DU MPhil Phd in Mathematics

Topic:- MATHS MPHIL

1) Consider the following:

A. Pythagorean theorem is mentioned in the Baudhayana Sulbha-sutra.

B. The famous mathematical treatise, Lilavati written by Brahmagupta, includes methods for computing multiplications, squares, and progressions of numbers.

C. Pell's equation, credited to the English mathematician John Pell, had already made appearance in Brahmagupta's work.
D. Madhava of 14th century was aware of infinite series and series expansion.

Choose the *correct* answer from the options given below:

[Question ID = 9308]

1. A and C only

   [Option ID = 37229]
2. A, B and D only

   [Option ID = 37230]
3. A, C and D only

   [Option ID = 37231]
4. A and D only

   [Option ID = 37232]

2) Which of the following mathematicians is not a recipient of SASTRA Ramanujan Prize?[Question ID = 9309]

1. Manjul Bhargava [Option ID = 37233]
2. Amalendu Krishna [Option ID = 37234]
3. Terence Tao [Option ID = 37235]
4. Akshay Venkatesh [Option ID = 37236]

3) Which of the following is not a mathematical society in India?[Question ID = 9310]

1. Ramanujan Mathematical Society [Option ID = 37237]
2. Indian Mathematical Olympiad Society [Option ID = 37238]
3. The Allahabad Mathematical Society [Option ID = 37239]
4. Calcutta Mathematical Society [Option ID = 37240]

4) Which of the following is a research institute of mathematics in India?[Question ID = 9311]

1. Agharkar Research Institute [Option ID = 37241]
2. Tata Institute of Fundamental Research [Option ID = 37242]
3. Bose Institute [Option ID = 37243]
4. Raman Research Institute [Option ID = 37244]

5)

Let $\{M_i\}_{i\in\Delta}$ be a family of left R-modules and
$\mu_j : M_j \to \coprod_{i\in\Delta} M_i$ be canonical injections for all $j \in \Delta$. Then

[Question ID = 9312]

1. $\coprod_{i\in\Delta} M_i \subsetneq \sum_{i\in\Delta} Im(\mu_i)$

   [Option ID = 37245]
2. $\sum_{i\in\Delta} Im(\mu_i) \subsetneq \coprod_{i\in\Delta} M_i$

   [Option ID = 37246]
3. $\coprod_{i\in\Delta} M_i = \sum_{i\in\Delta} Im(\mu_i) \neq \oplus_{i\in\Delta} Im(\mu_i)$

   [Option ID = 37248]
4. $\coprod_{i\in\Delta} M_i = \sum_{i\in\Delta} Im(\mu_i) = \oplus_{i\in\Delta} Im(\mu_i)$

   [Option ID = 46561]

6)

Let Rx and Ry be simple submodules of a left R-module M
for some $x, y \in M$. Then $f : Rx \to Ry$ given by
$f(ax) = ay$ for all $a \in R$ is

[Question ID = 9313]
1. not an R-homomorphism.

   [Option ID = 37249]
2. an R-homomorphism but not one one.

   [Option ID = 37250]
3. a one one R-homomorphism but not onto.

   [Option ID = 37251]
4. a one one, onto R-homomorphism.

   [Option ID = 37252]

7) If every cyclic module over a commutative ring R is free, then

[Question ID = 9314]
1. R has a nonzero maximal ideal.

   [Option ID = 37253]
2. R has exactly two maximal ideals.

   [Option ID = 37254]
3. R is a field.

   [Option ID = 37255]
4. R has nonzero zero divisors.

   [Option ID = 37256]

8) If R is a ring with unity such that $a^2 = a$ for all $a \in R$, then

[Question ID = 9315]
1. R may not be commutative but every element in R is the additive inverse of itself.

   [Option ID = 37257]
2. R is commutative but every element in R may not be the additive inverse of itself.

   [Option ID = 37258]
3. R is commutative and every element in R is the additive inverse of itself.

   [Option ID = 37259]
4. neither R is commutative nor every element in R is the additive inverse of itself.

   [Option ID = 37260]

9) If R is a commutative ring with unity and I, J are ideals in R such that $I + J = R$, then the ideal $I^5 + J^7$ is

[Question ID = 9316]
1. a maximal ideal of R.

   [Option ID = 37261]
2. a proper but may not be a maximal ideal of R.

   [Option ID = 37262]
3. a proper but may not be a prime ideal of R.

   [Option ID = 37263]
4. the whole ring R.

   [Option ID = 37264]

10) The degree of the field extension $\mathbb{Q} \subset \mathbb{Q}(\theta)$, where θ satisfies the equation $\theta^3(\theta^3 - p) = \theta^3 - 2$ for some odd prime p, is

[Question ID = 9317]
1. 2

   [Option ID = 37265]
2. 3

   [Option ID = 37266]
3. 4

   [Option ID = 37267]
4. 6

   [Option ID = 37268]

11) The quotient field of which of the following rings have zero characteristic:

A. $\frac{\mathbb{Z}[x]}{<2>}$

B. $\frac{\mathbb{Z}[x]}{<x>}$

C. $\frac{\mathbb{Z}[x]}{<x^2+1>}$

D. $\frac{\mathbb{Z}[x]}{<5,x,y>}$

Choose the ***correct*** answer from the options given below:

[Question ID = 9318]

1. A and B only

   [Option ID = 37269]
2. B and C only

   [Option ID = 37270]
3. A and D only

   [Option ID = 37271]
4. C and D only

   [Option ID = 37272]

12) The Galois group of the polynomial $x^3 - 3x - 3$ over the field $\mathbb{Q}$ is

[Question ID = 9319]

1. $\mathbb{Z}_3$

   [Option ID = 37273]
2. $\mathbb{Z}_2 \times \mathbb{Z}_3$

   [Option ID = 37274]
3. A_3

   [Option ID = 37275]
4. S_3

   [Option ID = 37276]

13) Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : If $\mathbb{Q} \subsetneq E \subsetneq \mathbb{Q}(x)$ then $\mathbb{Q}(x)$ is a finite field extension of E.

Reason R : Let f, g be two co-prime polynomials in $\mathbb{Q}[x]$ such that $f/g \notin \mathbb{Q}$ Then x is algebraic over $\mathbb{Q}(f/g)$.

In light of the above statements, choose the ***correct*** answer from the options given below:

[Question ID = 9320]

1. Both *A* and *R* are true and *R* is the correct explanation of *A*

   [Option ID = 37277]
2. Both *A* and *R* are true but *R* is NOT the correct explanation of *A*

   [Option ID = 37278]
3. *A* is true but *R* is false

   [Option ID = 37279]
4. *A* is false but *R* is true

   [Option ID = 37280]

14) The integral of $ydx + xdy - 2zdz = 0$ is

[Question ID = 9321]

1. $xy - z = C$

   [Option ID = 37281]

2. $xy - 2z = C$

[Option ID = 37282]

3. $xy - z^2 = C$

[Option ID = 37283]

4. $xy + z^2 = C$

[Option ID = 37284]

15) Solution(s) of the Dirichlet problem for Laplace equation is two-dimensions, if exists

[Question ID = 9322]

1. is unique.

[Option ID = 37285]

2. are finitely many but more than one.

[Option ID = 37286]

3. are infinitely many.

[Option ID = 37287]

4. is unique up to addition of a constant.

[Option ID = 37288]

16) The eigen values associated with the system $y_1' = 6y_1 - 7y_2;\ y_2' = y_1 - 2y_2$ **are**

[Question ID = 9323]

1. 5 and -1

[Option ID = 37289]

2. 6 and -1

[Option ID = 37290]

3. 7 and 2

[Option ID = 37291]

4. 7 and -2

[Option ID = 37292]

17)

Let u be a solution of the Neumann problem $\nabla^2 u = 0$ in D and $\frac{\partial u}{\partial n} = f(s)$ on B, where D is the interior of a simple, closed and smooth curve B, then the value of $\int_B f(s)ds$ is equal to

[Question ID = 9324]

1. 0

[Option ID = 37293]

2. $\frac{1}{2}$

[Option ID = 37294]

3. $\frac{3}{2}$

[Option ID = 37295]

4. 1

[Option ID = 37296]

18)

Given below are two statements:

Statement I: The solution of the one-dimensional heat equation $u_t = ku_{xx}$ is

$$u = \frac{1}{\sqrt{t}} e^{-\frac{x^2}{4kt}}$$

Statement II: $u = \frac{1}{2\sqrt{\pi kt}} e^{-\frac{(x-\xi)^2}{4kt}}$, where ξ is an arbitrary real constant , is also a solution of $u_t = ku_{xx}$.

In light of the above statements, choose the ***correct*** answer from the options given below:

[Question ID = 9325]

1. Both Statement I and Statement II are true

[Option ID = 37297]

2. Both Statement I and Statement II are false

[Option ID = 37298]
3. Statement I is true but Statement II is false

[Option ID = 37299]
4. Statement I is false but Statement II is true

[Option ID = 37300]

19)
Given below are two statements

Statement I: The set of all functions from {0,1,2} into $2^{\mathbb{Z}}$ is countable.

Statement II: The set of all functions from $2^{\mathbb{Z}}$ into {0,1,2} is countable.

In light of the above statements, choose the ***correct*** answer from the options given below

[Question ID = 9326]
1. Both Statement I and Statement II are true

[Option ID = 37301]
2. Both Statement I and Statement II are false

[Option ID = 37302]
3. Statement I is true but Statement II is false

[Option ID = 37303]
4. Statement I is false but Statement II is true

[Option ID = 37304]

20)
Given below are two statements

Statement I: The integral $\int_0^1 \frac{e^x}{x^3} dx$ converges.

Statement II: The integral $\int_1^\infty \frac{\sin^3 x}{x^3} dx$ converges.

In light of the above statements, choose the ***correct*** answer from the options given below

[Question ID = 9327]
1. Both Statement I and Statement II are true

[Option ID = 37305]
2. Both Statement I and Statement II are false

[Option ID = 37306]
3. Statement I is true but Statement II is false

[Option ID = 37307]
4. Statement I is false but Statement II is true

[Option ID = 37308]

21) Let X be a connected subset of the usual space $\mathbb{R}$ of real numbers and Y be a compact subset of $\mathbb{R}$. Then X U Y

[Question ID = 9328]
1. is always uncountable and has a limit point in $\mathbb{R}$

[Option ID = 37309]
2. is always uncountable but may not have a limit point in $\mathbb{R}$

[Option ID = 37310]
3. may be finite without a limit point in $\mathbb{R}$

[Option ID = 37311]
4. may be finite but has a limit point in $\mathbb{R}$

[Option ID = 37312]

22)
Let f be defined on the square: $\left\{(x,y): \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 1 \end{matrix}\right\}$ by the formula $f(x,y) = \left\{\begin{matrix} 1 \,; x \text{ is irrational} \\ 3y^2 \,; x \text{ is rational} \end{matrix}\right\}$

Given below are two statements

Statement I: $\int_0^1 \left(\int_0^1 f(x,y)dy\right) dx$ exists and equals to 1.

Statement II: $\int_0^1 \left(\int_0^1 f(x,y)dx\right) dy$ exists and equals to 1.

In light of the above statements, choose the **correct** answer from the options given below

[Question ID = 9329]

1. Both Statement I and Statement II are true

   [Option ID = 37313]
2. Both Statement I and Statement II are false

   [Option ID = 37314]
3. Statement I is true but Statement II is false

   [Option ID = 37315]
4. Statement I is false but Statement II is true

   [Option ID = 37316]

23) Consider the sequence $\langle f_n \rangle_{n=2}^{\infty}$ defined on [0,1] as

$$f_n(x) = \begin{cases} n^2 x; & 0 \le x \le \frac{1}{n} \\ -n^2 x + 2n; & \frac{1}{n} \le x \le \frac{2}{n} \\ 0; & \frac{2}{n} \le x \le 1 \end{cases}$$

Then,

[Question ID = 9330]

1. $\langle f_n \rangle$ is pointwise convergent to some f and $\lim_{n\to\infty} \int_0^1 f_n = \int_0^1 f$.

   [Option ID = 37317]
2. $\langle f_n \rangle$ is uniformly convergent to some f which it is continuous on [0,1].

   [Option ID = 37318]
3. $\langle f_n \rangle$ is pointwise convergent to some f but $\lim_{n\to\infty} \int_0^1 f_n \neq \int_0^1 f$.

   [Option ID = 37319]
4. $\langle f_n \rangle$ is not pointwise convergent.

   [Option ID = 37320]

24) Let A be an $n \times n$ complex matrix with $|a_{ii}| > \sum_{j=1, j\neq i}^{n} |a_{ij}|$, for $i = 1,2,3, \dots \dots n$.

Then,

[Question ID = 9331]

1. A has full rank.

   [Option ID = 37321]
2. A has at least one eigen value which is zero.

   [Option ID = 37322]
3. the image of the linear transformation $T(X) = AX$ has dimension less than n.

   [Option ID = 37323]
4. dimension of the null space of the linear transformation $T(X) = AX$ is strictly more than zero.

   [Option ID = 37324]

25) Let V be the real vector space of all 3×3 real matrices and let $A = \begin{pmatrix} a & b & 0 \\ 0 & a & 0 \\ 0 & 0 & c \end{pmatrix}$ where a, b, c are non zero real numbers. Which of the following is not a subspace of V?

[Question ID = 9332]

1. $\{X \in V \mid XA = AX\}$

   [Option ID = 37325]
2. $\{X \in V \mid X + A = A + X\}$

   [Option ID = 37326]
3. $\{X \in V \mid trace\ (AX) = 0\}$

   [Option ID = 37327]
4. $\{X \in V \mid det\ (AX) = 0\}$

   [Option ID = 37328]

26)

Consider the matrices $a = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{pmatrix}$ $b = \begin{pmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ $c = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 2 & 1 \\ 5 & 1 & 2 \end{pmatrix}$ $d = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

Which of the following statements is true?

[Question ID = 9333]

1. Matrix *d* is diagonalizable.

   [Option ID = 37329]
2. Matrices *a* and *c* are diagonalizable.

   [Option ID = 37330]
3. All the eigen values are real for the matrix b.

   [Option ID = 37331]
4. Matrix *a* is diagonalizable but *c* is not.

   [Option ID = 37332]

27)

Let $S = \left\{\frac{1}{2^n} | n \in \mathbb{N}\right\}$ and define $f: [0,1] \to \mathbb{R}$

by $f(x) = \sin\left(\frac{1}{x}\right)$, if $x \in S$, and $= 0$, otherwise. Then on [0,1], f is

[Question ID = 9334]

1. both Riemann integrable and Lebesgue integrable

   [Option ID = 37333]
2. Riemann integrable but not Lebesgue integrable

   [Option ID = 37334]
3. Lebesgue integrable but not Riemann integrable

   [Option ID = 37335]
4. neither Riemann integrable nor Lebesgue integrable

   [Option ID = 37336]

28) Given below are two statements

Statement I: There are infinitely many Lebesgue non-measurable subsets of $\mathbb{R}$.

Statement II: If $\{f_n\}$ is a sequence of nonnegative Lebesgue measurable functions on $\mathbb{R}$ such that $f_n \geq f_{n+1}$ for all n, and $f(x) = \lim_{n\to\infty} f_n(x)$ for all x in $\mathbb{R}$, then $\int_{-\infty}^{\infty} f(x)dx = \lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x)dx$.

In light of the above statements, choose the ***correct*** answer from the options given below

[Question ID = 9335]

1. Both Statement I and Statement II are true

   [Option ID = 37337]
2. Both Statement I and Statement II are false

   [Option ID = 37338]
3. Statement I is true but Statement II is false

   [Option ID = 37339]
4. Statement I is false but Statement II is true

   [Option ID = 37340]

29)

Given below are two statements

Statement I: If f is a Lebesgue integrable function on $\mathbb{R}$ such that $\int f(x)dx = 0$ then $f = 0$ a.e..

Statement II: If g is a nonnegative real valued function on $\mathbb{R}$ with finite range such that $\{x \in \mathbb{R} | g(x) = t\}$ is Lebesgue measurable for every $t \in \mathbb{R}$, then g is Lebesgue measurable.

In light of the above statements, choose the ***correct*** answer from the options given below

[Question ID = 9336]

1. Both Statement I and Statement II are true

   [Option ID = 37341]

2. Both Statement I and Statement II are false

[Option ID = 37342]

3. Statement I is true but Statement II is false

[Option ID = 37343]

4. Statement I is false but Statement II is true

[Option ID = 37344]

30) Let $S = \{z \in \mathbb{C} : |z - (4+3i)| = 2\}$ and let $\alpha = min\{|z| : z \in S\}$ Then the value of α is

[Question ID = 9337]

1. 3

[Option ID = 37345]

2. 4

[Option ID = 37346]

3. $\sqrt{13}$

[Option ID = 37347]

4. $\sqrt{45}$

[Option ID = 37348]

31) The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+2)} z^{n(n+2)}$ is

[Question ID = 9338]

1. 0

[Option ID = 37349]

2. ∞

[Option ID = 37350]

3. 1

[Option ID = 37351]

4. e^1

[Option ID = 37352]

32) The value of $\int_\gamma \frac{\exp(2z)}{z(z-2)} dz$ where γ is the anti-clockwise circle $|z| = 3$, is

[Question ID = 9339]

1. $-i\pi(e^4 - 1)$

[Option ID = 37353]

2. $i\pi(e^4 - 1)$

[Option ID = 37354]

3. $2\pi i(e^4 + 1)$

[Option ID = 37355]

4. $i\pi(e^4 + 1)$

[Option ID = 37356]

33) The $M\ddot{o}bius$ transformation which sends 1 to $0, 0\, to\ i$ and -1 to ∞ is

[Question ID = 9340]

1. $\frac{z-1}{i(z+1)}$

[Option ID = 37357]

2. $\frac{z+1}{i(z-1)}$

[Option ID = 37358]

3. $\frac{z-1}{z+1}$

[Option ID = 37359]

4. $\frac{z}{z+1}$

[Option ID = 37360]

34)

For a group homomorphism $\phi : G \to G'$, consider the following two statements :

Statement I: If $H \lhd G$, then $\phi(H) \lhd G'$.

Statement II: If $H \lhd G'$, then $\phi^{-1}(H) \lhd G$.

In light of the above statements, choose the ***correct*** answer from the options given below

[Question ID = 9341]

1. Both Statement I and Statement II are true

   [Option ID = 37361]
2. Both Statement I and Statement II are false

   [Option ID = 37362]
3. Statement I is true but Statement II is false

   [Option ID = 37363]
4. Statement I is false but Statement II is true

   [Option ID = 37364]

35)

Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: The commutator subgroup of $GL(n, \mathbb{R})$ is a subgroup of $SL(n, \mathbb{R})$.

Reason R: The commutator subgroup G' of a group G is the smallest normal subgroup such that G/G' is abelian.

In light of the above statements, choose the ***most appropriate*** answer from the options given below

[Question ID = 9342]

1. Both *A* and *R* are correct and *R* is the correct explanation of *A*

   [Option ID = 37365]
2. Both *A* and *R* are correct but *R* is NOT the correct explanation of *A*

   [Option ID = 37366]
3. *A* is correct but *R* is not correct

   [Option ID = 37367]
4. *A* is not correct but *R* is correct

   [Option ID = 37368]

36) The number of Sylow 5-subgroups of S_5 is

[Question ID = 9343]

1. 1

   [Option ID = 37369]
2. 6

   [Option ID = 37370]
3. 11

   [Option ID = 37371]
4. 21

   [Option ID = 37372]

37)

Let $\vec{q} = [0, -\omega z, \omega y]$, ($\omega$ a constant) be the velocity of an incompressible fluid. For such $\vec{q}$,

A. the motion is possible for every value of ω.

B. the motion is of potential kind for every value of ω.

C. the streamlines are concentric circles in yz–plane.

D. the velocity potential exists.

Choose the ***correct*** answer from the options given below:

[Question ID = 9344]

1. A and B only

   [Option ID = 37373]
2. A and C only

   [Option ID = 37374]
3. C and D only

   [Option ID = 37375]
4. B and D only

   [Option ID = 37376]

38) A two dimensional motion of a liquid has complex potential $\omega = U\left(z + \frac{1}{z}\right) + i\log z$, where U is real and positive and $i^2 = -1$. Then

A. the velocity at infinity is U in the positive sense of the real axis.

B. the unit circle $|z| = 1$ is a streamline.

C. there are in general two stagnation points.

Choose the ***correct*** answer from the options given below:

[Question ID = 9345]

1. A only

   [Option ID = 37377]
2. B and C only

   [Option ID = 37378]
3. A and B only

   [Option ID = 37379]
4. A and C only

   [Option ID = 37380]

39) A uniform flow with velocity $-Ui$ (where i is the unit vector in the positive x-direction) past a stationary cylinder $|z| = a$ has

A. complex potential $Uz + \frac{a^2}{z}, |z| \geq a$

B. velocity potential $U(r + \frac{a^2}{r}) \cos\theta$

C. stream fuinction $U(r - \frac{a^2}{r}) \sin\theta$.

where $z = re^{i\theta}$

Choose the ***correct*** answer from the options given below:

[Question ID = 9346]

1. A and B only

   [Option ID = 37381]
2. B and C only

   [Option ID = 37382]
3. A and C only

   [Option ID = 37383]
4. A only

   [Option ID = 37384]

40) Define $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by $d(x, y) = |x^2 - y^2|$, where $x, y \in \mathbb{R}$. Then, d is a

Statement I: metric on $\mathbb{R}$

Statement II: pseudometric on $\mathbb{R}$

In light of the above statements, choose the **correct** answer from the options given below

[Question ID = 9347]

1. Both Statement I and Statement II are true

   [Option ID = 37385]

2. Both Statement I and Statement II are false

   [Option ID = 37386]
3. Statement I is true but Statement II is false

   [Option ID = 37387]
4. Statement I is false but Statement II is true

   [Option ID = 37388]

41) Consider the following:

A. $I = [0, 1]$ is not a compact subset of $\mathbb{R}$ with discrete metric.

B. The intersection of a countable collection of dense open subsets of $\mathbb{R}$ with usual metric is dense in $\mathbb{R}$.

C. $\mathbb{R}$ with discrete metric is not a complete metric space.

D. Every finite subset of $\mathbb{R}$ with usual metric is a nowhere dense subset of $\mathbb{R}$.

Choose the **correct** answer from the options given below:

[Question ID = 9348]
1. A, B and C only

   [Option ID = 37389]
2. B, C and D only

   [Option ID = 37390]
3. A, B and D only

   [Option ID = 37391]
4. A, C and D only

   [Option ID = 37392]

42) Consider the following:

A. Intersection of two connected sets is connected.

B. If X is a connected metric space and $f: X \to \mathbb{R}$ is a continuous map such that $|f(x)| = 1, \forall x \in X$ then f is a constant function.

C. Interior of a connected subset is connected.

D. If constant functions are the only continuous functions from a metric space X into the usual discrete space $\mathbb{Z}$ of integers then X is connected.

Choose the **correct** answer from the options given below:

[Question ID = 9349]
1. A and B only

   [Option ID = 37393]
2. C and D only

   [Option ID = 37394]
3. A and C only

   [Option ID = 37395]
4. B and D only

   [Option ID = 37396]

43) Which of the following is correct?

[Question ID = 9350]
1. If K_i 's are nonempty compact subsets of $\mathbb{R}^n$ with $K_{i+1} \subseteq K_i$ then $\cap_{i=1}^{\infty} K_i \neq \emptyset$

   [Option ID = 37397]
2. A nonempty subset of the usual metric space $\mathbb{R}$ of real numbers which has largest and smallest elements is compact

   [Option ID = 37398]
3. If E_n 's are nonemtpy closed subsets of the usual metric space $\mathbb{R}$ with $E_{n+1} \subseteq E_n$ then $\cap_{n=1}^{\infty} E_n \neq \emptyset$

   [Option ID = 37399]
4. If F_n's are nonempty bounded subsets of the usual metric space $\mathbb{R}$ with $F_{n+1} \subseteq F_n$ then $\cap_{n=1}^{\infty} F_n \neq \emptyset$

[Option ID = 37400]

44) Given below are two statements

Statement I: If $\mathbb{R}$ is endowed with the lower limit topology then the closure of the set $K = \left\{\frac{1}{n} | n \in \mathbb{N}\right\}$ is $K \cup \{0\}$.

Statement II: If $\mathbb{R}$ is endowed with the finite complement topology then the closure of the set $K = \left\{\frac{1}{n} | n \in \mathbb{N}\right\}$ is $\mathbb{R}$.

In light of the above statements, choose the **correct** answer from the options given below:

[Question ID = 9351]

1. Both Statement I and Statement II are true

   [Option ID = 37401]
2. Both Statement I and Statement II are false

   [Option ID = 37402]
3. Statement I is true but Statement II is false

   [Option ID = 37403]
4. Statement I is false but Statement II is true

   [Option ID = 37404]

45) Given below are two statements

Statement I: The function $f: [0, 2\pi] \to \mathbb{S}^1$ defined by $f(t) = e^{it}$ is continuous and onto, where $\mathbb{S}^1$ is the usual unit circle.

Statement II: The function $f: [0, 2\pi] \to \mathbb{S}^1$ defined by $f(t) = e^{it}$ is open and onto, where $\mathbb{S}^1$ is the usual unit circle.

In light of the above statements, choose the ***correct*** answer from the options given below

[Question ID = 9352]

1. Both Statement I and Statement II are true

   [Option ID = 37405]
2. Both Statement I and Statement II are false

   [Option ID = 37406]
3. Statement I is true but Statement II is false

   [Option ID = 37407]
4. Statement I is false but Statement II is true

   [Option ID = 37408]

46) Given below are two statements

Statement I: Let $X_\lambda, \lambda \in \mathbb{A}$ be a family of topological spaces, where $\mathbb{A}$ is a nonempty index set. Then a function f from a space Y into the product space (under product topology) ΠX_λ is continuous if and only if the composition $P_\lambda of$ is continuous for every $\lambda \in \mathbb{A}$, where P_λ is the λ^{th} projection map.

Statement II: Let $X_\lambda, \lambda \in \mathbb{A}$ be a family of topological spaces, where $\mathbb{A}$ is a nonempty index set and let $E_\lambda \subseteq X_\lambda, \forall \lambda \in \mathbb{A}$. Then ΠE_λ is dense in the product space (under product topology) ΠX_λ if and only if each E_λ is dense in X_λ.

In light of the above statements, choose the **correct** answer from the options given below

[Question ID = 9353]

1. Both Statement I and Statement II are true

   [Option ID = 37409]
2. Both Statement I and Statement II are false

   [Option ID = 37410]
3. Statement I is true but Statement II is false

   [Option ID = 37411]
4. Statement I is false but Statement II is true

   [Option ID = 37412]

47)

Let $(X_1, \|.\|_1)$ and $(X_2, \|.\|_2)$ be normed spaces and $X = X_1 \times X_2$ be the product vector space. Which of the following does not define a norm on X, where $x = (x_1, x_2), x_1 \in X_1, x_2 \in X_2$?

[Question ID = 9354]

1. $\|x\| = max(\|x_1\|_1, \|x_2\|_2)$

   [Option ID = 37413]
2. $\|x\| = (\|x_1\|_1^4 + \|x_2\|_2^4)^{1/4}$

   [Option ID = 37414]
3. $\|x\| = min(\|x_1\|_1, \|x_2\|_2)$

   [Option ID = 37415]
4. $\|x\| = 3\|x_1\|_1 + 5\|x_2\|_2$

   [Option ID = 37416]

48)

Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : If x and y are two distinct elements of a normed space X, then there exists a bounded linear functional g on X such that $g(x) \neq g(y)$.

Reason R : A bounded linear functional defined on a subspace M of a normed space X has at least one extension, with the same norm, to the whole of X.

In light of the above statements, choose the ***correct*** answer from the options given below

[Question ID = 9355]

1. Both A and R are true and R is the correct explanation of A

   [Option ID = 37417]
2. Both A and R are true but R is NOT the correct explanation of A

   [Option ID = 37418]
3. A is true but R is false

   [Option ID = 37419]
4. A is false but R is true

   [Option ID = 37420]

49)

Let $X = \{x = (x_j) : x_j \in \mathbb{C} \; \forall \; j \; and \; \exists \; N \in \mathbb{N} \; such \; that \;\; x_j = 0 \; \forall \; j \geq N\}$ be a normed space with norm given by $\|x\|_\infty = Sup\{|x_j| : j \in \mathbb{N}\}$, where $x = (x_j)_{j=1}^{\infty} \in X$. Let $T : (X, \|.\|_\infty) \to \mathbb{C}$ be given by $T(x) = \sum_{k=1}^{\infty} \frac{x_k}{k^3}, x = (x_j)_{j=1}^{\infty}$.

Choose the correct option:

[Question ID = 9356]

1. $(X, \|.\|_\infty)$ is a Banach space.

   [Option ID = 37421]
2. T is a bounded linear functional on $(X, \|.\|_\infty)$.

   [Option ID = 37422]
3. There exists a unique $y = (y_1, y_2, \dots.) \in X$ such that $T(x) = \sum_{i=1}^{\infty} x_i \overline{y_i}$ for all $x \in X, x = (x_1, x_2, \dots.)$.

   [Option ID = 37423]
4. There exists a $y \in X$ such that T is not continuous at y

   [Option ID = 37424]

50)

Let H be a Hibert space and P be an orthogonal projection on H such that $P \neq 0, P \neq I$ and $P(H) = Y$ is a subset of H. Then

A. $P^2 = I$.

B. $Y^{\perp\perp} = Y$.

C. The restriction of P to Y is the identity operator on Y.

D. The null space of P is $Y^{\perp}$.

Choose the ***correct*** answer from the options given below:

[Question ID = 9357]

1. A and B only

   [Option ID = 37425]
2. B, C and D only

   [Option ID = 37426]
3. B and D only

   [Option ID = 37427]
4. B only

   [Option ID = 37428]