k^2
(C) k^k
(D) 2^k
(E) 2^{k+1}

9.

Let g be the function defined on the set of all real numbers by

$$g(x) = \begin{cases} 1 & \text{if } x \text{ is rational,} \\ e^x & \text{if } x \text{ is irrational.} \end{cases}$$

Then the set of numbers at which g is continuous is

(A) the empty set
(B) $\{0\}$
(C) $\{1\}$
(D) the set of rational numbers
(E) the set of irrational numbers

10.

For all real numbers x and y, the expression $\frac{x + y + |x - y|}{2}$ is equal to

(A) the maximum of x and y

(B) the minimum of x and y

(C) $|x + y|$

(D) the average of $|x|$ and $|y|$

(E) the average of $|x + y|$ and $x - y$

11.

Let B be a nonempty bounded set of real numbers and let b be the least upper bound of B. If b is not a member of B, which of the following is necessarily true?

(A) B is closed.
(B) B is not open.
(C) b is a limit point of B.
(D) No sequence in B converges to b.
(E) There is an open interval containing b but containing no point of B.

12.

A drawer contains 2 blue, 4 red, and 2 yellow socks. If 2 socks are to be randomly selected from the drawer, what is the probability that they will be the same color?

(A) $\frac{2}{7}$

(B) $\frac{2}{5}$

(C) $\frac{3}{7}$

(D) $\frac{1}{2}$

(E) $\frac{3}{5}$

13.

Let $\mathbb{R}$ be the set of real numbers and let f and g be functions from $\mathbb{R}$ into $\mathbb{R}$. The negation of the statement

"For each s in $\mathbb{R}$, there exists an r in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$."

is which of the following?

(A) For each s in $\mathbb{R}$, there does not exist an r in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$.
(B) For each s in $\mathbb{R}$, there exists an r in $\mathbb{R}$ such that $f(r) > 0$ and $g(s) \leq 0$.
(C) There exists an s in $\mathbb{R}$ such that for each r in $\mathbb{R}$, $f(r) > 0$ and $g(s) \leq 0$.
(D) There exists an s in $\mathbb{R}$ and there exists an r in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.
(E) For each r in $\mathbb{R}$, there exists an s in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.

14.

If g is a function defined on the open interval (a, b) such that $a < g(x) < x$ for all $x \in (a, b)$, then g is

(A) an unbounded function
(B) a nonconstant function
(C) a nonnegative function
(D) a strictly increasing function
(E) a polynomial function of degree 1

15.

For what value (or values) of m is the vector $(1, 2, m, 5)$ a linear combination of the vectors $(0, 1, 1, 1)$, $(0, 0, 0, 1)$, and $(1, 1, 2, 0)$?

(A) For no value of m
(B) -1 only
(C) 1 only
(D) 3 only
(E) For infinitely many values of m

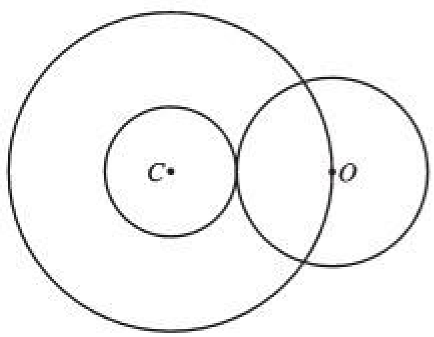
16.

For a function f, the finite differences $\Delta f(x)$ and $\Delta^2 f(x)$ are defined by $\Delta f(x) = f(x+1) - f(x)$ and $\Delta^2 f(x) = \Delta f(x+1) - \Delta f(x)$. What is the value of $f(4)$, given the following partially completed finite difference table?

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	−1	4	
2		−2	6
3			
4			

(A) −5
(B) −1
(C) 1
(D) 3
(E) 5

17.



In the figure above, the annulus with center C has inner radius r and outer radius 1. As r increases, the circle with center O contracts and remains tangent to the inner circle. If $A(r)$ is the area of the annulus and $a(r)$ is the area of the circular region with center O, then $\lim_{r \to 1^-} \frac{A(r)}{a(r)} =$

(A) 0

(B) $\frac{2}{\pi}$

(C) 1

(D) $\frac{\pi}{2}$

(E) ∞

18.

Which of the following are multiplication tables for groups with four elements?

I.

	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

II.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	a
d	d	c	a	b

III.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	c	d
d	d	c	d	c

(A) None
(B) I only
(C) I and II only
(D) II and III only
(E) I, II, and III

19.

Which of the following statements are true for every function f, defined on the set of all real numbers, such that $\lim_{x \to 0} \frac{f(x)}{x}$ is a real number L and $f(0) = 0$?

I. f is differentiable at 0.

II. $L = 0$

III. $\lim_{x \to 0} f(x) = 0$

(A) None
(B) I only
(C) III only
(D) I and III only
(E) I, II, and III

20.

What is the area of the region bounded by the coordinate axes and the line tangent to the graph of $y = \frac{1}{8}x^2 + \frac{1}{2}x + 1$ at the point $(0, 1)$?

(A) $\frac{1}{16}$

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) 1

(E) 2

21.

. Let $\mathbb{Z}$ be the group of all integers under the operation of addition. Which of the following subsets of $\mathbb{Z}$ is NOT a subgroup of $\mathbb{Z}$?

(A) $\{0\}$
(B) $\{n \in \mathbb{Z}: n \geq 0\}$
(C) $\{n \in \mathbb{Z}: n \text{ is an even integer}\}$
(D) $\{n \in \mathbb{Z}: n \text{ is divisible by both 6 and 9}\}$
(E) $\mathbb{Z}$

22.

. In the Euclidean plane, point A is on a circle centered at point O, and O is on a circle centered at A. The circles intersect at points B and C. What is the measure of angle BAC ?

(A) 60°
(B) 90°
(C) 120°
(D) 135°
(E) 150°

23.

Which of the following sets of vectors is a basis for the subspace of Euclidean 4-space consisting of all vectors that are orthogonal to both $(0, 1, 1, 1)$ and $(1, 1, 1, 0)$?

(A) $\{(0, -1, 1, 0)\}$
(B) $\{(1, 0, 0, 0), (0, 0, 0, 1)\}$
(C) $\{(-2, 1, 1, -2), (0, 1, -1, 0)\}$
(D) $\{(1, -1, 0, 1), (-1, 1, 0, -1), (0, 1, -1, 0)\}$
(E) $\{(0, 0, 0, 0), (-1, 1, 0, -1), (0, 1, -1, 0)\}$

24.

. Let f be the function defined by $f(x, y) = 5x - 4y$ on the region in the xy-plane satisfying the inequalities $x \leq 2$, $y \geq 0$, $x + y \geq 1$, and $y - x \leq 0$. The maximum value of f on this region is

(A) 1
(B) 2
(C) 5
(D) 10
(E) 15

25.

. Let f be the function defined by

$$f(x) = \begin{cases} -x^2 + 4x - 2 & \text{if } x < 1, \\ -x^2 + 2 & \text{if } x \geq 1. \end{cases}$$

Which of the following statements about f is true?

(A) f has an absolute maximum at $x = 0$.
(B) f has an absolute maximum at $x = 1$.
(C) f has an absolute maximum at $x = 2$.
(D) f has no absolute maximum.
(E) f has local maxima at both $x = 0$ and $x = 2$.

26.

Let f be a function such that $f(x) = f(1 - x)$ for all real numbers x. If f is differentiable everywhere, then $f'(0) =$

(A) $f(0)$
(B) $f(1)$
(C) $-f(0)$
(D) $f'(1)$
(E) $-f'(1)$

27.

If V_1 and V_2 are 6-dimensional subspaces of a 10-dimensional vector space V, what is the smallest possible dimension that $V_1 \cap V_2$ can have?

(A) 0
(B) 1
(C) 2
(D) 4
(E) 6

28.

Assume that p is a polynomial function on the set of real numbers. If $p(0) = p(2) = 3$ and $p'(0) = p'(2) = -1$, then $\int_0^2 xp''(x)\,dx =$

(A) -3
(B) -2
(C) -1
(D) 1
(E) 2

29.

Suppose B is a basis for a real vector space V of dimension greater than 1. Which of the following statements could be true?

(A) The zero vector of V is an element of B.
(B) B has a proper subset that spans V.
(C) B is a proper subset of a linearly independent subset of V.
(D) There is a basis for V that is disjoint from B.
(E) One of the vectors in B is a linear combination of the other vectors in B.

30.

Which of the following CANNOT be a root of a polynomial in x of the form $9x^5 + ax^3 + b$, where a and b are integers?

(A) -9

(B) -5

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) 9

31.

When 20 children in a classroom line up for lunch, Pat insists on being somewhere ahead of Lynn. If Pat's demand is to be satisfied, in how many ways can the children line up?

(A) 20!

(B) 19!

(C) 18!

(D) $\frac{20!}{2}$

(E) $20 \cdot 19$

32.

How many integers from 1 to 1,000 are divisible by 30 but not by 16 ?

(A) 29
(B) 31
(C) 32
(D) 33
(E) 38

33.

Suppose f is a differentiable function for which $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f'(x)$ both exist and are finite. Which of the following must be true?

(A) $\lim_{x\to\infty} f'(x) = 0$

(B) $\lim_{x\to\infty} f''(x) = 0$

(C) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} f'(x)$

(D) f is a constant function.

(E) f' is a constant function.

34.

In xyz-space, an equation of the plane tangent to the surface $z = e^{-x} \sin y$ at the point where $x = 0$ and $y = \frac{\pi}{2}$ is

(A) $x + y = 1$
(B) $x + z = 1$
(C) $x - z = 1$
(D) $y + z = 1$
(E) $y - z = 1$

35.

For each real number x, let $\mu(x)$ be the mean of the numbers 4, 9, 7, 5, and x; and let $\eta(x)$ be the median of these five numbers. For how many values of x is $\mu(x) = \eta(x)$?

(A) None
(B) One
(C) Two
(D) Three
(E) Infinitely many

36.

Which of the following integrals on the interval $\left[0, \frac{\pi}{4}\right]$ has the greatest value?

(A) $\int_0^{\frac{\pi}{4}} \sin t \, dt$

(B) $\int_0^{\frac{\pi}{4}} \cos t \, dt$

(C) $\int_0^{\frac{\pi}{4}} \cos^2 t \, dt$

(D) $\int_0^{\frac{\pi}{4}} \cos 2t \, dt$

(E) $\int_0^{\frac{\pi}{4}} \sin t \cos t \, dt$

37.

Which of the following integrals on the interval $\left[0, \frac{\pi}{4}\right]$ has the greatest value?

(A) $\int_0^{\frac{\pi}{4}} \sin t \, dt$

(B) $\int_0^{\frac{\pi}{4}} \cos t \, dt$

(C) $\int_0^{\frac{\pi}{4}} \cos^2 t \, dt$

(D) $\int_0^{\frac{\pi}{4}} \cos 2t \, dt$

(E) $\int_0^{\frac{\pi}{4}} \sin t \cos t \, dt$

38.

Consider the function f defined by $f(x) = e^{-x}$ on the interval $[0, 10]$. Let $n > 1$ and let $x_0, x_1, \ldots, x_n$ be numbers such that $0 = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = 10$. Which of the following is greatest?

(A) $\sum_{j=1}^{n} f(x_j)(x_j - x_{j-1})$

(B) $\sum_{j=1}^{n} f(x_{j-1})(x_j - x_{j-1})$

(C) $\sum_{j=1}^{n} f\left(\frac{x_j + x_{j-1}}{2}\right)(x_j - x_{j-1})$

(D) $\int_0^{10} f(x)\, dx$

(E) 0

39.

A fair coin is to be tossed 8 times. What is the probability that more of the tosses will result in heads than will result in tails?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{87}{256}$

(D) $\frac{23}{64}$

(E) $\frac{93}{256}$

40.

The function $f(x, y) = xy - x^3 - y^3$ has a relative maximum at the point

(A) (0, 0)

(B) (1, 1)

(C) (−1, −1)

(D) (1, 3)

(E) $\left(\frac{1}{3}, \frac{1}{3}\right)$