Sample Paper 1

ANSWERKEY																			
1	(c)	2	(b)	3	(d)	4	(c)	5	(a)	6	(d)	7	(a)	8	(d)	9	(c)	10	(d)
11	(b)	12	(b)	13	(c)	14	(d)	15	(a)	16	(b)	17	(b)	18	(d)	19	(a)	20	(a)
21	(d)	22	(a)	23	(b)	24	(b)	25	(c)	26	(a)	27	(c)	28	(a)	29	(b)	30	(a)
31	(d)	32	(c)	33	(b)	34	(d)	35	(b)	36	(a)	37	(b)	38	(b)	39	(c)	40	(d)
41	(c)	42	(d)	43	(c)	44	(d)	45	(c)	46	(a)	47	(c)	48	(b)	49	(d)	50	(a)

SOLUTIONS

1. **(c)** Here, the two triangles are similar.

Ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

So, $\frac{h_1^2}{h_2^2} = \frac{25}{36}$

$\therefore \quad \frac{h_1}{h_2} = \frac{5}{6}$

2. **(b)** n(S) = 6 × 6 = 36
E= {(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4), (5, 6), (6, 5)}
n(E) = 10

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

3. **(d)** $(\cos^4 A - \sin^4 A) = (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$
$= (\cos^2 A - \sin^2 A)(1) = \cos^2 A - (1 - \cos^2 A)$
$= 2\cos^2 A - 1$

4. **(c)** For reflection of a point with respect to x-axis change sign of y-coordinate and with respect to y-axis change sign of x-coordinate.

5. **(a)** It is given that AD is the bisector of ∠A.

$\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow AC = \frac{6 \times 3}{4} = 4.5$ cm

6. **(d)** Given, $\tan\theta = \frac{a}{b}$

$\therefore \quad \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2 - b^2}{a^2 + b^2}$

7. **(a)** Let the age of father be 'x' years and the age of son be 'y' years

According to question, $x + y = 65$ ...(i)

and $2(x - y) = 50 \Rightarrow x - y = 25$ ...(ii)

Adding eqs. (i) and (ii), we get, $2x = 90 \Rightarrow x = 45$

Hence, the age of father = 45 years

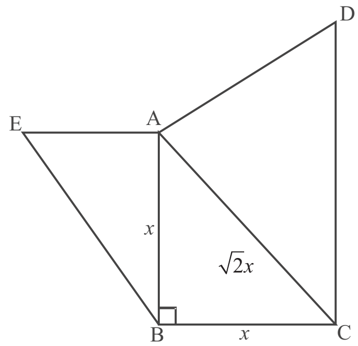
8. **(d)** $P(6, 2) = \left(\frac{4\times3+1\times6}{3+1}, \frac{3\times y+1\times5}{3+1}\right)$

$\because \quad 6 \neq \frac{18}{4}$ (Question is wrong)

$2 = \frac{3y+5}{4} \Rightarrow 3y + 5 = 8$

$3y = 3 \Rightarrow y = 1$

9. **(c)**



Let AB = BC = x.

Since, ΔABC is right-angled with ∠B = 90°

$\therefore \quad AC^2 = AB^2 + BC^2 = x^2 + x^2 = 2x^2$

$\Rightarrow \quad AC = \sqrt{2}x$

Since, $\Delta ABE \sim \Delta ACD$

$$\therefore \quad \frac{\text{Area}(\Delta ABE)}{\text{Area}(\Delta ACD)} = \frac{AB^2}{AC^2} = \frac{x^2}{2x^2} = \frac{1}{2}.$$

Thus, $\frac{\text{Area (ABE)}}{\text{Area (ACD)}} \quad \frac{1}{2}$

Thus, required ratio is 1 : 2.

10. **(d)** We know that $\sec^2\theta - \tan^2\theta = 1$ and $\sec\theta = \frac{x}{p}$, $\tan\theta = \frac{y}{q}$

$\therefore \quad x^2q^2 - p^2y^2 = p^2q^2$

11. **(b)** Substitute $x = 1$ in $f(x)$ and $x = -2$ in $g(x)$, and add

$f(1) = 2(1) - 6(1) + 4(1) - 5 = -5 \Rightarrow g(-2) = 3(4) - 9 = 3$

$f(1) + g(-2) = -2$

12. **(b)** S = {1, 2, 3, ..., 100}

n(S) = 100

E = {11, 22, 33, 44, 55, 66, 77, 88, 99}

n(E) = 9

$\therefore \quad P(E) = \frac{9}{100}$

13. **(c)**

14. **(d)** Let AB be the chord of circle such that $\angle AOB = 90°$

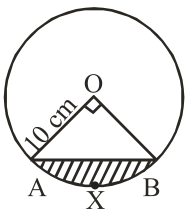
Let OA = 10 cm

$\therefore \; AB = 10\sqrt{2}$ cm

Area of minor segment A X B

= Area of the sector AOB – Area of ΔAOB

$$= \frac{90°}{360°} \times \pi(10)^2 - \frac{1}{2} \times 10 \times 10$$



$= 25\pi - 50 = 25 \times 3.14 - 50 = 78.5 - 50 = 28.5$ cm^2.

15. **(a)** Let the speeds of the cars starting from *A* and *B* be *x* km/hr and *y* km/hr respectively

According to problem,

$9x - 90 = 9y$ .... (i)

$\frac{9}{7}x + \frac{9}{7}y = 90$ .... (ii)

Solving we get $x = 40$ km/hr, $y = 30$ km/hr,

speed of car *A* = 40 km/hr

& speed of car *B* = 30 km/hr

16. **(b)** Polynomial should not have terms with variables whose powers are negative integers or fractions.

17. **(b)** Given $(x)^2 + (x + 2)^2 = 290$

$\Rightarrow \; x^2 + x^2 + 4x + 4 = 290$

$\Rightarrow \; 2x^2 + 4x - 286 = 0$

$\Rightarrow \; x^2 + 2x - 143 = 0$

$\Rightarrow \; x^2 + 13x - 11x - 143 = 0$

$\Rightarrow \; (x + 13)(x - 11) = 0$

$\Rightarrow \; x = -13, \; x = 11$

x cannot be negative, discard $x = -13$, so $x = 11$

Hence the two consecutive positive integers are 11, 13

18. **(d)** 1 + 3 + 5 + ... + (2n – 1)

$1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

$1 + 3 + 5 + ... + (2n - 1) = n^2$

Thus the sum of first n odd natural numbers $= n^2$

2 + 4 + 6 + ... + 2n

2 = 1 (1 + 1)

2 + 4 = 6 = 2 (2 + 1)

2 + 4 + 6 = 12 = 3 (3 + 1)

2 + 4 + 6 + 8 = 20 = 4(4 + 1)

2 + 4 + 6 + ... + 2n = n (n + 1)

Thus, the sum of first 'n' even natural numbers = n(n + 1)

According to given condition

$n(n + 1) = n^2.k$

$$\Rightarrow \; k = \frac{n(n+1)}{n^2} = \frac{n+1}{n}$$

19. **(a)** Given equations are :

$7x - y = 5$ and $21x - 3y = k$

Here $a_1 = 7, b_1 = -1, c_1 = 5$

$a_2 = 21, b_2 = -3, c_2 = k$

We know that the equations are consistent with unique solution

if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Also, the equations are consistent with many solutions

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\therefore \frac{7}{21} = \frac{-1}{-3} = \frac{5}{k} \Rightarrow \frac{1}{3} = \frac{5}{k} \Rightarrow \; k = 15$$

Hence, for $k = 15$, the system becomes consistent.

20. **(a)** Let the numbers be 37a and 37b. Then

$37a \times 37b = 4107 \Rightarrow ab = 3$

Now, co-primes with product 3 are (1, 3)

So, the required numbers are

$(37 \times 1, 37 \times 3)$ i.e., (37, 111).

$\therefore$ Greater number = 111

21. **(d)** If two similar triangles have equal area then triangles are necessarily congruent.

22. **(a)** Here, the two lines are $2x + 3y = 7$ and $2ax + (a + b)y = 28$. The above lines are coincident.

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, $\frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{28}$

$\Rightarrow$ $a = 4, b = 8$

$\therefore$ $b = 2a$

23. **(b)** $2ax - 2by + a + 4b = 0$ .... (i)

and $2bx + 2ay + b - 4a = 0$ .... (ii)

Multiplying eq. (i) with b and eq. (ii) with a, we get

$2abx - 2b^2y + ab + 4b^2 = 0$ .... (iii)

and $2abx + 2a^2y + ab - 4a^2 = 0$ .... (iv)

Subtracting (iv) from (iii), we get

$-(2b^2 + 2a^2)\,y + 4b^2 + 4a^2 = 0$

$\Rightarrow -(2b^2 + 2a^2)\,y = -4b^2 - 4a^2 \Rightarrow y = 2$

Substituting y = 2 in eq. (1), we get

$2ax - 2b \times 2 + a + 4b = 0$

$\Rightarrow x = -1/2 \qquad \therefore x = -1/2,\ y = 2$

24. **(b)** Put $x + 1 = 0$ or $x = -1$ and $x + 2 = 0$ or $x = -2$ in $p(x)$

Then, $p(-1) = 0$ and $p(-2) = 0$

$\Rightarrow p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta = 0$

$\Rightarrow -1 + 3 + 2\alpha + \beta = 0 \Rightarrow \beta = -2\alpha - 2$ .... (i)

$p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$

$\Rightarrow -8 + 12 + 4\alpha + \beta = 0 \Rightarrow \beta = -4\alpha - 4$ .... (ii)

By equalising both of the above equations, we get

$-2\alpha - 2 = -4\alpha - 4$

$\Rightarrow 2\alpha = -2 \Rightarrow \alpha = -1$

put $\alpha = -1$ in eq. (i)

$\Rightarrow \beta = -2(-1) - 2 = 2 - 2 = 0$

Hence, $\alpha = -1,\ \beta = 0$

25. **(c)** Let α, β be two zeroes of $2x^2 - 8x - m$, where $\alpha = \frac{5}{2}$.

$\therefore \alpha + \beta = \frac{(-\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

$\Rightarrow \frac{5}{2} + \beta = \frac{8}{2}$

$\Rightarrow \beta = \frac{8}{2} - \frac{5}{2} = \frac{3}{2}$.

26. **(a)** Let $f(x) = 2x^3 - 5x^2 + ax + b$

$f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow 16 - 20 + 2a + b = 0 \Rightarrow 2a + b = 4$

$f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0 \Rightarrow b = 0$

$\Rightarrow 2a = 4 \Rightarrow a = 2, b = 0$

27. **(c)** $\cos A = \frac{3}{5} \Rightarrow \sin A = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

consider

$$9\cot^2 A - 1 = \frac{9\cos^2 A}{\sin^2 A} - 1 = \frac{9\cos^2 A - \sin^2 A}{\sin^2 A}$$

$$= \frac{9\left(\frac{9}{25}\right) - \left(\frac{16}{25}\right)}{\frac{16}{25}} = \frac{(81 - 16)}{25} \times \frac{25}{16} = \frac{65}{16}$$

28. **(a)** All isosceles triangles are not similar.

29. **(b)** Let α and 6α be roots of equation.

Sum of roots : $\alpha + 6\alpha = \frac{14}{p}$

$\Rightarrow 7\alpha = \frac{14}{p} \Rightarrow p = \frac{2}{\alpha}$

Product of roots : $(\alpha)(6\alpha) = \frac{8}{p} \Rightarrow p = \frac{4}{3\alpha^2}$

$\Rightarrow \frac{2}{\alpha} = \frac{4}{3\alpha^2}$

$\Rightarrow \alpha = \frac{2}{3}$

Therefore, $p = \frac{2}{\alpha} = 3$

30. **(a)** The equations $3x - (a + 1)y = 2b - 1$

$5x + (1 - 2a)y = 3b$

The system will have infinite number of solutions

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Here, $a_1 = 3,\ b_1 = -(a + 1),\ c_1 = 2b - 1$

$a_2 = 5,\ b_2 = 1 - 2a,\ c_2 = 3b$

$\therefore \frac{3}{5} = \frac{-(a+1)}{1-2a} = \frac{2b-1}{3b}$

Taking I and II

$\frac{3}{5} = \frac{-(a+1)}{1-2a}$

$\Rightarrow -5a - 5 = 3 - 6a$

$\Rightarrow -5a + 6a = 3 + 5$

$a = 8$

Taking I and III

$\frac{3}{5} = \frac{2b-1}{3b}$

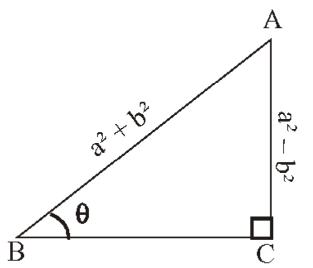
$\Rightarrow 10b - 5 = 9b$

$\Rightarrow 10b - 9b = 5$

$b = 5$

$\therefore a = 8, b = 5$

31. **(d)** $\sin\theta = \frac{a^2 - b^2}{a^2 + b^2}$



Since, $\sin\theta = \frac{\text{perpendicular}}{\text{base}}$

$\therefore \frac{AC}{AB} = \frac{a^2-b^2}{a^2+b^2}$

Now in ΔABC,

$\angle B = \theta$ and $\angle C = 90°$

$(a^2+b^2)^2 = BC^2 + (a^2-b^2)^2$

$\therefore BC = 2ab$

$\text{cosec }\theta = \frac{a^2+b^2}{a^2-b^2}$,

$\cot\theta = \frac{BC}{AC} = \frac{2ab}{a^2-b^2}$

$\text{cosec }\theta + \cot\theta = \frac{a^2+b^2}{a^2-b^2} + \frac{2ab}{a^2-b^2} = \frac{a+b}{a-b}$

32. **(c)** 3, because it is the exponent of the highest degree term in the polynomial $y^3 - 2y^2 - \sqrt{3}y + \frac{1}{2}$.

33. **(b)** Since α, β are roots of $x^2 + x\sqrt{a} + \beta = 0$

$\Rightarrow \alpha^2 + \alpha\sqrt{\alpha} + \beta = 0$ ...(i)

and $\beta^2 + \beta\sqrt{\alpha} + \beta = 0$ ...(ii)

Multiply equation (*i*) by β and equation (*ii*) by α and subtract

$\alpha^2\beta + \alpha\beta\sqrt{\alpha} + \beta^2 = 0$

$\alpha\beta^2 + \alpha\beta\sqrt{\alpha} + \alpha\beta = 0$

(–) (–) (–)

$\alpha\beta(\alpha-\beta) + \beta(\beta-\alpha) = 0$

$\Rightarrow (\alpha\beta - \beta)(\alpha - \beta) = 0$

$\Rightarrow \alpha\beta - \beta = 0$

($\because \alpha - \beta = 0 \Rightarrow \alpha = \beta$ which is not possible)

$\Rightarrow (\alpha - 1)\beta = 0$

$\Rightarrow \alpha - 1 = 0$

$\Rightarrow \alpha = 1$

Put $\alpha = 1$ in (i) $\Rightarrow \beta = -2$

34. **(d)** $ax + by = c,\ bx - ay = c$

Using the cross-multiplication method,

$\frac{x}{-ac-bc} = \frac{y}{ac-bc} = \frac{1}{-a^2-b^2}$

$\Rightarrow x = \frac{-ac-bc}{-a^2-b^2} = \frac{-c(a+b)}{-(a^2+b^2)} = \frac{c(a+b)}{a^2+b^2}$

and

$y = \frac{ac-bc}{-a^2-b^2} = \frac{c(a-b)}{-(a^2+b^2)} = -\frac{c(a-b)}{a^2+b^2}$

Therefore, $x = \frac{c(a+b)}{a^2+b^2},\ y = -\frac{c(a-b)}{a^2+b^2}$

35. **(b)** In ΔKPN and ΔKLM, we have

∠KNP = ∠KML = 46°

∠K = ∠K (Common)

∴ ΔKNP ~ ΔKML

(By A-A criterion of similarity)

$\Rightarrow \frac{KN}{KM} = \frac{NP}{ML} \Rightarrow \frac{c}{b+c} = \frac{x}{a}$

36. **(a)**

$$\frac{\sec\theta.\text{cosec }\theta(90°-\theta) - \tan\theta\cot(90°-\theta) + \sin^2 55° + \sin^2 35°}{\tan 10° \tan 20° \tan 60° \tan 70° \tan 80°}$$

$$= \frac{\sec\theta.\sec\theta - \tan\theta.\tan\theta + \sin^2(90°-35°) + \sin^2 35°}{\tan 10° \tan 20°.\sqrt{3}.\tan(90°-20°)\ \tan(90°-10°)}$$

[Using $\text{cosec }(90°-\theta) = \sec\theta,\ \cot(90°-\theta) = \tan\theta$]

$$= \frac{(\sec^2\theta - \tan^2\theta) + (\cos^2 35° + \sin^2 35°)}{\sqrt{3}\tan 10° \tan 20° \cot 20° \cot 10°}$$

[Using $\sin(90°-\theta) = \cos\theta,\ \tan(90°-\theta) = \cot\theta$]

$$= \frac{1+1}{\sqrt{3}.(\tan 10° \cot 10°)(\tan 20° \cot 20°)}$$

[Using $\sec^2\theta - \tan^2\theta = 1,\ \sin^2\theta + \cos^2\theta = 1$]

$$= \frac{2}{\sqrt{3}\times 1\times 1} = \frac{2}{\sqrt{3}}$$ [Using $\tan\theta.\cot\theta = 1$]

37. **(b)** Let $f(x) = 6x^3 - 11x^2 + kx - 20$

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$$

$\Rightarrow 6.\frac{64}{27} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$

$\Rightarrow 128 - 176 + 12k - 180 = 0$

$\Rightarrow 12k + 128 - 356 = 0 \Rightarrow 12k = 228$

$\Rightarrow k = 19$

38. **(b)** $\frac{21}{45} = \frac{21}{9\times 5} = \frac{21}{3^2\times 5}$.

Clearly, 45 is not of the form $2^m \times 5^n$. So the decimal expansion of $\frac{21}{45}$ is non-terminating and repeating.

39. **(c)** Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr then speed of boat in downstream is $(x + y)$ km/hr and the speed of boat upstream is $(x - y)$ km/hr.

Ist case : Distance covered upstream = 12 km

$\therefore$ time $= \frac{12}{x-y}$ hr

Distance covered downstream = 40 km

$\therefore$ time $= \frac{40}{x+y}$ hr

Total time is 8 hr $\therefore\ \frac{12}{x-y}+\frac{40}{x+y}=8$ ...(i)

IInd case :
Distance covered upstream = 16 km

$\therefore$ time = $\frac{16}{x-y}$ hr

Distance covered downstream

= 32 km $\therefore$ time = $\frac{32}{x+y}$ hr

Total time taken = 8 hr

$\therefore\ \frac{16}{x-y}+\frac{32}{x+y}=8$ ...(ii)

Solving (i) and (ii), we get,
x = speed of boat in still water = 6 km/hr,
y = speed of stream = 2 km/hr

40. (d) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$\Rightarrow \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\sec A \cos A$

$\Rightarrow (\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + 2 + \sec^2 A + 2$

$\Rightarrow 1 + 4 + 1 + \cot^2 A + 1 + \tan^2 A$

$\Rightarrow 7 + \cot^2 A + \tan^2 A$

$\therefore (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$
$= 7 + \cot^2 A + \tan^2 A$

Hence, a = 7

41. (c), 42. (d), 43. (c), 44. (d), 45. (c)

46. (a) Radius of inner semicircular end

$= \frac{60}{2} = 30$ m

47. (c) Radius of outer semicircular end

= 30 + 10 = 40 m

48. (b) The distance arounded the track along its inner edge

$= 106 \times 2 + 2 \times \pi r$

$= 212 + 2 \times \frac{22}{7} \times 30 = 212 + 188.57$

= 400.57 m

49. (d) The distance arounded the track along its outer edge

$= 106 \times 2 + 2 \times \pi r$

$= 212 + 2 \times \frac{22}{7} \times 40 = 212 + 251.43$

= 463.43 m

50. (a) The area of the track

= 2 × Area of ractangle + 2 × Area of semicircular ring.

$= 2(10 \times 106) + 2 \times \frac{1}{2} \times \frac{22}{7} \times (40^2 - 30^2)$

$= 2120 + 2200 = 4320\ m^2$