QUESTION PAPER CODE 65/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 225 \Rightarrow |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 225$ $\quad\frac{1}{2}$

$\Rightarrow \quad (5)^2 |\vec{b}|^2 = 225 \Rightarrow |\vec{b}| = 3$ $\quad\frac{1}{2}$

2. $\int \frac{3x}{3x-1}dx = \int \frac{3x-1+1}{3x-1}dx$ $\quad\frac{1}{2}$

$= x + \frac{1}{3}\log|3x-1| + C$ $\quad\frac{1}{2}$

3. $\lim_{x\to 0} \frac{\sin\frac{3x}{2}}{2} = \lim_{x\to 0} \frac{3}{2}\cdot\frac{\sin\frac{3x}{2}}{\frac{3x}{2}} = \frac{3}{2}$ $\quad\frac{1}{2}$

$\Rightarrow \quad k = \frac{3}{2}$ $\quad\frac{1}{2}$

4. $|A^{-1}| = \frac{1}{|A|}$ $\quad\frac{1}{2}$

$= \frac{1}{4}$ $\quad\frac{1}{2}$

SECTION B

5. Given differential equation can be written as

$\frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$ $\quad 1$

Getting integrating factor $= e^{x - \log x}$ or $\frac{e^x}{x}$ $\quad 1$

6. For coplanarity of vectors $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ \lambda & 7 & 3 \end{vmatrix} = 0$ $\quad 1$

Solving to get $\lambda = 0$ $\quad 1$

7. Let, Number of executive class tickets be x and economy class tickets be y.

∴ LPP is Maximise Profit $P = 1500x + 1000y$ 1

Subject to: $x + y \leq 250,\ x \geq 25,\ y \geq 3x$ 1

8. Getting $\begin{pmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$ 1

$2x + 3 = 7$ and $2y - 4 = 14$

$\Rightarrow x = 2,\ y = 9$ 1

9. $f(x) = \sin 2x - \cos 2x$

$\Rightarrow f'(x) = 2\cos 2x + 2\sin 2x$ 1

$f'\left(\frac{\pi}{6}\right) = 2\left[\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right] = (1 + \sqrt{3})$ 1

10. $\frac{y}{y+2}dy = \frac{(x+2)}{x}dx \Rightarrow \int\left(1 - \frac{2}{y+2}\right)dy = \int\left(1 + \frac{2}{x}\right)dx$ $\frac{1}{2} + \frac{1}{2}$

$y - 2\log|y+2| = x + 2\log|x| + C$ 1

11. Put $\sin x = t \Rightarrow \cos x\, dx = dt$ $\frac{1}{2}$

Integral reduces to $\int \frac{dt}{\sqrt{8 - t^2}} = \sin^{-1}\left(\frac{t}{\sqrt{8}}\right) + C$ $\frac{1}{2} + \frac{1}{2}$

$= \sin^{-1}\left(\frac{\sin x}{\sqrt{8}}\right) + C$ $\frac{1}{2}$

12. $\frac{dr}{dt} = 5$ cm/min, $\frac{dh}{dt} = -4$ cm/min

$V = \pi r^2 h$ $\frac{1}{2}$

$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2hr\frac{dr}{dt}\right)$ 1

$\left.\frac{dV}{dt}\right)_{r=8,\, h=6} = 224\pi\ \text{cm}^3/\text{min}$ $\frac{1}{2}$

∴ Volume is increasing at the rate of $224\pi\ \text{cm}^3$/min.

SECTION C

13. Let the award for regularily be ₹ x and for hard work be ₹ y.

$\therefore$ $x + y = 6000$ and 1

$x + 3y = 11000$

$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$ or $A.X = B$ 1

$\therefore$ $X = A^{-1}B$ as $|A| = 2 \neq 0$.

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3500 \\ 2500 \end{pmatrix}$ $\therefore$ x = ₹ 3500, y = ₹ 2500 1

Any two values like obedience, respect for elders,... 1

14. $f'(x) = 6x^2 - 6x - 36$ $\frac{1}{2}$

$= 6(x^2 - x - 6) = 6(x - 3)(x + 2)$

$f'(x) = 0 \Rightarrow x = -2, x = 3$ 1

$\therefore$ the intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ $\frac{1}{2}$

getting $f'(x)$ +ve in $(-\infty, -2) \cup (3, \infty)$
and –ve in $(-2, 3)$ } $1\frac{1}{2}$

$\therefore$ f(x) is strictly increasing in $(-\infty, -2) \cup (3, \infty)$, and $\frac{1}{2}$

strictly decreasing in $(-2, 3)$

15. For $\int \frac{x^2+x+1}{(x+1)^2(x+2)}dx = \int \left[\frac{3}{x+2} - \frac{2}{x+1} + \frac{1}{(x+1)^2}\right]dx$ $2\frac{1}{2}$

$= 3\log|x+2| - 2\log|x+1| - \frac{1}{x+1} + C$ $1\frac{1}{2}$

OR

$I = \int (x-3)\sqrt{3-2x-x^2}\,dx = \int \left[-\frac{1}{2}(-2-2x) - 4\right]\sqrt{3-2x-x^2}\,dx$ 1

$= -\frac{1}{2}\int(-2-2x)\sqrt{3-2x-x^2}\,dx - 4\int\sqrt{4-(x+1)^2}\,dx$ $\frac{1}{2}+1$

$= -\frac{1}{3}(3-2x-x^2)^{3/2} - 4\left[\frac{(x+1)}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right)\right] + C$ $\frac{1}{2}+1$

16. Let the vector $\vec{p} = (2\vec{a}+\vec{b}+2\vec{c})$ makes angles α, β, γ respectively with the vector $\vec{a}, \vec{b}, \vec{c}$

Given that $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}\cdot\vec{b} = \vec{c}\cdot\vec{a} = 0$ 1

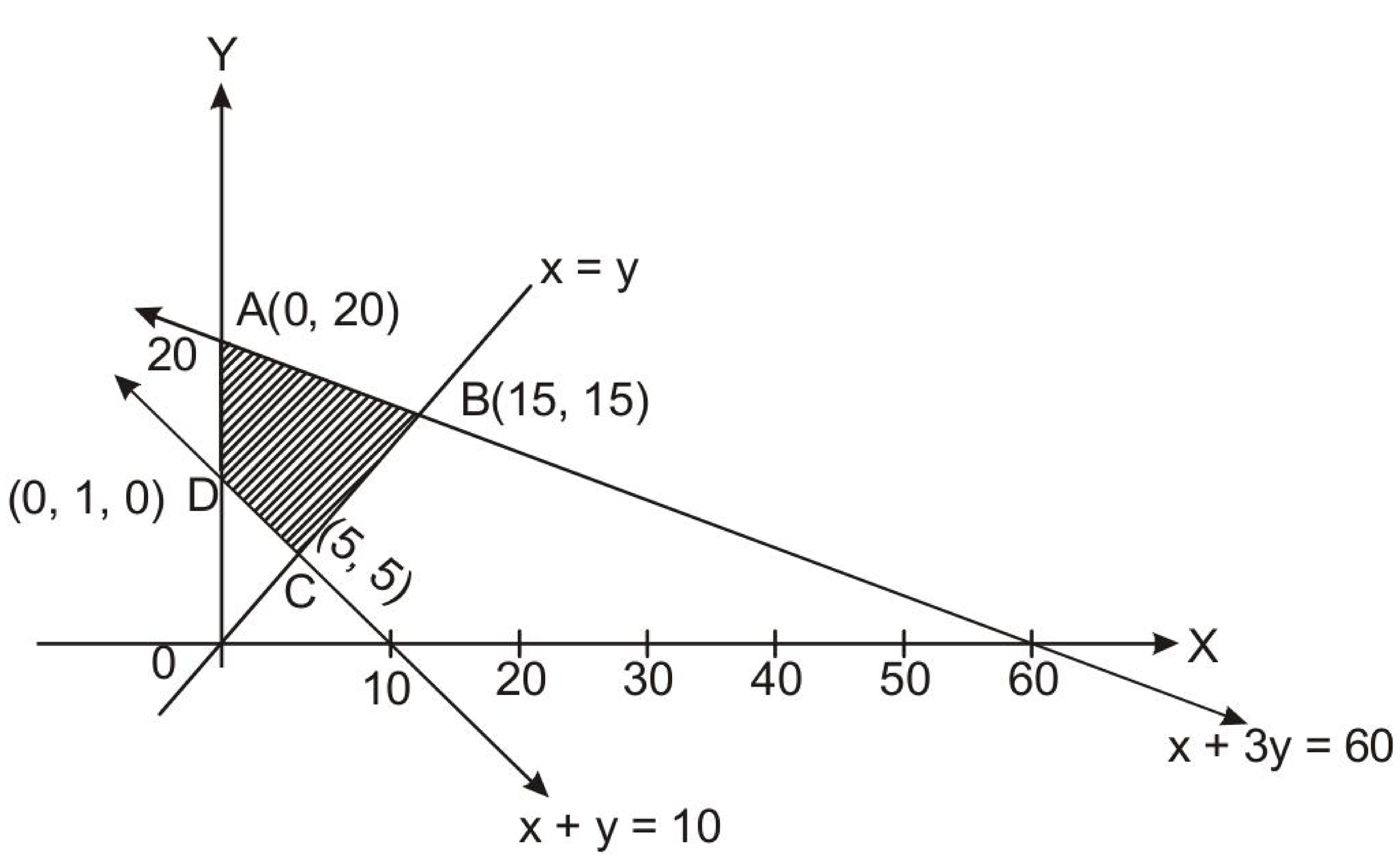
$\cos\alpha = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{a}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|}$ $\frac{1}{2}$

$= \frac{2|a|^2}{3|\vec{a}||\vec{a}|} = \frac{2}{3} \Rightarrow \alpha = \cos^{-1}\frac{2}{3}$ 1

$\cos\beta = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{b}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{b}|} = \frac{|\vec{b}|^2}{3|\vec{b}||\vec{b}|} = \frac{1}{3} \Rightarrow \beta = \cos^{-1}\frac{1}{3}$ 1

$\cos\gamma = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{c}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{c}|} = \frac{2|\vec{c}|^2}{3|\vec{c}||\vec{c}|} = \frac{2}{3} \Rightarrow \gamma = \cos^{-1}\frac{2}{3}$ $\frac{1}{2}$

17.



Correct graph of three lines $1\frac{1}{2}$

Correct shading 1

Vertices of feasible region are

A(0, 20), B(15, 15), C(5, 5), D(0, 10)

Z(A) = 180

Z(B) = 180

Z(C) = 60 1

Z(D) = 90

∴ Z = 60 is minimum at x = 5, y = 5 $\frac{1}{2}$

18. Let the events be

E_1: tansferring a red ball from A to B

E_2: transferring a black ball from A to B

A: Getting a red ball from bag B $\frac{1}{2}$

$$P(E_1) = \frac{3}{5},\ P(E_2) = \frac{2}{5}$$ $\frac{1}{2}$

$$P(A/E_1) = \frac{1}{2},\ P(A/E_2) = \frac{1}{3}$$ 1

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$ $\frac{1}{2}$

$$= \frac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2} + \frac{2}{5}\cdot\frac{1}{3}} = \frac{9}{13}$$ $1+\frac{1}{2}$

OR

Required probability $= P(A \cup B)$ 1

$$= P(A) + P(B) - P(A)\cdot P(B)$$ $\frac{1}{2}$

$$= P(A)\,[1 - P(B)] + 1 - P(B')$$ $\frac{1}{2}$

$$= P(A)\,P(B') - P(B') + 1$$ 1

$$= (1 - P(B'))\,(1 - P(A)) = 1 - P(A')\,P(B')$$ 1

19. Given equation can be written as $\tan^{-1}(1) - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{3}{2}\tan^{-1}x = \frac{\pi}{4} \quad \text{or} \quad \tan^{-1}x = \frac{\pi}{6}$$ $1\frac{1}{2}$

$$\Rightarrow \quad x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$ 1

20. Let $u = (\cos x)^x \Rightarrow \log u = x.\log\cos x$ $\frac{1}{2}$

$\Rightarrow \frac{du}{dx} = (\cos x)^x.[-x\tan x + \log\cos x]$ $1\frac{1}{2}$

$\therefore \quad y = (\cos x)^x + \sin^{-1}\sqrt{3x} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-3x}}.\frac{\sqrt{3}}{2\sqrt{x}}$ $1\frac{1}{2}$

$\therefore \quad \frac{dy}{dx} = (\cos x)^x[-x\tan x + \log\cos x] + \frac{\sqrt{3}}{2\sqrt{x}}.\frac{1}{\sqrt{1-3x}}$ $\frac{1}{2}$

OR

$y = (\sec^{-1}x)^2 \Rightarrow \frac{dy}{dx} = 2\sec^{-1}x.\frac{1}{x\sqrt{x^2-1}}$ 1

$\therefore \quad x\sqrt{x^2-1}\cdot\frac{dy}{dx} = 2\sec^{-1}x$ $\frac{1}{2}$

$\Rightarrow \quad x\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}\right) = \frac{2}{x\sqrt{x^2-1}}$ $1\frac{1}{2}$

$\Rightarrow \quad x^2(x^2-1)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot x(2x^2-1) = 2$ 1

i.e., $\quad x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3-x)\frac{dy}{dx} = 2$

21. Let $I = \int_0^\pi \frac{x\tan x\,dx}{\tan x + \sec x} = \int_0^\pi \frac{(\pi - x)\tan(\pi - x)}{\tan(\pi - x) + \sec(\pi - x)}dx$ $\frac{1}{2}$

So, $I = \int_0^\pi \frac{(\pi - x)(-\tan x)}{-\tan x - \sec x}dx \Rightarrow I = \pi\int_0^\pi \frac{\tan x}{\sec x + \tan x}dx - I$ $\frac{1}{2} + \frac{1}{2}$

$\Rightarrow \quad 2I = \pi\int_0^\pi \frac{\tan x}{\sec x + \tan x}dx = \pi\int_0^\pi (\sec x\tan x - \tan^2 x)dx$ $\frac{1}{2}$

$= \pi\int_0^\pi \sec x\tan x\,dx - \pi\int_0^\pi \sec^2 x\,dx + \pi\int_0^\pi dx$ 1

$= \pi|\sec x|_0^\pi - \pi|\tan x|_0^\pi + \pi^2$

$= \pi^2 - 2\pi$ 1

$\Rightarrow \quad I = \left(\frac{\pi^2}{2} - \pi\right)$

22. Writing the given differential equation in the form

$$\frac{dx}{dy} = \frac{xe^{x/y} + y^2}{ye^{x/y}}$$

$$= \frac{x}{y} + \frac{y}{e^{x/y}}$$ 1

Put $\frac{x}{y} = v$ so, $x = vy$

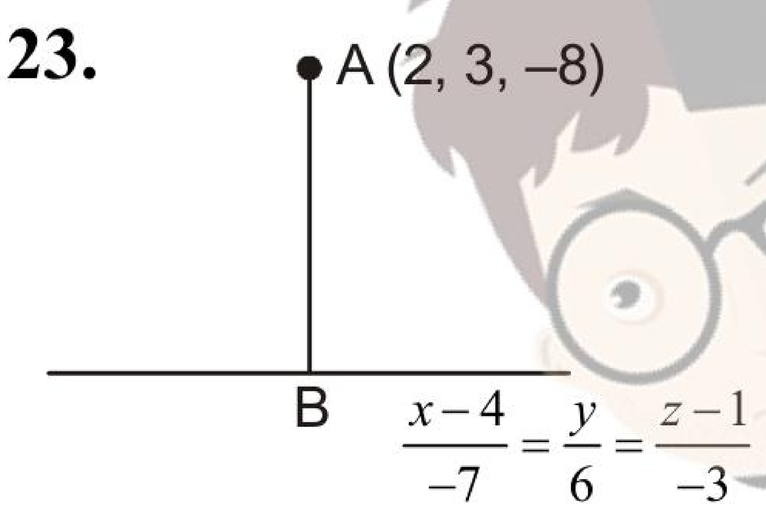
and $\frac{dx}{dy} = v + y\frac{dv}{dy}$ 1

$\therefore \quad v + y\frac{dv}{dy} = v + \frac{y}{e^v} \Rightarrow \frac{dv}{dy} = \frac{1}{e^v}$ 1

$\Rightarrow \quad \int e^v dv = \int dy \quad$ so, $y = e^v + C$

hence, $y = e^{\frac{x}{y}} + C$ 1

23.



Writing line in symmetric form $\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$ $\frac{1}{2}$

$\Rightarrow \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$ gives co-ordinates of B as

$x = -2\lambda + 4,\ y = 6\lambda,\ z = -3\lambda + 1$ for some λ. 1

So, direction ratios of AB are $-2\lambda + 2,\ 6\lambda - 3,\ -3\lambda + 9$ $\frac{1}{2}$

Since AB is perpendicular to the given line

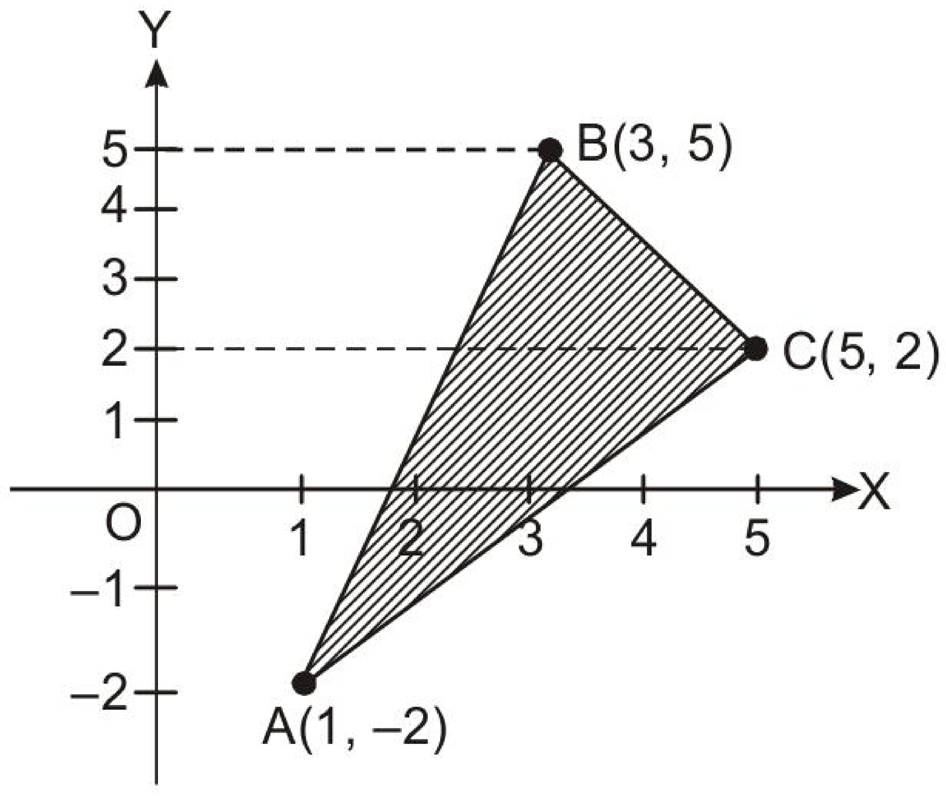
$-2(-2\lambda + 2) + 6(6\lambda - 3) + -3(-3\lambda + 9) = 0$

$\Rightarrow \lambda = 1$ 1

So, foot of perpendicular is B(2, 6, –2) 1

SECTION D

24. **For Correct Figure** 1



Equation of AB: $x = \frac{1}{7}(2y + 11)$

Equation of BC: $x = \frac{1}{3}(19 - 2y)$ $1\frac{1}{2}$

Equation of AC: $x = y + 3$

Required area $= \int_{-2}^{2}(y+3)dy + \frac{1}{3}\int_{2}^{5}(19-2y)dy - \frac{1}{7}\int_{-2}^{5}(2y+11)dy$ $1\frac{1}{2}$

$\Rightarrow \quad A = \left.\frac{(y+3)^2}{2}\right]_{-2}^{2} + \frac{1}{3}\left.\frac{(19-2y)^2}{-4}\right]_{2}^{5} - \frac{1}{7}\left.\frac{(2y+11)^2}{4}\right]_{-2}^{5}$ 1

$= \frac{1}{2}(25-1) - \frac{1}{12}(81-225) - \frac{1}{28}(441-49) = 10$ sq.units 1

OR

Here $h = \frac{4}{n}$ or $nh = 4$, $f(x) = 3x^2 + 2x + 1$ $\frac{1}{2}$

$\int_0^4 (3x^2 + 2x + 1)dx = \lim_{h \to 0} h[f(0) + f(0+h) + f(0+2h) + ... + f(0 + \overline{n-1}h)]$ 1

$= \lim_{h \to 0} h[(1) + (3h^2 + 2h + 1) + (3.2^2h^2 + 2.2h + 1) + ... + (3(n-1)^2h^2 + 2(n-1)h + 1)]$ $1\frac{1}{2}$

$= \lim_{h \to 0} h\left[n + 3h^2\frac{n(n-1)(2n-1)}{6} + 2h\frac{n(n-1)}{2}\right]$ 1

$= \lim_{h \to 0}\left[nh + \frac{(nh)(nh-h)(2nh-h)}{2} + (nh)(nh-h)\right]$ 1

$= 4 + 64 + 16 = 84$ 1

25. $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$ 1

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow \quad (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0$$ 1+1

$\Rightarrow \quad -(a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca) = 0$ 1

$\Rightarrow \quad \frac{-1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ 1

$\Rightarrow \quad a - b = 0 = b - c = c - a$ as $a + b + c \neq 0$ 1

$\Rightarrow \quad a = b = c$

26. (i) for any $A, B \in P(X)$, $A*B = A \cap B$ and $B*A = B \cap A$

as $A \cap B = B \cap A \therefore A*B = B*A$ 2

$\Rightarrow$ * is commutative

(ii) for any $A, B, C \in P(X)$

$(A*B)*C = (A \cap B)*C = (A \cap B) \cap C$

and $A*(B*C) = A*(B \cap C) = A \cap (B \cap C)$

Since $(A \cap B) \cap C = A \cap (B \cap C) \Rightarrow$ * is associative 2

(iii) for every $A \in P(X)$, $A*X = A \cap X = A$

$X*A = X \cap A = A$ 1

$\Rightarrow$ X is the identity element

(iv) $X*X = X \cap X = X \Rightarrow X$ is the only invertible element. $\because$ it is true only for X. 1

OR

$$f(x) = \frac{4x}{3x+4}$$

for $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$, $f(x_1) = f(x_2) \Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$

$\therefore \quad 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$ 2

$\therefore \quad$ f is a 1 – 1 function.

for $y = \frac{4}{3}$, there is no x such that $f(x) = \frac{4}{3}$

$\therefore \quad$ f is not invertible 1

But $f : R - \left\{-\frac{4}{3}\right\} \to$ Range of f is ONTO so invertible. 1

and $f^{-1}(y) = \frac{4y}{4-3y}$ 2

27. Let given volume of cone be, $V = \frac{1}{3}\pi r^2 h$ ...(i) $\frac{1}{2}$

$\therefore \quad$ Surface area (curved) $S = \pi r l = \pi r\sqrt{r^2 + h^2}$ $\frac{1}{2}$

or $\quad A = S^2 = \pi r^2(r^2 + h^2)$

$$A = S^2 = \pi^2 r^2\left[r^2 + \left(\frac{3V}{\pi r^2}\right)^2\right] \quad \text{[using (i)]}$$

$$= \pi^2\left[r^4 + \frac{9V^2}{\pi^2 r^2}\right]$$ $1\frac{1}{2}$

$$\frac{dA}{dr} = \pi^2\left[4r^3 - \frac{18V^2}{\pi^2 r^3}\right]$$ 1

$$\frac{dA}{dr} = 0 \quad \Rightarrow \quad 4\pi^2 r^6 = 18 \cdot \frac{1}{9} \pi^2 r^4 h^2$$

$\Rightarrow \quad 2r^2 = h^2$ or $h = \sqrt{2}r$ $\quad 1\frac{1}{2}$

$$\frac{d^2A}{dr^2} = \pi^2 \left[12r^2 + \frac{54V^2}{\pi^2 r^4} \right] > 0$$ 1

$\Rightarrow$ for least curved surface area, height $= \sqrt{2}$ (radius)

OR

$$x = a\cos\theta + a\theta\sin\theta \quad \Rightarrow \quad \frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta$$

$= a\theta\cos\theta$ 1

$$y = a\sin\theta - a\theta\cos\theta \quad \Rightarrow \quad \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta$$

$= a\theta\sin\theta$ 1

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$ $\frac{1}{2}$

Equation of tangent is

$y - (a\sin\theta - a\theta\cos\theta) = \tan\theta(x - a\cos\theta - a\theta\sin\theta)$ 1

Equation of normal is

$$y - (a\sin\theta - a\theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\theta\sin\theta)$$ 1

$\Rightarrow \quad y\sin\theta + x\cos\theta = a$ $\quad \frac{1}{2}$

distance of normal from origin $= \dfrac{|-a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a| =$ constant 1

28. Let X denote the number of defective bulbs drawn

$\Rightarrow \quad p = \frac{1}{5}, \; q = \frac{4}{5}$ 1

$X = 0, 1, 2, 3.$ 1

$P(X=0) = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{64}{125}$ $\frac{1}{2}$

$P(X=1) = 3 \times \frac{4}{5} \times \frac{4}{5} \times \frac{1}{5} = \frac{48}{125}$ $\frac{1}{2}$

$P(X=2) = 3 \times \frac{4}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{12}{125}$ $\frac{1}{2}$

$P(X=3) = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{125}$ $\frac{1}{2}$

$\text{Mean} = \frac{48}{125} + \frac{24}{125} + \frac{3}{125} = \frac{75}{125} = \frac{3}{5}$ 1

$\text{Variance} = \left(\frac{48}{125} + \frac{48}{125} + \frac{9}{125}\right) - \left(\frac{3}{5}\right)^2 = \frac{60}{125} = \frac{12}{25}$ 1

29. Equation of the plane passing through three points is.

$$\begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$ 2

or $2x + 3y - 3z - 5 = 0$ 1

Since $2(3) + 3(1) - 3(3) = 0 \Rightarrow$ lines is parallel to the plane 1

$\therefore$ $\text{Distance} = \left| \frac{2(3) + 3(5) + (-3)(-2) - 5}{\sqrt{(2)^2 + (3)^2 + (-3)^2}} \right| = \sqrt{22}$ $1\frac{1}{2} + \frac{1}{2}$