JECA-2019

Subject : MATHEMATICS

(Booklet Number)

Duration : 2 Hours **Full Marks : 100**

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks 1. In case of incorrect answer or any combination of more than one answer, ¼ marks will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked
   A, B, C or D.
3. Use only **Black/Blue ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Mark the answers only in the space provided. Do not make any stray mark on the OMR.
5. Write question booklet number and your roll number carefully in the specified locations of the **OMR**. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your full signature in appropriate boxes in the OMR.
7. The OMR is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/signature of the candidate, name of the examination centre. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will be **reported against** & his/her candidature will be summarily cancelled.
9. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.
10. Hand over the OMR to the invigilator before leaving the Examination Hall.

SPACE FOR ROUGH WORK

MATHEMATICS

1. Let $\frac{1-ix}{1+ix} = a - ib$, where x, a, b $\in \mathbb{R}$. Then

(A) $a^2 + b^2 = 0$
(B) $a^2 + b^2 = 1$
(C) $a^2 + b^2 > 1$
(D) $a^2 + b^2 < 1$

2. The product of the values of $(1+i\sqrt{3})^{3/4}$ is

(A) 4
(B) 16
(C) 8
(D) 32

3. If a, b are real numbers between 0 and 1 such that the points $z_1 = a + i$, $z_2 = 1 + bi$ and $z = 0$ form an equilateral triangle, then (a, b) =

(A) $(2-\sqrt{3}, 2+\sqrt{3})$
(B) $(2-\sqrt{3}, 2-\sqrt{3})$
(C) $\left(2-\sqrt{3}, \frac{1}{2}\right)$
(D) $\left(\frac{1}{2}, 2-\sqrt{3}\right)$

4. If $|z^2-1| = |z|^2+1$ then z lies on

(A) the real axis
(B) a parabola
(C) the imaginary axis
(D) a circle

5. If $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{a^2+c^2}{b} \end{vmatrix} = \lambda abc$, then λ is

(A) 1
(B) 2
(C) 3
(D) 4

6. For a fixed positive integer n, if $\Delta = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$, then the term independent of n in $\frac{\Delta}{(n!)^3}$ is

(A) 1
(B) 2
(C) 3
(D) 4

7. Let $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$ and let $\Delta_1 = \begin{vmatrix} bc-f^2 & gf-ch & hf-bg \\ gf-hc & ac-g^2 & gh-af \\ hf-bg & gh-af & ab-b^2 \end{vmatrix}$, then

(A) $\Delta_1 \neq 0$
(B) $\Delta_1 > 0$
(C) $\Delta_1 < 0$
(D) $\Delta_1 = 0$

8. The square of any determinant is
(A) symmetric
(B) skew symmetric
(C) neither symmetric nor skew symmetric
(D) may be symmetric or skew symmetric

9. Let A, B be two square matrices of same order and AB = A, BA =B. Then
(A) $A^2 = A$
(B) $A^2 \neq A$
(C) $A^2 = I$
(D) $A^2 = \underline{0}$
where I is the identity matrix and $\underline{0}$, corresponding null matrix.

10. Choose the correct statement:
(A) Every non-singular matrix is orthogonal.
(B) Every orthogonal matrix is non-singular.
(C) An orthogonal matrix is skew-symmetric.
(D) The det-value of every orthogonal matrix is positive.

11. Let A be an orthogonal matrix of order 3. Then
(A) det A > 0
(B) det A < 0
(C) det A = ±1
(D) det A ≠ ±1

12. The positive values of α, β, γ for which $\begin{pmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{pmatrix}$ is orthogonal, are given by

(A) $\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{3}}, 1$
(B) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}$
(C) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{5}}, 1$
(D) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}$

13. Let A, B, C be three square matrices of same order such that AB = AC. Then

(A) B = C always holds

(B) B = C only when A is orthogonal

(C) B = C only when A is non-singular

(D) B = C only when A is identity matrix

14. If $A = (a_{ij})_{n\times n}$ and $\text{Adj}(KA) = K^r \text{ Adj } A$, where K is a scalar, then r =

(A) n (B) $\frac{n}{2}$

(C) $n-1$ (D) $\frac{n-1}{2}$

15. The value of k for which the rank of the matrix $\begin{pmatrix} 3 & 4 & -5+k \\ 2 & -1 & 7 \\ 1 & -2 & 8 \end{pmatrix}$ is less than 3 is

(A) 1 (B) -1

(C) 2 (D) -2

16. The matrix $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & x \end{pmatrix}$ is orthogonal if x is

(A) ± 1 (B) 2

(C) -2 (D) 3

17. Given the matrix $M = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ and its inverse $N = [n_{ij}]_{3\times 3}$, then the element n_{23} of matrix N is

(A) 2 (B) -2

(C) 1 (D) -1

18. Let $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, where I is 2×2 identity matrix, then x and y are respectively

(A) $-\frac{1}{11}, \frac{2}{11}$
(B) $-\frac{1}{11}, -\frac{2}{11}$
(C) $\frac{1}{11}, \frac{2}{11}$
(D) $\frac{1}{11}, -\frac{2}{11}$

19. The equation $x^n - nqx + (n-1)\,r = 0$ $(q, r \in \mathbb{R})$ has a pair of equal roots. Then

(A) $q^{n-1} = r^n$
(B) $q^n = r^{n-1}$
(C) $q^{2n} = r$
(D) $q^{2n-3} = r^2$

20. To remove the 2[nd] term of the equation $x^3 - 3x^2 + 12x + 16 = 0$ we have to increase the roots by

(A) 1
(B) -1
(C) -2
(D) 3

21. If p, q and r are positive then the equation $x^4 + px^3 + qx - r = 0$ has

(A) exactly one positive real root
(B) exactly two positive real roots
(C) no real root
(D) two negative real roots

22. Let A, B, C denote non-void subsets of set S. Then

(A) $A \cap C = B \cap C \Rightarrow A = B$
(B) $A \cup C = B \cup C \Rightarrow A = B$
(C) $A \cap C = B \cap C, A \cap C' = B \cap C' \Rightarrow A = B$
(D) $A \cap C' = B \cap C' \Rightarrow A = B$

23. In $\mathbb{Z}$, the set of all integers, the inverse of -7 with respect to '*' defined by $a * b = a+b+7$ for all $a, b, \in \mathbb{Z}$ is

(A) -14
(B) 7
(C) 14
(D) -7

24. Let (B, +) be an abelian group and let multiplication '.' be defined by $x \cdot y = x$ for all $x, y \in B$. Then

(A) $(B, +, \cdot)$ is a ring.
(B) distributive properties do not hold in $(B, +, \cdot)$
(C) one distributive property does not hold in $(B, +, \cdot)$
(D) y is inverse of x in $(B, \cdot)$

25. Let $f : \mathbb{Q} \rightarrow \mathbb{Q}$ ($\mathbb{Q}$ is a set of rationals) be defined by $f(x) = ax + b$ $(a, b \in \mathbb{Q}, a \neq 0)$ Then
(A) f is bijective
(B) f is injective but not surjective
(C) f is surjective but not injective
(D) f is neither injective nor surjective

26. If $f : \mathbb{R} \rightarrow S$ defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$ is onto, then S =
(A) [0, 1]
(B) [−1, 1]
(C) [0, 3]
(D) [−1, 3]

27. The number of subsets of $\{1, 2, 3.....9\}$ containing at least one odd number is
(A) 324
(B) 394
(C) 496
(D) 512

28. On $\mathbb{N}$, the set of natural numbers, the relation ρ be defined as a ρ b iff $a^2 - 4ab + 3b^2 = 0$. Then
(A) ρ is an equivalence relation
(B) ρ is reflexive but neither symmetric nor transitive
(C) ρ is symmetric but neither reflexive nor transitive
(D) ρ is transitive but neither reflexive nor symmetric

29. Let $S = \{-1, 0, 1\}$. Then
(A) S is a group with respect to addition
(B) S is not a group with respect to addition
(C) S is a group with respect to subtraction
(D) S is a group with respect to multiplication

30. Let A, B, C be any three non-void sets. Then A − (B ∪ C) is
(A) (A − B) ∪ (A − C)
(B) (A − B) ∩ (A − C)
(C) A ∪ (B − C)
(D) A ∩ (B − C)

31. Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = e^x$, $g(x) = x^2$, $\forall x \in \mathbb{R}$
(A) g ∘ f and g are injective but f is not so
(B) g ∘ f and f are injective but g is not so
(C) g ∘ f , f, g all are injective
(D) None of them is injective

32. Let the binary operation $\circ$ be defined on Q^+, the set of positive rational numbers, as $a \circ b = \frac{ab}{2}$, for all a, $b \in Q^+$. Then the inverse of $a \in Q^+$ with respect to the operation $\circ$ is

(A) $\frac{1}{a}$ (B) $\frac{a}{4}$

(C) $\frac{4}{a}$ (D) $\frac{a}{2}$

33. In the three-element group {e, a, b} under multiplication, e is the identity element, then $a^5b^4 =$

(A) a (B) e

(C) ab (D) b

34. If a, b, c are any three elements of a group $(G, *)$ and $(a * b) * x = c$, then $x =$

(A) $c*(a^{-1} * b^{-1})$ (B) $c*(b^{-1} * a^{-1})$

(C) $(a^{-1} * b^{-1})*c$ (D) $(b^{-1} * a^{-1})*c$

35. Let S be non-void subset of Q, the set of rationals. Let $a * b = a + b + ab$ for all a, $b \in S$. Given that $(S, *)$ is a semi group with unique identity element. Then

(A) Every element of S has its inverse in S

(B) Elements of S have inverses in S but for -1

(C) Elements of S have inverses in S but for 1

(D) No element of S has its inverse in S

36. Let O be the origin and P be the point at a distance 3 units from origin. If direction ratios of OP are (1, – 2, –2), then co-ordinates of P are given by

(A) (1, 2, 2) (B) (1, – 2, –2)

(C) (3, 6, 5) (D) (–3, –6, 5)

37. The equation $r^2 \cos^2\left(\theta - \frac{\pi}{3}\right) = 2$ (in polar co-ordinate) represents

(A) a hyperbola (B) a pair of straight lines

(C) a parabola (D) an ellipse

38. By shifting the origin to the point (α, β) without changing the direction of axes, each of the equations $x - y + 3 = 0$ and $2x - y + 1 = 0$ is reduced to the form $ax + by = 0$, $(ab \neq 0)$ then $(\alpha, \beta) =$

(A) $(2, 5)$ (B) $(5, 2)$
(C) $(-2, -5)$ (D) $(-5, -2)$

39. The pair of lines $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$ are rotated about the origin by $\frac{\pi}{6}$ in the anti-clockwise sense. The equation of the pair in the new system will be

(A) $\sqrt{3}x^2 - xy = 0$ (B) $\sqrt{3}y^2 - xy = 0$
(C) $x^2 - y^2 = 0$ (D) $\sqrt{3}x^2 + xy = 0$

40. If the gradient of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is four times that of the other, then

(A) $4h^2 = 5ab$ (B) $5h^2 = 4ab$
(C) $16h^2 = ab$ (D) $16h^2 = 25ab$

41. If $y = mx$ bisects the angle between the lines $x^2(\tan^2\theta + \cos^2\theta) + 2xy\tan\theta - y^2\sin^2\theta = 0$ when $\theta = \pi/3$, then value of m is

(A) $\frac{-2 \pm \sqrt{5}}{\sqrt{3}}$ (B) $\frac{-2 \pm \sqrt{6}}{\sqrt{7}}$
(C) $\frac{-2 \pm \sqrt{7}}{\sqrt{3}}$ (D) $\frac{-2 \pm \sqrt{3}}{\sqrt{7}}$

42. If the planes $x = cy + bz$, $y = az + cx$ and $z = bx + ay$ have a common line of intersection and $a^2 + b^2 + c^2 = 1 + kabc$, then k is

(A) 1 (B) -1
(C) 2 (D) -2

43. The line $y = 3x + 2$ intersects the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ at P and Q. Let O be the origin. Then the angle $\angle POQ$ is

(A) $\frac{\pi}{2}$ (B) $\tan^{-1}\frac{2\sqrt{2}}{3}$
(C) $\pi - \tan^{-1}\frac{2\sqrt{3}}{5}$ (D) $\sin^{-1}\frac{2\sqrt{3}}{5}$

44. The value of k, so that the equation $xy + 5x + ky + 15 = 0$ may represent a pair of straight lines is

(A) -3 (B) 3
(C) 2 (D) -2

45. The polar of a point P with respect to the parabola $y^2 = 4ax$ is parallel to the line $\ell x + my = 1$. The locus of P is

(A) $\ell x + 2am = 0$ (B) $\ell y + 2am = 0$
(C) $\ell x - 2am = 0$ (D) $\ell y - 2am = 0$

46. The locus of the poles of focal chords of a parabola $y^2 = 4ax$, $a > 0$, is the

(A) axis of the parabola (B) tangent at the vertex of the parabola
(C) line $y = x$ (D) directrix of the parabola

47. If the polar of P with respect to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be equally inclined to the co-ordinate axes, then locus of P is

(A) $b^2x = a^2y$ (B) $a^2x = b^2y$
(C) $b^2x + a^2y = 1$ (D) $a^2x + b^2y = 1$

48. If tangents be drawn from any point on $x + 2 = 0$ to the parabola $y^2 = 8x$, then the angle between the tangents is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

49. The director circle of a conic in a plane

(A) is the locus of point of intersection of the tangents to the conic which are inclined at an angle of $\frac{\pi}{4}$.
(B) is the locus of point of intersection of chords of the conic that are perpendicular to each other.
(C) is the locus of point of intersection of the tangents to the conic that are inclined at an angle of $\frac{\pi}{2}$.
(D) A parabola has no director circle.

50. The distance of the plane $x - y + z = 5$ from $(1, -2, 3)$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

(A) 4
(B) 3
(C) 2
(D) 1

51. The three planes $x + 2y + kz - 8 = 0$, $2x - y + z - 3 = 0$ and $3x + y - 2z + 1 = 0$ have single common point if k is **not** equal to

(A) 3
(B) –3
(C) 5
(D) –5

52. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through mid-point of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then the value of 'a' is

(A) 1
(B) 2
(C) –2
(D) 0

53. The angle between the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$ and the plane $x + y + 4 = 0$ is

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(A) $\frac{\pi}{3}$
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{2}$

54. Consider the circles

$$\Gamma_1 : x^2 + y^2 + 4x - 10y - 2 = 0$$

$$\Gamma_2 : x^2 + y^2 - 8x + 6y + 16 = 0$$

Then

(A) They touch each other at a point
(B) They touch each other internally
(C) Their common chord is $12x - 16y - 18 = 0$
(D) Their centres are at a distance of 20 units

55. A plane passes through the point (2, 2, 2) and cuts the axes at A, B, C. If the locus of the centre of the sphere OABC is $x^{-1} + y^{-1} + z^{-1} = \alpha$, then $\alpha =$

(A) 1
(B) 2
(C) 4
(D) 8

56. Let $\vec{\alpha} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\vec{\beta} = -\hat{i} + 2\hat{j} + 2\hat{k}$ be two vectors and θ is the angle between them. An acute angle ψ be such that $\theta + \psi = 90°$. Then cosine of the angle ψ is

(A) $\frac{8}{9}$ (B) $\frac{8}{3}$

(C) $\frac{\sqrt{17}}{9}$ (D) $\frac{\sqrt{11}}{9}$

57. Let $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$ be three non-collinear vectors, no two of which are collinear. If the vectors $\vec{\alpha} + 2\vec{\beta}$ is collinear with $\vec{\gamma}$ and $\vec{\beta} + 3\vec{\gamma}$ is collinear with $\vec{\alpha}$ then $\vec{\alpha} + 2\vec{\beta} + 6\vec{\gamma}$ is equal to

(A) $\lambda\vec{\alpha}$ (B) $\lambda\vec{\beta}$

(C) $\lambda\vec{\gamma}$ (D) $\vec{0}$

where $\lambda \neq 0$ is a scalar

58. For any vector $\vec{a}$, $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ equals

(A) $(\vec{a})^2$ (B) $2(\vec{a})^2$

(C) $3(\vec{a})^2$ (D) $4(\vec{a})^2$

59. If $\vec{a}, \vec{b}, \vec{c}$ are perpendicular to $\vec{b} + \vec{c}$, $\vec{c} + \vec{a}$, $\vec{a} + \vec{b}$ respectively, and if $|\vec{a} + \vec{b}| = 6$, $|\vec{b} + \vec{c}| = 8$ and $|\vec{c} + \vec{a}| = 10$, then $|\vec{a} + \vec{b} + \vec{c}| =$

(A) $5\sqrt{2}$ (B) 50

(C) $10\sqrt{2}$ (D) 10

60. $\hat{a}, \hat{b}$ are unit vectors such that $[\hat{a}\ \hat{b}\ \hat{a} \times \hat{b}] = \frac{1}{4}$, then the angle between $\hat{a}$ and $\hat{b}$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

61. The equation $|x| = x - 5$

(A) is solvable for all $x \in R$ (B) has no solution

(C) is solvable for $|x| < \frac{2}{3}$ (D) is solvable for $|x| < \frac{3}{2}$

62. $\lim_{x\to 0}\frac{3^x-2^x}{4^x-3^x}=$

(A) $\frac{3}{4}$

(B) $\log_{4/3}\frac{3}{2}$

(C) $\log_e\frac{3}{2}$

(D) $\frac{2}{3}$

63. If, $f(x) = 1, -\infty < x < 0$

$= 1 + \sin x, \quad 0 \le x < \frac{\pi}{2}$

$= 2 + \left(x-\frac{\pi}{2}\right)^2, \quad \frac{\pi}{2} \le x < \infty$

then,

(A) $f(x)$ is continuous everywhere

(B) $f(x)$ is continuous at all points except $x = 0$

(C) $f(x)$ is continuous at all points except $x = \frac{\pi}{2}$

(D) $f(x)$ is continuous at all points except $x = 0$ and $x = \frac{\pi}{2}$

64. If $f(x) = (x-2)(x-4)(x-6)\cdots(x-2n)$, then $f'(2)$ is

(A) $(-1)^n . 2^{n-1}(n-1)!$

(B) $(-2)^{n-1}(n-1)!$

(C) $(-2)^n n!$

(D) $2^{n-1}(n-1)!$

65. If for $y = \sin(m \sin^{-1}x)$; $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + ky = 0$, then k is

(A) m^2

(B) $-m^2$

(C) $2m^2$

(D) $-2m^2$

66. If $x(1-x)\,y_2-(4-12x)\,y_1-36y=0$ then

$x(1-x)\,y_{n+2}-\{4-n-(12-2n)\,x\}\,y_{n+1}-\varphi(n)y_n=0$ where $\varphi(n)=$

(A) $(4-n)(9+n)$

(B) $(4-n)(9-n)$

(C) $(4+n)(9-n)$

(D) $(4+n)(9+n)$

67. Let $f(x) = x^n$, n be a natural number, then $\frac{f^{(1)}(1)}{1!} + \frac{f^{(2)}(1)}{2!} \ldots + \frac{f^{(n)}(1)}{n!}$ will be

(A) 1
(B) 2^n
(C) 3^n
(D) $2^n - 1$

68. Let $f : \mathbb{R} \to \mathbb{R}$ be such that $f'''(x)$ exist. Suppose $f(a) = f'(a) = f(b) = f'(b) = 0$ for some $a < b$, then

(A) $f'''(c) = 0$ for some $c \in \mathbb{R}$
(B) $f'''(x)$ is never zero
(C) $f'''(x) > 0$ for all x
(D) $f'''(x) < 0$ for all x

69. Let f, g be differentiable on [0, 2] such that $f(0) = 2$, $f(2) = 5$, $g(0) = 0$, $g(2) \neq 0$, $f'(x) = g'(x) (\neq 0)$ in (0, 2), then

(A) g(2) may have any non-zero value
(B) $g(2) = 3$
(C) $g(2) = 5$
(D) $g(2) = -1$

70. If $c_0 + \frac{c_1}{2} + \frac{c_2}{3} + \cdots + \frac{c_n}{n+1} = 0$, where $c_0, c_1, \cdots c_n \in \mathbb{R}$ then of the equation $c_n x^n + c_{n-1} x^{n-1} + \ldots c_2 x^2 + c_1 x + c_0 = 0$ $[c_0 \neq 0]$

(A) solvability cannot be ascertained
(B) all the roots are complex
(C) at least one real root exists
(D) 0 is only one real root

71. Let $y = \cos^2 x$, then the expansion of f(x) in the neighbourhood of zero is

(A) $x + \frac{x^2}{2} - \frac{x^3}{3} + \ldots\ldots$
(B) $1 - x^2 + \frac{x^4}{3} - \ldots\ldots$
(C) $x^2 - \frac{x^4}{4!} + \frac{x^6}{6!} - \ldots\ldots$
(D) $x - \frac{x^3}{3} + \frac{x^7}{7} - \ldots\ldots$

72. Let $f(x) = \frac{x-1}{e^x}$, then

(A) $f(x)$ has no extrema at $x = 2$
(B) $f(x)$ is decreasing in $(-\infty, 2)$ and increasing in $[2, \infty)$
(C) $f(x)$ is maximum for $x = 2$
(D) $f(x)$ is minimum for $x = 2$

73. If $y = \cos^4 x + \sin^4 x$ is greatest for $x = \theta$ and least for $x = \varphi$, then

(A) $\theta = \frac{k\pi}{2}$, $\varphi = (2k+1)\pi/4$
(B) $\theta = k\pi$, $\varphi = (2k+1)\pi/2$
(C) $\theta = (2k+1)\pi/2$, $\varphi = 2k\pi$
(D) $\theta = \varphi = (2k+1)\pi/3$

where k is integer.

74. The equation of the normal at $\theta = \frac{\pi}{2}$ for the curve $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ is

(A) $x + y - \frac{a\pi}{2} = 2a$

(B) $x - y - \frac{a\pi}{2} = 0$

(C) $x + y + \frac{a\pi}{2} = 0$

(D) $x - y - 3\frac{a\pi}{2} = 0$

75. The tangents at the extremities of any focal chord of the parabola $y^2 = 4ax$ intersect at an angle

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

76. Consider the ellipse $2x^2 + 3y^2 = 1$. Equation(s) of tangent(s) which is/are parallel to $2x - y + 3 = 0$ are given by

(A) $y = 2x \pm \sqrt{\frac{5}{3}}$

(B) $y = 2x \pm \sqrt{\frac{7}{3}}$

(C) $y = x \pm \sqrt{\frac{5}{3}}$

(D) $x = y \pm \sqrt{\frac{4}{3}}$

77. If $\lim_{x \to 0} \frac{x(1 - a\cos x) + b\sin x}{x^3} = \frac{1}{3}$, then 'a' and 'b' are respectively

(A) $\frac{1}{2}, -\frac{1}{2}$

(B) $-\frac{1}{2}, \frac{1}{2}$

(C) $\frac{1}{2}, \frac{1}{2}$

(D) $-\frac{1}{2}, -\frac{1}{2}$

78. $\lim_{x \to 0} \frac{\cosh x - \cos x}{x \sin x}$

(A) does not exist

(B) is equal to $\frac{1}{2}$

(C) is equal to 1

(D) is equal to -1

79. For $f(x, y) = x \sin\frac{1}{y} + y \sin\frac{1}{x}, xy \neq 0$

Let $\lim\limits_{x\to 0} \lim\limits_{y\to 0} f(x, y)$ .... (1)

$\lim\limits_{y\to 0} \lim\limits_{x\to 0} f(x, y)$ .... (2)

$\lim\limits_{(x,y)\to(0,0)} f(x, y)$ .... (3)

then

(A) (1) exists but (3) does not exist

(B) (2) exists but (3) does not exist

(C) neither (1) nor (2) exists but (3) exists

(D) (1), (2), (3) does not exist

80. Let $u = \log\frac{x^2 + y^2}{x + y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) u

81. Let $u(x, y) = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}, xy \neq 0$ then $\frac{\partial^2 u}{\partial x \partial y}$ is

(A) $x + y$

(B) $\frac{x^2 + y^2}{x^2 - y^2}$

(C) $\frac{x^2 - y^2}{x^2 + y^2}$

(D) $x^2 + y^2$

82. Let $z = z(x, y)$ be a differentiable function and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}$ is equal to

(A) $x\frac{\partial z}{\partial y} - y\frac{\partial z}{\partial x}$

(B) $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$

(C) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$

(D) $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x}$

83. If $u = f(y - z, z - x, x - y)$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \alpha$ where α is

(A) 0
(B) f
(C) 2f
(D) $\frac{1}{2}f$

84. If $z = \log(\tan x + \tan y)$, then $\sin 2x \frac{\partial z}{\partial x} + \sin 2y \frac{\partial z}{\partial y}$ is

(A) 1
(B) 2
(C) 3
(D) 4

85. If $u = \frac{x}{a} + f(ay - bx)$; then $a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y}$ is

(A) a
(B) 1
(C) b
(D) a/b

86. If $z(x, y) = (x-1)^2 + 2y^2$ then

(A) z is minimum for $x = 1, y = 0$
(B) z is maximum for $x = 1, y = 0$
(C) z has no extrema
(D) z has extrema at (0, 0)

87. If $x^x y^y z^z = 1$ then $\frac{\partial z}{\partial x} =$

(A) $-\frac{1 + \log x}{1 + \log z}$
(B) $\frac{1 + \log x}{1 + \log z}$
(C) $-\frac{1 + \log z}{1 + \log x}$
(D) $\frac{1 + \log z}{1 + \log x}$

88. If $I_n = \int \frac{\sin nx}{\sin x} dx$, then $I_n = \frac{2}{n-1} f(x) + I_{n-2}$, (n>2), where $f(x)$ is

(A) $\sin nx$
(B) $\cos nx$
(C) $\sin(n-1)x$
(D) $\cos(n-1)x$

89. Let $I_n = \int \sec^n x\, dx$. If $I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \alpha I_{n-1}$, then α is

(A) $\frac{n}{n-1}$
(B) $\frac{n+1}{n-1}$
(C) $\frac{n-2}{n-1}$
(D) $\frac{n+2}{n-1}$

90. Let $f : [0, a] \to \mathbb{R}$ be a continuous function such that $f(a)\, f(a - x) = 1$, then $\int_0^a \frac{dx}{1 + f(x)}$ is equal to

(A) 0 (B) 1

(C) a (D) a/2

91. $\int_0^2 [x^2]\, dx$ is equal to

(A) 1 (B) $5 - \sqrt{2} - \sqrt{3}$

(C) $3 - \sqrt{2}$ (D) 8/3

92. If $\int_0^\alpha e^{-kx} . x^{n-1}\, dx = \frac{\alpha}{k^n}$ where $k > 0$ and n is a positive integer, then α is

(A) n! (B) n[illegible]

(C) $(n - 1)!$ (D) $n - 1$

93. $\int_{-\alpha}^{\alpha} 5^{-x^2}\, dx =$

(A) $\sqrt{\frac{\pi}{\log 5}}$ (B) $\sqrt{\frac{\log 5}{\pi}}$

(C) $\frac{1}{2}\sqrt{\frac{\pi}{\log 5}}$ (D) $\frac{\pi}{\sqrt{\log 5}}$

94. $\int_0^1 \frac{dx}{x^{1/2}(1 - x)^{1/3}}$

(A) does not exist (B) exists and is equal to 1

(C) exists and is equal to $\frac{6\sqrt{\pi}\,\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{6}\right)}$ (D) is equal to $\frac{\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{1}{3}\right)}{5}$

where the symbols have their usual meaning.

95. Tangents are drawn from the origin to the curve $y = \sin x$. Their points of contact lie on the curve

(A) $y = x$ (B) $\frac{x^2}{2} + y^2 = 1$

(C) $x^2y^2 = x^2 - y^2$ (D) $y^2 = 4x$

96. The orthogonal trajectories of the family of parabolas $y = cx^2$ is a family of

(A) circles (B) ellipses

(C) parabolas (D) straight lines

97. The non- zero value of n for which the differential equation $(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)dy = 0,\ x \neq 0$, becomes exact is

(A) -3 (B) -2

(C) 2 (D) 3

98. $z = \sin x$ transforms the differential equation $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$ into

(A) $\frac{d^2y}{dz^2} + y = 0$ (B) $\frac{d^2y}{dz^2} - y = 0$

(C) $\frac{d^2y}{dz^2} + \frac{dy}{dz} + y = 0$ (D) $\frac{d^2y}{dz^2} + 4y = 0$

99. If the subnormal of a curve is a constant, then the curve must be a

(A) circle (B) parabola

(C) ellipse (D) hyperbola

100. General solution of the differential equation $\frac{d^2y}{dx^2} + 3y = -2x$ is

(A) $y = c_1 \cos x + c_2 \sin x - \frac{2x}{3}$

(B) $y = c_1 \cos\sqrt{3}x + c_2 \sin x + \frac{x}{3}$

(C) $y = c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x - \frac{2x}{3}$

(D) $y = c_1 \cos x + c_2 \sqrt{3} \sin x + \frac{x}{3}$

where c_1, c_2 are real constants.

SPACE FOR ROUGH WORK