Question: If a wire of length $l$ has a resistance of $R$, is stretched by 25%. The percentage change in its resistance is?

Options: 
(a) 25%  
(b) 50%  
(c) 45.25%  
(d) 56.25%  
Answer: (d)  
Solution: 
\[ R = \rho \frac{l}{A} = \rho \frac{l^2}{V} \quad (\because V = AI) \]  
\[ R' = \rho \frac{(1.25)^2 l^2}{V} = 1.5625R \]  
\[ \%R = \left( \frac{R' - R}{R} \right) \times 100 = (1.5625 - 1) \times 100 \]  
\[ = 56.25\% \]

Question: A chord is tied to a wheel of moment of inertia $I$ and radius $r$. The other end is attached to a mass ‘$m$’ as shown. If the mass ‘$m$’ falls by a height ‘$h$’ then the square of angular of speed of the wheel is?

Options: 
(a) $\frac{mgh}{I + mr^2}$  
(b) $\frac{2mgh}{I + mr^2}$  
(c) $\frac{2mgh}{2I + mr^2}$  
(d) $\frac{mgh}{2I + mr^2}$  
Answer: (b)  
Solution: 

Considering no slipping between chord and wheel and considering no energy loss due to friction.
So, by Energy Conservation:-

\[ mgh = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 \]

\[ \Rightarrow mgh = \frac{1}{2} m (\omega r)^2 + \frac{1}{2} I \omega^2 \quad \therefore \omega = \omega r \]

\[ \Rightarrow \omega^2 = \frac{2mgh}{mr^2 + I} \]

**Question:** What is the recoil velocity of Hydrogen atom when a photon is emitted due to corresponding transition from \( n = 5 \) to \( n = 1 \). (\( R = \text{Rydberg’s constant.} \ m_H = \text{mass of hydrogen atom} \))

**Options:**

(a) \( \frac{hR}{m_H} \)

(b) \( \frac{hR}{25 m_H} \)

(c) \( \frac{4hR}{25 m_H} \)

(d) \( \frac{24hR}{25 m_H} \)

**Answer:** (d)

**Solution:**

Energy released during transition of \( e^- \) from \( n = 5 \) to \( n = 1 \)

\[ E = E_5 - E_1 = Rhc \left( -\frac{1}{(5)^2} - \left(-\frac{1}{(1)^2}\right) \right) \]

\[ \Rightarrow E = \frac{24}{25} Rhc \ldots (i) \]

So momentum of Photon released would be:-

\[ E = mc^2 = (mc).c = pc \]

Using equation (i)

\[ p = \frac{E}{c} = \frac{24}{25} Rh \]

So Recoil velocity of H-atom would be:

By conservation of linear Momentum.

\[ \Rightarrow m_H v_H = p = \frac{24}{25} Rh \]

\[ v_H = \frac{24}{25} \frac{Rh}{m_H} \]

**Question:** For earth’s gravitation

Given: \([g_A = g_C < g_B]\). Find \( \frac{OA}{AB} \).
Options:
(a) 1 : 1
(b) 2 : 3
(c) 4 : 5
(d) 4 : 9
Answer: (c)
Solution:
\[
g_c = \frac{GM}{(R + R/2)^2} = \frac{4GM}{9R^2} = \frac{4}{9}g
\]
\[
g_A = g \frac{x}{R} \quad \text{(where } x = OA\text{)}
\]
So, if \( g_A = g_C \)
\[
\Rightarrow 4g = g \frac{x}{R}
\]
\[
\Rightarrow x = \frac{4}{9}R
\]
To find :- \( \frac{OA}{AB} = \frac{x}{R-x} = \frac{4}{5} \) (Where \( AB = OB - OA \) and \( OB = R \))

Question: Find Dimension of \( \frac{C}{V} \)?

Options:
(a) \( [M^{-2}L^4T^{-7}A^2] \)
(b) \( [M^2L^4T^{-6}A^{-2}] \)
(c) \( [M^2L^4T^6A^2] \)
(d) \( [M^2L^4T^{-6}A^2] \)
Answer: (a)
Solution:
\[
\frac{C}{V} = \frac{Q}{V^2} = \frac{Q}{(W/Q)^2} = \frac{Q^3}{W^2} = \left( \frac{It}{W^2} \right)^2
\]
\[
\left[ \frac{C}{V} \right] = \left[ \frac{It}{W^2} \right] = \frac{A^4T^3}{(ML^2T^{-2})^2}
\]
\[
= M^{-2}L^4T^{-7}A^3
\]
**Question:** An aeroplane with its wings spread 10 m is flying with speed 180 kph in horizontal direction. The total intensity of earth’s field is $2.5 \times 10^{-4}$ Tesla and angle of dip is $60^\circ$. Then find emf induced between the tips of the plane wings.

**Options:**
(a) 108 mV  
(b) 54 mV  
(c) 216 mV  
(d) 140 mV  

**Answer:** (a)

**Solution:**

![Diagram](image)

$B = 2.5 \times 10^{-4} T$

$\delta = 60^\circ$

$B_v = B \sin \delta$

$= 2.5 \times 10^{-4} \sin 60 = \frac{2.5\sqrt{3}}{2} \times 10^{-4} T$

$l = 10 m$

$V = 180 km/h = \frac{180 \times 5}{18} = 50 m/s$

$|E| = Bvlv = \frac{2.5\sqrt{3}}{2} \times 10^{-4} \times 10 \times 50$

$= 1082 \times 10^{-4} V = 108 \times 10^{-3} V = 108 mV$

**Question:** A person walks parallel to a 50 cm wide plane mirror as shown. How much distance will he be able to see the image of a source placed 60 cm in point of it?

**Options:**
(a) 50 cm  
(b) 100 cm  
(c) 150 cm
(d) 200 cm
Answer: (c)
Solution:

Man can see image by while traversing MM'
Now,
\[
\frac{25}{60} = \frac{x}{180} \Rightarrow x = 75
\]
\[MM' = 2x = 150 \text{ cm}\]

**Question:** Find the time taken by the block to reach the bottom of inclined plane. \(E = 200 \text{ i N/C}, M = 1 \text{ kg}, q = 5 \text{ mC}, g = 10 \text{ m/s}^2, \mu = 0.2\)

**Options:**
(a) 1.35 s
(b) 1.65 s
(c) 1.9 s
(d) 2.3 s
Answer: (a)
Solution:

Net force along the incline
\[ F = mg \sin \theta - (\mu N + qE \cos \theta) \]
\[ = mg \sin \theta - \mu (mg \cos \theta + qE \sin \theta) - qE \cos \theta \]
\[ = 1 \times 10 \sin 30 - 0.2 (1 \times 10 \cos 30 + 200 \times 5 \times 10^{-3} \times \sin 30) - 200 \times 5 \times 10^{-3} \cos 30 \]
\[ = 5 - 0.2 \left(5\sqrt{3} + 0.5\right) - \sqrt{3}/2 \]
\[ = 2.3N \]
\[ a = \frac{F}{m} = \frac{2.3}{1} = 2.3 \text{ m/s}^2 \]

Time taken to slide down 2 m long

Incline \( t = \sqrt{\frac{25}{a}} = \sqrt{\frac{2 \times 2}{2.3}} = 1.32 \text{s} \)

**Question:** Statement 1: A seconds pendulum, has a time period of 1 second.

**Statement 2:** It takes precisely 1 second to move between the two extreme positions.

**Options:**
(a) Statement 1 is false, Statement 2 is true
(b) Statement 1 is true, Statement 2 is true
(c) Statement 1 is true, Statement 2 is false
(d) Statement 1 is false, Statement 2 is false

**Answer:** (a)

**Solution:**
[Statement 1 is false, Statement 2 is true]

A **seconds pendulum** is a pendulum whose **period** is precisely two **seconds**; one **second** for a swing in one direction and one second for the return swing.

So it will take 1 second to move between two extreme positions.

Thus statement 1 is false and statement 2 is true.

**Question:** Velocity v/s position graph of a body performing SHM is

**Options:**
(a) ellipse
(b) circle
(c) parabola
(d) straight line

**Answer:** (a)

**Solution:**
For SHM
\[ x = A \sin \omega t \]  
\[ v = \frac{d(x)}{dt} = A\omega \cos \omega t \]

From equation (i)
\[ \sin \omega t = \frac{x}{A} \Rightarrow \sin^2 \omega t = \frac{x^2}{A^2} \]  

From equation (ii)
\[ \cos \omega t = \frac{v}{A\omega} \Rightarrow \cos^2 \omega t = \frac{v^2}{A^2\omega^2} \]

Adding equation (iii) and (iv)
\[
\sin^2 \omega t + \cos^2 \omega t = \frac{x^2}{A^2} + \frac{v^2}{A^2 \omega^2}
\]
\[
\Rightarrow 1 = \frac{x^2}{A^2} + \frac{v^2}{A^2 \omega^2}
\]
This is clearly on equation of ellipse.

**Question:** A body starts from rest and moves with constant acceleration \(a\), for time \(t_1\), then it retards uniformly with \(a_2\) in time \(t_2\). Find \(t_1/t_2\).

**Options:**
(a) \(\frac{a_1}{a_2}\)  
(b) \(\frac{a_2}{a_1}\)  
(c) 1  
(d) None of these

**Answer:** (b)

**Solution:**
For acceleration period,
\[
u = 0, \quad v = u, \quad a = a_1, \quad t = t_1
\]
So, \(v = u = at \Rightarrow v = 0 + a_1t_1 \Rightarrow t_1 = \frac{v}{a_1} \) \(\ldots(i)\)

For retardation period,
\[
u = v, \quad v = 0, \quad a = -a_2, \quad t = t_2
\]
So, \(v = u + at \Rightarrow 0 = v - a_2t_2 \Rightarrow t_2 = \frac{v}{a_2} \) \(\ldots(ii)\)

On dividing equation (i) by (ii)
\[
\frac{t_1}{t_2} = \frac{a_2}{a_1}
\]

**Question:** A wire has length \(l_1\) when tension in it is \(T_1\) & \(l_2\) when tension is \(T_2\). Find the natural length of wire.

**Options:**
(a) \(\frac{T_1l_1 - T_2l_2}{T_1 - T_2}\)  
(b) \(\frac{T_2l_2 - T_1l_1}{T_1 - T_2}\)  
(c) \(\frac{T_1l_1 + T_2l_2}{T_1 + T_2}\)  
(d) \(\frac{T_2l_2 + T_1l_1}{T_1 + T_2}\)

**Answer:** (b)

**Solution:**
Let the natural length of wire be \(l_0\).
Using Hooke’s law, \( Y = \frac{TI_0}{A\Delta l} \)

Where \( \Delta l = l - l_0 \)

We get \( l - l_0 = \frac{TI_0}{AY} \)

Case 1: Tension \( T_1 \) and length of wire \( l = l_1 \)

\[ \therefore l_1 - l_0 = \frac{T_1l_0}{AY} \ldots (1) \]

Case 2: Tension is \( T_2 \) and length of wire \( l = l_2 \)

\[ \therefore l_2 - l_0 = \frac{T_2l_0}{AY} \ldots (2) \]

Dividing both equations \( \frac{l_1 - l_0}{l_2 - l_0} = \frac{T_1}{T_2} \)

\[ \Rightarrow l_0 = \frac{l_1T_2 - l_2T_1}{T_2 - T_1} \]

**Question:** A radioactive sample is undergoing \( \alpha \)-decay. At time \( t_1 \), its activity is \( A \) and at another time \( t_2 \), the activity is \( \frac{A}{5} \). What is the average life time for the sample

**Options:**
(a) \( \frac{t_2 - t_1}{\ln 2} \)
(b) \( (t_2 - t_1) \ln 5 \)
(c) \( \frac{t_2 - t_1}{\ln 5} \)
(d) \( \frac{t_2 - t_1}{2} \)

**Answer:** (c)

**Solution:**

Activity \( = \left| \frac{dN}{dt} \right| \)

At time \( t_1 \)

\[ A = N_0\lambda e^{-\lambda t_1} \ldots (1) \]

At time \( t_2 \)

\[ \frac{A}{5} = N_0\lambda e^{-\lambda t_2} \ldots (2) \]

From eq (1) & (2)

\[ 5 = \frac{e^{-\lambda t_1}}{e^{-\lambda t_2}} \]

\[ 5 = e^{-\lambda(t_2-t_1)} \]

\[ \ln 5 = -\lambda(t_2-t_1) \]

From eq (1) & (2)

\[ \lambda = \frac{\ln 5}{t_1 - t_2} \]
Mean lifetime given by $\tau = \frac{1}{\lambda}$

$$\tau = \frac{t_2 - t_1}{\ln 5}$$

**Question:** A bike starts from rest and accelerates uniformly at ‘a’ m/s$^2$ for time ‘$t_1$’ seconds. Then it retards with deceleration ‘a’ for time ‘$t_2$’ seconds with till it comes to rest. Find the average speed for the entire duration.

**Options:**
(a) $a\left(t_1 + t_2\right)$
(b) $\frac{at_1}{2}$
(c) $\frac{at_2^2}{2}$
(d) $at_1$

**Answer:** (b)

**Solution:**
Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

Initial speed is zero.
And acceleration is a.

$v = u + at$
$v = at_1$ \hspace{1cm} after time $t_1$

& $S_1 = \frac{1}{2}at_1^2$

Now,
$v = u + at$
$o = at_1 - at_2$
$at_1 = at_2 \Rightarrow t_1 = t_2$

$S_2 = at_1t_2 - \frac{1}{2}at_2^2$

Total distance $= S_1 + S_2$

$= \frac{1}{2}at_1^2 + at_1t_2 - \frac{1}{2}at_2^2$

$= \frac{1}{2}at_1^2 + at^2 - \frac{1}{2}at^2$

$S = at^2$
$t_1 + t_2 = 2t$

$\langle v \rangle = \frac{at^2}{2t}$

$= \frac{at}{2} = \frac{at_2}{2}$
**Question:** If incident say, refracted ray and normal say are represented by unit vectors $\hat{a}, \hat{b}$ and $\hat{c}$ then relation between them is?

**Options:**
(a) $\hat{a} - \hat{b} = \hat{c}$
(b) $\hat{a} \cdot (\hat{b} \times \hat{c}) = 0$
(c) $\hat{a} + \hat{c} = 2\hat{b}$
(d) $\hat{a} \times (\hat{b} \times \hat{c}) = 0$

**Answer:** (b)

**Solution:**
Let $\mu_1 < \mu_2$

All three unit vectors are coplanar, we can say this from first law of refraction
Scalar triple product is given by $\vec{A} \cdot (\vec{B} \times \vec{C})$

If $\vec{A}, \vec{B} \& \vec{C}$ vectors are coplanar then $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0 \tag{i}$

From eq. (i) we have $\hat{a} \cdot (\hat{b} \times \hat{c}) = 0$

**Question:** If the internal energy of a gas is $U = 3PV + 4$, then the gas can be?

**Options:**
(a) Monoatomic
(b) Diatomic
(c) Polyatomic
(d) Either mono or diatomic

**Answer:** (c)

**Solution:**
Given, $U = 3PV + 4$
We have $PV = nRT$
$U = 3(nRT) + 4$
Differentiating wrt temperature
$dU = 3(nRdT) + 0$

$\frac{nRdT}{2} = 3(nRdT)$

$\frac{f}{2} = 3 \Rightarrow f = 6$

It would be triatomic, suitable option is Polyatomic.
**Chemistry**

**Question:** Increasing order of $\Delta_e$H of the following elements: O, S, Se, Te (Consider both sign and magnitude)

**Options:**
(a) $S < Se < Te < O$
(b) $O < S < Se < Te$
(c) $S < O < Se < Te$
(d) $O < Te < Se < S$

**Answer:** (a)

**Solution:**
The values are,
$S = -200$ kJ/mol
$Se = -195$ kJ/mol
$Te = -190$ kJ/mol
$O = -141$ kJ/mol

So, considering both sign and magnitude, the order should be $S < Se < Te < O$

**Question:** Hybridisation order of the carbon atom from left to right is $\text{CH}_2=\text{C}=\text{CH}$–$\text{CH}_3$

**Options:**
(a) sp$^2$, sp, sp$^2$, sp$^3$
(b) sp$^2$, sp$^2$, sp$^2$, sp$^3$
(c) sp$^2$, sp, sp, sp$^3$
(d) sp, sp, sp$^2$, sp$^3$

**Answer:** (a)

**Solution:**

\[
\begin{align*}
\text{H} & \text{C} = \text{C} = \text{C} - \text{C} - \text{H} \\
\text{H} & \text{1} \text{2} \text{3} \text{4} \text{H}
\end{align*}
\]
Question: Match the following

<table>
<thead>
<tr>
<th>Column-I</th>
<th>Column-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Siderite</td>
<td>(P) Fe</td>
</tr>
<tr>
<td>(B) Calamine</td>
<td>(Q) Al</td>
</tr>
<tr>
<td>(C) Cryolite</td>
<td>(R) Zn</td>
</tr>
<tr>
<td>(D) Malachite</td>
<td>(S) Cu</td>
</tr>
</tbody>
</table>

Options:
(a) A → P; B → R; C → S; D → Q
(b) A → P; B → S; C → Q; D → Q
(c) A → P; B → R; C → Q; D → S
(d) A → Q; B → R; C → P; D → S

Answer: (c)

Solution:
Siderite (FeCO₃) is an ore of iron
Calamine (ZnCO₃) is an ore of zinc
Cryolite (Na₃AlF₆) is an ore of Aluminium
Malachite (CuCO₃, Cu(OH)₂) is an ore of copper.

Question: Which of the following groups contains both acidic oxides:

Options:
(a) N₂O, BaO
(b) CaO, SiO₂
(c) B₂O₃, SiO₂
(d) B₂O₃, CaO

Answer: (c)

Solution:
N₂O → Neutral
BaO, CaO → Basic
B₂O₃, SiO₂ → Acidic

**Question:** Match the following.

<table>
<thead>
<tr>
<th>Column-I</th>
<th>Column-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) <img src="image1.jpg" alt="Image" /></td>
<td>(P) Wurtz Reaction</td>
</tr>
<tr>
<td>(B) <img src="image2.jpg" alt="Image" /></td>
<td>(Q) Sandmeyer Reaction</td>
</tr>
<tr>
<td>(C) <img src="image3.jpg" alt="Image" /></td>
<td>(R) Fittig Reaction</td>
</tr>
<tr>
<td>(D) <img src="image4.jpg" alt="Image" /></td>
<td>(S) Gattermann Reaction</td>
</tr>
</tbody>
</table>

**Options:**
(a) A → P; B → Q; C → R; D → S
(b) A → Q; B → S; C → P; D → R
(c) A → Q; B → S; C → R; D → P
(d) A → S; B → Q; C → P; D → R

**Answer:** (b)

**Solution:** Sandmeyer takes place with Cu⁺
Gattermann takes place with CuO
Alkyl halide coupling is Wurtz
Aryl halide coupling is Fittig Reaction

**Question:** Match the following.
<table>
<thead>
<tr>
<th>Molecule</th>
<th>Bond order</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Ne₂</td>
<td>(P) 1</td>
</tr>
<tr>
<td>(B) N₂</td>
<td>(Q) 2</td>
</tr>
<tr>
<td>(C) F₂</td>
<td>(R) 0</td>
</tr>
<tr>
<td>(D) O₂</td>
<td>(S) 3</td>
</tr>
</tbody>
</table>

Options:
(a) A → R; B → S; C → Q; D → P
(b) A → R; B → S; C → P; D → Q
(c) A → R; B → P; C → Q; D → S
(d) A → R; B → Q; C → P; D → P

Answer: (b)

Solution: B.O = \( \frac{\text{No. of bonding } e^- - \text{No. of antibonding } e^-}{2} \)

a) Ne₂ = \( \frac{10 - 10}{2} = 0 \)

b) N₂ = \( \frac{10 - 4}{2} = 3 \)

c) F₂ = \( \frac{10 - 8}{2} = 1 \)

d) O₂ = \( \frac{10 - 6}{2} = 2 \)

Question: Final product of the reaction

Options:
(a)
Answer: (c)

Solution: Allylic position is reactive for nucleophilic substitution reaction

Question: False statement about Calgon is:

Options:
(a) Calgon is also called as graham’s salt
(b) Calgon method does not precipitate Ca\(^{2+}\)
(c) Calgon contains metal which is 2\(^{nd}\) most abundant in earth’s crust
(d) Calgon is polymeric and water soluble
Answer: (c)

Solution:

* Calgon (Sodium hexametaphosphate) is also known as Graham’s salt. It has a polymeric chain structure and is water soluble

* When added to hard water, the following reaction takes place

\[
\begin{align*}
\text{Na}_6\text{P}_6\text{O}_{18} & \rightarrow 2\text{Na}^+ + \text{Na}_4\text{P}_6\text{O}_{18}^{2-} \\
\text{Calgon} & \\
\text{M}^{2+} + \text{Na}_6\text{P}_6\text{O}_{18}^{2-} & \rightarrow [\text{Na}_2\text{MP}_6\text{O}_{18}]^{2-} + 2\text{Na}^+ \\
(M = \text{Mg, Ca})
\end{align*}
\]

The complex ion keeps the Mg\(^{2+}\) and Ca\(^{2+}\) ion in the solution and not precipitated

* Second most abundant metal in earth’s crust is iron and is not present in Calgon

**Question:** Final product of the reaction is

**Options:**

(a)
Answer: (c)

Solution:
Question: 2,4 DNP test is given by:

Options:
(a) Aldehyde
(b) Amine
(c) Ester
(d) Halogens

Answer: (a)

Solution:

Both aldehyde and ketones gives the 2,4 DNP test
Question:

\[
\begin{align*}
&\text{(1) HCHO, NaOH} \\
&\text{(2) CH}_3\text{CH}_2\text{Br, DMF} \\
&\text{(3) HI, Heat}
\end{align*}
\]

Options:

(a) \[
\begin{align*}
&\text{CH}_2-\text{I} \\
&\text{CH}_3 \\
&\text{OH}
\end{align*}
\]

(b) \[
\begin{align*}
&\text{CH}_2-\text{CH}_2-\text{I} \\
&\text{CH}_3 \\
&\text{OH}
\end{align*}
\]

(c) \[
\begin{align*}
&\text{CH}_2-\text{CH}_2-\text{I} \\
&\text{OH}
\end{align*}
\]

(d) \[
\begin{align*}
&\text{CH}_2-\text{I} \\
&\text{O-CH}_3
\end{align*}
\]

Answer: (a)

Solution:
Question: Match the following:

<table>
<thead>
<tr>
<th>Column-I</th>
<th>Column-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Sodium carbonate</td>
<td>(P) Deacon</td>
</tr>
<tr>
<td>(B) Titanium</td>
<td>(Q) Castner-kellner</td>
</tr>
<tr>
<td>(C) Chlorine</td>
<td>(R) Van-arkel</td>
</tr>
<tr>
<td>(D) Sodium Hydroxide</td>
<td>(S) Solvay</td>
</tr>
</tbody>
</table>

Options:
(a) A → S; B → R; C → Q; D → P
(b) A → Q; B → R; C → P; D → S
(c) A → S; B → R; C → P; D → Q
(d) A → Q; B → P; C → R; D → S

Answer: (c)

Solution:

Sodium carbonate is manufactured by Solvay process.

Titanium is refined by Van-Arkle method.

Chlorine is manufactured by Deacon’s process.

Sodium hydroxide is manufactured by Castner-kellner process.

Question:
(1) Zn - Hg, HCl
(2) Cr₂O₃, 473 K
10-20 atm

Options:
(a)
(b)
(c)
(d)

Answer: (d)

Solution:
**Question:** Match the following.

<table>
<thead>
<tr>
<th>Column-I</th>
<th>Column-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Sucrose</td>
<td>(P) α-D glucose and β-D fructose</td>
</tr>
<tr>
<td>(B) Lactose</td>
<td>(Q) β-D galactose and β-D glucose</td>
</tr>
<tr>
<td>(C) Maltose</td>
<td>(R) α-D glucose and α-D glucose</td>
</tr>
<tr>
<td>(D) Cellulose</td>
<td>(S) β-D glucose and β-D glucose</td>
</tr>
</tbody>
</table>

**Options:**

(a) A → P; B → Q; C → R; D → S
(b) A → Q; B → R; C → S; D → P
(c) A → R; B → S; C → P; D → Q
(d) A → S; B → P; C → Q; D → R

**Answer:** (a)

**Solution:**
Sucrose

α-D-Glucose

β-D-Fructose

Maltose

α-D-Glucose

β-D-Galactose

Lactose

β-D-Glucose
Question: Seliwanoff and Xanthoproteic test are respectively used for the identification of:
Options:
(a) Proteins, Ketoses
(b) Ketoses, Proteins
(c) Aldoses, Ketoses
(d) Ketoses, Aldoses
Answer: (b)

Solution:
1) Seliwanoff test is for carbohydrate. It distinguishes between aldoses and ketose sugar
2) Xanthoproteic test is for protein

Question: What is the ratio of number of octahedral voids per unit cell in HCP/CCP?
Answer: 1.50

Solution: The number of octahedral voids is equal to effective number of atoms in both HCP and CCP structures
Thus,
number of octahedral voids in HCP = 6
number of octahedral voids in CCP = 4
Ratio = \( \frac{6}{4} = 1.5 \)
JEE-Main-26-02-2021-Shift-2 (Memory Based)

MATHEMATICS

Question: \( \tan^{-1} a + \tan^{-1} b = \frac{\pi}{4} \).

Find the value of \( a + b - \left( \frac{a^2 + b^2}{2} \right) + \left( \frac{a^3 + b^3}{3} \right) - \left( \frac{a^4 + b^4}{5} \right) \ldots \)

Options:
(a) 
(b) 
(c) 
(d) 

Answer: ()

Solution:
\[
\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4} \\
\tan^{-1} \left[ \frac{a + b}{1 - ab} \right] = \frac{\pi}{4} \\
\text{At } b = 1 - ab \\
(1 + a)(1 + b) = 2 \\
\text{Now, } (a + b) - \left( \frac{a^2 + b^2}{2} \right) + \left( \frac{a^3 + b^3}{3} \right) - \left( \frac{a^4 + b^4}{5} \right) \ldots \\
= \left[ a - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} + \ldots \right] + \left[ b - \frac{b^2}{2} + \frac{b^3}{3} - \frac{b^4}{4} + \ldots \right] \\
= \log(1 + a) + \log(1 + b) \\
= \log(1 + a)(1 + b) \\
= \log 2
\]

Question: 3, 3, 4, 4, 4, 5, 5 find the probability for 7 digit number such that number is divisible by 2

Options:
(a) \( \frac{1}{7} \) 
(b) \( \frac{3}{7} \) 
(c) \( \frac{4}{7} \) 
(d) \( \frac{6}{7} \)

Answer: (b)

Solution:
Numbers given are 3, 3, 4, 4, 4, 5, 5

Total number of 7 digit number is \( \frac{7!}{2! \cdot 3! \cdot 2!} = 210 \)

Number divisible by ‘2’ has ‘4’ at unit place

\( \therefore \) Total favourable case = \( \frac{1 \times 66}{2! \cdot 2!} = 90 \)

\( \therefore \) Required probability = \( \frac{90}{210} = \frac{3}{7} \)

**Question:** Mirror image of \((1, 3, 5)\) w.r.t plane \(4x - 5y + 2z = 8\) is \((\alpha, \beta, \gamma)\), then \(5(\alpha + \beta + \gamma) = ?\)

**Options:**
(a)
(b)
(c)
(d)

**Answer:** ()

**Solution:**
Equation of line perpendicular to plane \(4x - 5y + 2z = 8\) and passing through \((1, 3, 5)\) is

\[ \frac{x - 1}{4} = \frac{y - 3}{-5} = \frac{z - 5}{2} = \lambda \]

Any general point on this line is \(P(4\lambda + 1, 3 - 5\lambda, 2\lambda + 5)\)

Let P lies on plane,

\( \therefore 4(4\lambda + 1) - 5(-5\lambda + 3) + 2(2\lambda + 5) = 8 \)

\(45\lambda = 9\)

\( \Rightarrow \lambda = \frac{1}{5} \)

\( \therefore P = \left(\frac{9}{5}, 2, \frac{27}{5}\right) \)

As P is mid point of \((1, 3, 5)\) and \((\alpha, \beta, \gamma)\)

\( \therefore \alpha = \frac{13}{5}, \beta = 1, \gamma = \frac{29}{5} \)
\[ \alpha + \beta + \gamma = \frac{47}{5} \]
\[ 5(\alpha + \beta + \gamma) = 47 \]

**Question:** \( f(x) \) is differentiable function at \( x = a \), such that \( f'(a) = 2, \ f(a) = 4 \). Find 
\[ \lim_{x \to a} \frac{xf(a) - af(x)}{x - a} \]
**Options:**
(a) 
(b) 
(c) 
(d) 
**Answer:** ()
**Solution:**
\[ \lim_{x \to a} \frac{xf(a) - af(x)}{x - a} \]
On applying L-Hospital’s Rule
\[ \lim_{x \to a} \frac{f(a) - af'(x)}{1} = f(a) - af''(a) = 4 - 2a \]

**Question:** The locus of mid point of the line segment from (3, 2) to the circle \( x^2 + y^2 = 1 \) which touch the circle at point P is a circle with radius r. what is the value of r
**Options:**
(a) 1 
(b) \( \frac{1}{2} \) 
(c) \( \frac{1}{3} \) 
(d) \( \frac{1}{4} \) 
**Answer:** (b)
**Solution:**
Question: The slope of the tangent to curve is \( \frac{xy^2 + y}{x} \) and it intersects the line \( x + 2y = 4 \) at \( x = -2 \), then the value of ‘y’ is \((3, y)\) lies on the curve?

Options:
(a) 
(b) 
(c) 
(d) 

Answer: ()

Solution:
\[
\frac{dy}{dx} = \frac{xy^2 + y}{x} \Rightarrow \frac{dy}{dx} - \frac{y}{x} = y^2 \\
\frac{1}{y^2} \frac{dy}{dx} - \frac{1}{yx} = 1 \\
\text{Put } -\frac{1}{y} = t \Rightarrow \frac{1}{y^2} \frac{dy}{dx} = \frac{dt}{dx} \\
\Rightarrow \frac{dt}{dx} + \frac{t}{x} = 1 \\
I.F. = e^{\int \frac{dx}{x}} = \ln x = x
\]
\[ t(x) = \int x \, dx = \frac{x^2}{2} + C \]
\[ \Rightarrow -\frac{x}{y} = \frac{x^2}{2} + C \] (passes through (3, y))
\[ \Rightarrow -\frac{3}{y} = \frac{9}{2} + C \Rightarrow -\frac{3}{y} = \frac{9}{2} \]
\[ \Rightarrow \frac{x}{y} = \frac{x^2}{2} - \frac{9}{2y} \]
\[ \Rightarrow \frac{3-x}{y} = \frac{x^2}{2} - \frac{9}{2} \]
\[ \Rightarrow y = \frac{2(3-x)}{x^2-9} \]

At \( x = -2 \); \( y = \frac{2 \times 5}{(-5)} = -2 \)

**Question:** \( f(x) = \int_{1}^{x} \frac{\ln(1+t)}{t} \, dt \), \( f(e) + f\left( \frac{1}{e} \right) = \)

**Options:**
(a)
(b)
(c)
(d)

**Answer:**

**Solution:**

\[ f(x) = \int_{1}^{x} \frac{\ln(1+t)}{t} \, dt \]

Let \( t = \frac{1}{u} \Rightarrow dt = -\frac{1}{u^2} \, du \)

\[ \therefore f\left( \frac{1}{x} \right) = \int_{1}^{\frac{1}{x}} \frac{\ln(1+\frac{1}{u})}{\frac{1}{u}} \left( -\frac{1}{u} \right) du = -\int_{1}^{\frac{1}{x}} \frac{\ln\left( \frac{1+u}{u} \right)}{u} \, du \]

\[ \therefore f(x) + f\left( \frac{1}{x} \right) = \int_{1}^{x} \left[ \frac{\ln(1+t)}{t} + \ln(1+t) - \ln(1+t) \right] dt \]

\[ f(x) + f\left( \frac{1}{x} \right) = \int_{1}^{x} \frac{\ln t}{t} \, dt = \left[ \ln(\ln t) - \frac{(\ln t)^2}{2} \right]_{1}^{x} = \frac{1}{2} \ln^2 x \]
\[ \therefore f(e) + f\left(\frac{1}{e}\right) = \frac{1}{2} \ln^2 e = \frac{1}{2} \]

**Question:** \( f(x) = \int_1^x e^{t} f(t) \, dt + e^x; \) \( f(x) \) is a differentiable function \( x \in \mathbb{R} \). Then \( f(x) = \)

**Options:**
(a) 
(b) 
(c) 
(d) 

**Answer:** ()

**Solution:**
\[
\begin{align*}
\int_1^x e^{t} f(t) \, dt + e^x & = f(x) \\
\frac{dy}{dx} & = e^x (y + 1) \\
\int \frac{dy}{y+1} & = \int e^x \, dx \\
\log |y + 1| & = e^x + C \\
y + 1 & = \pm e^{e^x} \\
y + 1 & = k e^{e^x} \quad \left(\text{Put } \pm e^C = k\right)
\end{align*}
\]

At \( x = 1, y = 0 \)
\[ \Rightarrow 1 = ke^e \]
\[ k = \frac{1}{e^e} \]
\[ \Rightarrow y + 1 = \frac{e^e}{e^e} \]
\[ \Rightarrow f(x) = \frac{e^x - 1}{e^e} \]
Question: If $A_1$ is area between the curves $y = \sin x$, $y = \cos x$ and $y$-axis for $0 \leq x \leq \frac{\pi}{2}$ and $A_2$ is area between $y = \sin x$ and $y = \cos x$ and $x$-axis for $0 \leq x \leq \frac{\pi}{2}$, find the value of $\frac{A_2}{A_1}$.

Options:
(a) 
(b) 
(c) 
(d)

Answer: ()

Solution:
\[
A_1 = \int_0^{\pi/4} (\cos x - \sin x) \, dx = (\sin x + \cos x)\frac{\pi}{4} = \sqrt{2} - 1
\]
\[
A_2 = \int_0^{\pi/4} \sin x \, dx + \int_{\pi/4}^{\pi/2} \cos x \, dx = (\cos x)\frac{\pi}{4} + (\sin x)\frac{\pi}{4}
\]
\[
= \left( -\frac{1}{\sqrt{2}} + 1 \right) + \left( 1 - \frac{1}{\sqrt{2}} \right) = 2 - \sqrt{2}
\]
\[
\frac{A_2}{A_1} = \frac{2 - \sqrt{2}}{\sqrt{2} - 1} = \sqrt{2}
\]

Question: $P_n = \alpha^n + \beta^n$, $\alpha + \beta = 1$, $\alpha\beta = -1$, $P_{n-1} = 11$, $P_{n+1} = 29$, then $(P_n)^2$.

Answer: 324.00

Solution:
\[
(\alpha + \beta)P_n = (\alpha + \beta)(\alpha^n + \beta^n)
\]
\[
(\alpha + \beta)P_n = \alpha^{n+1} + \beta^{n+1} + \alpha\beta(\alpha^{n-1} + \beta^{n-1})
\]
\[
\Rightarrow (1)P_n = P_{n+1} - P_{n-1} = 29 - 11 = 18
\]
\[
\Rightarrow P_n^2 = 324
\]
**Question:** Let $A(1, 4)$ and $B(1, -5)$ be two points let $P$ be the point on $(x-1)^2 + (y-1)^2 = 1$. Find maximum value of $(PA)^2 + (PB)^2$.

**Answer:** 53.00

**Solution:**
Let $P(1 + \cos \theta, 1 + \sin \theta)$

\[
PA^2 + PB^2 = \cos^2 \theta + (\sin \theta - 3)^2 + \cos^2 \theta + (\sin \theta + 6)^2
\]

\[
= 2 \cos^2 \theta + 2 \sin^2 \theta + 6 \sin \theta + 45
\]

\[
= 47 + 6 \sin \theta
\]

So, it will be maximum when $\sin \theta = 1$

\[
(PA^2 + PB^2)_{\text{max}} = 47 + 6 = 53
\]

**Question:** Let $L$ be a line of intersection of $x + 2y + z = 0$ and $y + z = 4$. If $P(\alpha, \beta, \gamma)$ is foot of perpendicular from $(3, 2, 1)$ on $L$. Find $21(\alpha + \beta + \gamma)$.

**Options:**
(a)
(b)
(c)
(d)

**Answer:** 98.00

**Solution:**
D.R. of line $L$:

\[
\begin{vmatrix}
i & j & k \\
1 & 2 & 1 \\
0 & 1 & 1
\end{vmatrix}
= (1, -1, 1)
\]

Put $z = 0$ in both planes \(\Rightarrow y = 4, x = -2\)

\[
\therefore \text{Equation of line } L : \frac{x+2}{1} = \frac{y+4}{-1} = \frac{z-0}{1} = \lambda
\]

Let point $P$ on line is $(\lambda - 2, 4 - \lambda, \lambda)$ & $A(3, 2, 1)$

\[
\therefore AP \perp \text{line}
\]

\[
(1)(\lambda - 5) + (-1)(2 - \lambda) + (1)(\lambda - 1) = 0
\]

\[
\Rightarrow 3\lambda = 8 \Rightarrow \lambda = \frac{8}{3}
\]

\[
\Rightarrow P\left(\frac{2}{3}, \frac{4}{3}, \frac{8}{3}\right)
\]

\[
\Rightarrow 21(\alpha + \beta + \gamma) = 21\left(\frac{2}{3} + \frac{4}{3} + \frac{8}{3}\right)
\]
\[ 21 \left( \frac{14}{3} \right) = 98 \]

**Question:** How many four digit numbers are there where g.c.d. with 18 is ‘3’.

**Options:**

(a) 
(b) 
(c) 
(d) 

**Answer:** 1000.00

**Solution:**

Number of required numbers

\[ = 4 \text{digit numbers divisible by } 3 - 4 \text{digit numbers divisible by } 6 - 4 \text{digit numbers divisible by } 9 + 4 \text{digit numbers divisible by } 18 \]

\[ = 3000 - 1500 - 1000 + 200 \]

\[ = 1000 \]

**Question:** The prime factorization of a number ‘n’ is given as \( n = 2^2 \times 3^3 \times 5^1 \), \( y + z = 5 \) and \( y^{-1} + z^{-1} = \frac{5}{6} \). Find out the odd divisors of n including 1.

**Answer:** 12.00

**Solution:**

\[ y + z = 5 : \frac{1}{y} + \frac{1}{z} = \frac{5}{6} \Rightarrow (y, z) = (2, 3) \text{ or } (3, 2) \]

\[ \therefore \text{Number of odd divisors of } n = 2^x \times 3^y \times 5^z \text{ is } (y+1)(z+1) \]

\[ = 3 \times 4 = 12 \]

**Question:** -16, 8, -4, 2, ….., A.M and G.M of \( p^{th} \) and \( q^{th} \) terms are roots of \( 4x^2 - 9x + 5 = 0 \) then \( p + q = \)

**Answer:** 10.00

**Solution:**

Given sequence is -16, 8, -4, 2

It is a GP with common ratio \( r = -\frac{1}{2} \)

Its \( n^{th} \) term is \( a_n = (-16) \left( -\frac{1}{2} \right)^{n-1} \)

Roots of \( 4x^2 - 9x + 5 = 0 \) are 1, \( \frac{5}{4} \)

\[ \therefore \text{GM} \leq \text{AM} \Rightarrow \therefore \text{GM} = 1 \]
Now GM of $p^{th}$ and $q^{th}$ term $= \sqrt{(-16)(-\frac{1}{2})^{p-1}(-16)(-\frac{1}{2})^{q-1}}$

$\Rightarrow 16\left(-\frac{1}{2}\right)^{\frac{p+q-2}{2}} = 1$

$\Rightarrow \left(-\frac{1}{2}\right)^{\frac{p+q-2}{2}} = \frac{1}{16}$

$\Rightarrow \frac{p + q - 2}{2} = 4$

$\Rightarrow p + q = 10$

**Question:** The value of square of slope of the common tangent to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$

**Options:**
(a) 
(b) 
(c) 
(d) 

**Answer:** 3.00

**Solution:**
Given ellipse are $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and $\frac{(31)}{4} + \frac{(31)}{4} = 1$

Let equation of common tangent to ellipse with slope ‘$m$’ is

$y = mx + \sqrt{9m^2 + 4}$ and $y = mx + \frac{(31)}{4}m^2 + \frac{(31)}{4}$

$\therefore 9m^2 + 4 = \frac{(31)}{4}m^2 + \frac{(31)}{4}$

$\Rightarrow \frac{5m^2}{4} = \frac{15}{4}$

$\Rightarrow m^2 = 3$

**Question:** $\sum_{n=1}^{18}(x_i - \alpha) = 36; \sum_{n=1}^{18}(x_i - \beta)^2 = 90$ and the standard deviation is

Find $|\beta - \alpha|$

**Answer:** 0.00

**Solution:**
Let $\alpha = \beta$
\[ \therefore \text{Standard deviation remains unchanged if observations are added or subtracted by a fixed number} \]

\[ S.D. = \sqrt{\frac{\sum_{i=1}^{18} (x_i - \alpha)^2}{18} - \left(\frac{\sum_{i=1}^{18} (x_i - \alpha)}{18}\right)^2} \]

\[ = \sqrt{\left(\frac{90}{18}\right) - \left(\frac{36}{18}\right)^2} = 1, \text{ which is given} \]

Hence, \( \alpha = \beta \) according to the conditions

\[ \therefore |\beta - \alpha| = 0 \]