# **BTech Mathematics and Computing (MnC) Curriculum - 2019 Intake Onward**

# **##** Department Core Curriculum

## Semester 1

Course code	L-T-P-C	Course Name
MC100	1-0-0-1	Introduction to Profession

**Total Credits 23** 

Refer to Institute Core Course List for semester-1 for the remaining courses in semester-1.

# Semester 2

Course code	L-T-P-C	Course Name
CS102	2-0-2-3	Software Tools

Total Credits 21

Refer to Institute Core Course List for semester-2 for the remaining courses in semester-2.

## Semester 3

Course code	L-T-P-C	Course Name
MTH211	3-1-0-4	Real Analysis
MTH221	3-1-0-4	Discrete Mathematics
MTH222	3-1-0-4	Linear Algebra and Applications
CS230	3-0-0-3	Probability and Statistics for Computer Science
CS220	3-0-2-4	Data Structures and Algorithms

**Total Credits 19** 

Course code	L-T-P-C	Course Name
MTH212	3-0-3-4	Multivariate Calculus
MTH223	3-0-3-4	Algebra
MTH213	3-1-0-4	Numerical Analysis
CS222	3-0-3-4	Algorithm Design
	3-0-0-3	Open Elective

Total Credits 19

# Semester 5

Course code	L-T-P-C	Course Name
MTH332	3-0-0-3	Stochastic Processes
MTH3141	3-1-0-2	Differential Equations I
MTH3142	3-1-0-2	Differential Equations II
MTH3151	3-1-0-2	Complex Analysis I
MTH3152	3-1-0-2	Complex Analysis II
	3-0-0-3	Open Elective
	3-0-0-3	Open Elective

Total Credits 17

# Semester 6

Course code	L-T-P-C	Course Name
MTH316	3-0-0-3	Measure and Probability
MTH317	3-0-0-3	Topology
CS331	3-0-3-4	Machine Learning
CS321	3-0-0-3	Theory of Computation
	3-0-0-3	Program Elective

Course code	L-T-P-C	Course Name
	3-0-0-3	Open Elective

**Total Credits 19** 

# Semester 7

Course code	L-T-P-C	Course Name
	12 Credits	Internship
OR		
	3-0-0-3	Program Elective
	3-0-0-3	Program Elective
	3-0-0-3	Open Elective
	3-0-0-3	Open Elective
OR		
	6 Credits	BTP
	3-0-0-3	Open Elective
	3-0-0-3	Open Elective

Total Credits 12

# Semester 8

Course code	L-T-P-C	Course Name
	3-0-0-3	Program Elective
	3-0-0-3	Program Elective
	3-0-0-3	Open Elective
	3-0-0-3	Open Elective
OR		

Course code	L-T-P-C	Course Name
	6 Credits	BTP
	3-0-0-3	Open Elective
	3-0-0-3	Open Elective

Total Credits 12 PE - Program Elective

**Program Total Credits 142** 

BTech project (BTP) and external internship

Of the 24 credits in Semester 7 & 8, 12 can be acquired via a BTech Project spanning both the semesters (6+6 credits) or a semester long internship in Semester 7 (12 credits).

# Courses

#### Course numbering

A course with numbering MTHXYZ is a course in the year X of the 4-year degree program.

## **Course Descriptions**

## MC100

Introduction to Profession 1-0-0-1

## Content

• The spirit of the course is to provide a window into the future of the students who have opted for B.Tech in Mathematics and Computing. Every week, the class will be given a seminar, as a general talk by experts, who will be the faculty of the Department, as well as some invited speakers. Among topics and themes we will have lectures in Mathematics, Applications of Mathematics, Computing and Computer Science, Machine Learning, Data Science etc.

## CS102

#### Software Tools 2-0-2-3

*Objective:* The course familiarises the student with the standard tools on a GNU/Linux system and enables one to perform tasks like creating and managing sourcecodes, documents, webpages, slides, images etc.

#### Prerequisite: None

## Contents:

- Basic GNU/Linux commands
- Shell scripting, awk, sed
- Web development tools, HTML, CSS, Javascript, PHP
- Basics of typography, typesetting using TeX, LaTeX, Beamer
- Raster and vector image editing, basics of color theory

Suggested Textbooks:

None

Reference Texts:

- 1. The Unix Programming Environment, B. Kernighan, R. Pike, 2015
- 2. The Not So Short Introduction to LATEX2e

## MTH211

## Real Analysis 3-1-0-4

Contents:

- Review: The real number system; Archimedian property, Completeness, Convergence of sequences and series, limits. Continuity, Uniform Continuity.
- Metric Spaces. Introduce distance, define metric. Metric spaces, Examples in  $\mathbb{R}^N$ ,  $l^2$ ,  $l^p$  (Holder and Minkowski inequalities), C[a, b] (Uniform Convergence). Open sets, closed sets, and examples of these in

different metrics.cantor set. Complete Metric Spaces, completeness of C[a, b]. Compactness, with many examples. Finite Intersection property. Compact subsets in  $\mathbb{R}^n$ . Brouwer's Fixed point Theorem in  $\mathbb{R}^2$  .(Applications). Connectedness, IVP, path connectedness. Continuous functions on connected sets.

- Differentiation. Derivatives of functions, Taylor's theorem. Monotonic functions, Functions of bounded variation; Absolutely continuous functions. Riemann Integration. Properties of Riemann integral, characterization of Riemann integrable functions. Improper integrals. Pointwise convergence, uniform convergence of functions, relation with convergence of functions in the mean, differentiation, integration; Examples.
- Polynomial Approximations Power series, Taylor series. Weierstrass Approximation Theorem, Bernstein Polynomials. Fourier Series, computation of Fourier coefficients; smoothness and decay. Different kinds of convergence. Fejer's Theorem (averaging). Another proof of Weierstrass Theorem. Rates of convergence and comparison. (Use MATLAB...)

- 1. T. Apostol, Mathematical Analysis.
- 2. W. Rudin, Principles of Mathematical Analysis.
- 3. Terence Tao, Analysis I and II. Trim Series.
- 4. R. R. Goldberg: Methods of Real Analysis.
- 5. N.L. Carothers, Real Analysis.

6. Kenneth R. Davidson and Allan P. Donsig, Real Analysis and Applications- Theory and Practice.

## MTH221

## **Discrete Mathematics 3-1-0-4**

## Contents:

- Sets and Relations. Sets, examples and non-examples, set building operations, various identities and how to prove them, Venn diagram, size of sets, finite and infinite sets, countable sets, countability of rationals, algebraic numbers, Cantor's diagonalisation and uncountability of reals, irrationals, continuum. Schroeder-Bernstein Theorem, Relations, equivalence relations and partitions, order relations, partial orders, Dilworth's theorem.
- Combinatorics: Basic counting techniques, ordered and unordered sets, permutations and combinations, binomial and multinomial theorem, binomial identities, Kruskal–Katona theorem, principle of inclusion and exclusion, Bell numbers, Stirling numbers of first and second kind (twelve fold theory), pigeonhole principle, recurrence relations, Fibonacci numbers, generating functions and their use in solving recurrence relations, Burnside's Lemma, Polya's counting theorem, Ramsey's Theorem
- Graph Theory: Simple graphs, Adjacency Matrices, Example of graphs: Cycles, Trees, Moore graphs, Strongly regular graphs, basic properties of graphs, connectivity, Paths, Cycles, Eulerian walks, Hamiltonian cycles, Euler's theorem, Cliques and Co-cliques (Independent set), Colourings, Chromatic index, various bounds, Chromatic polynomial, Graph matching, Hall's theorem, Planarity, Euler's formula, Kuratowski's theorem, Statement of 4 colour theorem.

#### Suggested Textbooks:

- 1. C.L. Liu. Elements of Discrete Mathematics.
- 2. Rosen, Discrete Mathematics for Computer Science
- 3. D B West, Introduction to graph theory, Prentice Hall Inc.
- 4. J.H van Lint and R.M Wilson, A course in Combinatorics, Cambridge University Press

## CS230

#### Probability and Statistics for Computer Science 3-0-0-3

*Objective*: The course gives an elementary-level introduction to probability and statistics for engineers and scientists. Along with theory and methods, this course focuses on applications in real-life using statistical computing and graphics tools, e.g., R programming language.

#### Prerequisite: None

#### Contents:

- Data visualisation tools and techniques
- Discrete experiments, Probability space, Equally-likely outcomes and combinatorial problems, Nonequally likely outcomes

- Conditional probability, Bayes formula, Independent events
- Random variables, Binomial and Poisson distributions, Expectation, Variance, Linearity of expectation
- Markov and Chebyshev inequalities, simple applications
- Joint distributions, joint densities, correlation
- Statistics, sampling, central limit theorem, hypothesis testing

## Suggested Textbooks:

1. Introduction to Probability and Statistics for Engineers and Scientists, S. Ross, 2007

## **Reference Texts:**

1. An Introduction to probability theory and its applications. (Vols. 1, 2), W Feller, 3/e

## **MTH222**

## Linear Algebra And Applications 3-1-0-4

## Contents:

- Notion of a field. Vector spaces over fields, subspaces, bases and dimension.
- Linear transformations. Representation of linear transformations by matrices, effect of change-ofbasis, rank-nullity theorem, Gaussian elimination revisited with application to determination of rank, bases for row-space, column-space of a matrix, and solution space of a corresponding system of homogeneous linear equations. Applications to graphs and networks. Elementary matrices, LU decomposition. Equivalence of matrices.
- Eigenvalues and eigenvectors, characteristic polynomials, minimal polynomials, Cayley- Hamilton Theorem, triangulation, diagonalization, matrix exponentials, rational canonical form, Jordan canonical form.
- Positive definite matrices, minorant characterization, Singular value decomposition. Finite element method. Bilinear forms, symmetric and skew-symmetric bilinear forms, real quadratic forms, Sylvester's law of inertia.
- Applications: Page-rank algorithm. Linear programming, Network models, game theory

- 1. K. Hoffman and R. Kunze, Linear Algebra.
- 2. G. Strang, Linear Algebra and its Applications.
- 3. D. C. Lay, Linear Algebra and its Applications.
- 4. S. Lang, Linear Algebra.
- 5. P. Lax, Linear Algebra and its Applications.
- 6. A. Ramachandra Rao and P Bhimasankaram, Linear Algebra, TRIM.

## Multivariate Calculus 3-1-0-4

## Contents:

- Cartesian and Polar coordinate systems for  $\mathbb{R}^n$  , Volume element.
- Functions of several variables, Continuity. Several examples.
- Differentiation. Partial derivatives and the Tangent space. The Chain Rule. Inverse function Theorem and Implicit function Theorem.
- Higher Derivatives. Extrema of Functions in several variables.
- Applications. Planetary Motion.
- Review of Riemann Integration on  $\mathbb{R}^n$ . Iterated Integrals, change of variables, Jacobian.
- Line integrals, Surface Integrals. Green's Theorem, Divergence Theorem, Stokes' Theorem.
- Applications; Conservation Laws.

## Suggested Textbooks:

- 1. P.D. Lax and M.S. Terrell. Multivariate Calculus with applications. 2017. (Course Textbook)
- 2. M. Spivak. Calculus on Manifolds.
- 3. J.R. Munkres. Manifolds.

## MTH213

## Numerical Analysis 3-1-0-4

## Contents:

- System of linear equations: matrix norms, Gaussian Elimination, LU decomposition, QR decomposition, Gauss Jacobi and Gauss Seidel methods with convergence analysis, condition number, Gershgorin theorem for locating eigenvalues, power method to approximate the eigenvalues.
- Nonlinear equations/systems: Bisection method, Regula Falsi, Secant method, Newton's method, fixed point iteration and order of convergence.
- Interpolation/Approximation: Polynomial interpolation, Hermite interpolation, spline interpolation, error analysis.
- Numerical Integration: Trapezoidal and Simpsons rules, Gaussian quadrature formulae and error analysis.
- Numerical Differentiation: Forward, backward and central difference approximations, single and multistep methods for initial value problems. Numerical solutions to PDE using finite difference method.

- 1. S. D. Conte and Carl de Boor, Elementary Numerical Analysis- An Algorithmic Approach.
- 2. K. E. Atkinson, Introduction to Numerical Analysis.
- 3. Steven C. Chapra, Applied Numerical Methods with MATLAB for Engineers and Scientists.
- 4. D. Kincaid and W. Cheney, Numerical Analysis: Mathematics of Scientific Computing. (2002).
- 5. Richard L. Burden and J. Douglas Faires, Numerical Analysis, (2010).

## Algebra 3-1-0-4

Contents:

- Equivalence relation and partitions, principle of induction, binary operation, groups, basic examples of groups: integers, rationals, real numbers, complex numbers, vector spaces, modular arithmetic, symmetric groups, matrix groups, dihedral groups, quaternion groups.
- Subgroups, Lagrange's Theorem, homomorphisms, kernel, cokernel, normal subgroups, quotient groups. Order of an element, generators and relations, cyclic groups, automorphism of a group, Group actions, Sylow Theorems, Direct products and direct sum, structure of finite abelian group, statement of the structure theorem of finitely generated abelian groups, simple groups. (If time permits: Semi direct product, groups of small order.)
- Rings, polynomial rings, formal power series rings, matrix rings, group rings, Gausian integers, ideals and quotient rings, prime ideal, integral domain, Zorn's Lemma, maximal ideals, Chinese remainder theorem, Gauss lemma.
  - (If time permits: ED, UFD, PID)
- Basic definitions and examples of fields, field extensions, characteristic of a field, finite field.

## Suggested Textbooks:

- 1. David S Dummit, Richard M. Foote, Abstract Algebra, John Wiley and sons (main reference)
- 2. Michael Artin, Algebra, Prentice Hall
- 3. I. N. Herstein, Topics in Algebra
- 4. N. Jacobson, Basic Algebra Vol. I, W.H. Freeman and Co.
- 5. S. Lang, Undergraduate Algebra.
- 6. N S Gopalakrishnan, University Algebra, Wiley Eastern.

## CS222

## Algorithm Design 3-0-3-4

*Objective:* The course aims to train students in a few standard approaches to the design and analysis of algorithms for a variety of problems arising in computer science and its applications. It also equips students with a suite of programs/algorithms which may be adapted to various situations.

## Prerequisite: CS220 Data Structures and Algorithms

## Contents:

- Asymptotic analysis of algorithms
- Basic bit-wise computations and their analysis
- Number theoretic algorithms such as the GCD and modulo computations
- Introduction to graph algorithms
- Algorithm design strategies such as Greedy, Divide and Conquer, and Dynamic programming
- Examples of modelling of real-life problems from application areas.

- 1. Algorithms, S. Dasgupta, C. H. Papadimitriou, U. V. Vazirani, 2017.
- 2. Algorithm Design, J. Kleinberg, E. Tardos, 2013.

## Reference Texts:

1. Introduction to Algorithms, T. H. Cormen, C. E. Leiserson, R. Rivest, C. Stein, 2010.

## CS220

#### Data Structures and Algorithms 3-0-2-4

*Objective:* Given an algorithm using abstract data types the student should be able to implement it efficiently in a programming language like C. Also given a program the student should be able to find its time and space complexity.

## Prerequisite: None

- C language basics, programming with pointers
- Time and space complexity of algorithms
- Basic data structures, lists, stacks, queues, binary search trees, heaps, hashing, trees, tree traversals, graphs, self balancing trees
- Graph search, BFS, DFS, minimum spanning tree algorithms, shortest path algorithms.

## Suggested Textbooks:

- 1. The C Programming Language, B. W. Kernighan, D. Ritchie, 2015.
- 2. Data Structures & Algorithms, A. Aho, 2002

#### Reference Texts:

1. Introduction to Algorithms, T. H. Cormen, C. E. Leiserson, R. Rivest, C. Stein, 2010.

## **MTH332**

#### Stochastic Processes 3-0-0-3

## Contents:

- Simple symmetric random walk, recurrence in dimensions one/two and transience in three.
- Finite state Markov chains, examples. recurrence, transience and aperiodicity, fundamental theorem for irreducible chains; (Renewal equation), gamblers ruin, Ehrenfest urn model, simulations, card shuffling.

Infinite state chains: Random walks, Markov chains with countable state space. criterion for recurrence, Birth and Death chains.

- Poisson processes and various ways of looking (axiomatic definition; construction using exponential variables; via differential equations), Com- pound Poisson Process, M/M/1 Queue.
- Brownian motion; brief introduction.

#### Suggested Textbooks:

1. H M Taylor, S Karlin: An introduction to stochastic modelling.

- 2. P Bremaud: Discrete Probability Models and Methods.
- 3. Olle Haggstrom; Finite Markov Chains and algorithmic applications.
- 4. P Bremaud: Markov chains: Gibbs fields, MonteCarlo simulation and Queues.
- 5. P G Hoel, S C Port, C J Stone: Introduction to Stochastic processes.
- 6. D P Bertsekas, J N Tsitsiklis: Introduction to Probability (MIT notes).
- 7. W Feller: An Introduction to probability theory and its applications Vol. 1

## Complex Analysis I 3-1-0-2

## Contents:

- Complex numbers: algebraic properties, graphical representation, Riemann Sphere, limits and continuity, Differentiability, CR equations,
- Analytic functions. Elementary functions, integration on contours, Cauchy's theorem, Cauchy's integral formula and its applications, Morera's theorem. Series of complex numbers, Taylor's theorem, Sequence of analytic functions, Schwarz reflection principle, Runge's approximation theorem, Identity theorem, Maximum modulus principle, Laurent series, Cauchy's Residue theorem.

## MTH3152

## Complex Analysis II 3-1-0-2

#### Contents:

- Classification of isolated singularities, Riemann's theorem on removable singularities, Casorati-Weierstrass theorem.
- Meromorphic functions, Argument principle, winding number, Rouche's theorem and its applications, Open mapping theorem.
- Evaluation of certain improper integrals, Conformal mapping, Mobius transformation, Schwarz Lemma, Picks lemma, Weierstrass theorem for infinite products.
- Harmonic functions, Poisson integral formula, Mean value Property, Dirichlet problem.

- 1. Churchill, Brown, Complex variables and Applications. (2009)
- 2. Stein and Shakarchi, Complex analysis. (2013).
- 3. Ahlfors, Complex Analysis.
- 4. Serge Lang, Complex Analysis.
- 5. Conway, Functions of One Complex Variable.
- 6. W. Rudin, Real and complex analysis

## Differential Equations I 3-1-0-2

## Contents:

• System of ordinary differential equations, Local existence and uniqueness of solutions, Global existence, Stability theory, First order PDEs- Linear and quasi linear PDE's, Methods of characteristics, Cauchy problems.

## MTH3142

## **Differential Equations II 3-1-0-2**

## Contents:

• Sturm-Liouville problems, Power series solution, Second-order PDEs, and their classification. Laplace, Heat and Wave equations- Fourier series, Method of separation of variables

## Suggested Textbooks:

- 1. Vladimir I. Arnold, Ordinary Differential Equations.
  - 2, Earl A. Coddington, Norman Levinson, Theory of Ordinary Differential Equations.
- 2. Lawrence Perko, Differential Equations and Dynamical Systems. 3rd edition, 2000.
- 3. William E. Boyce, Richard C. DiPrima, 2009. Elementary Differential Equations and Boundary Value Problems
- 4. Walter A. Strauss, Partial Differential Equations: An Introduction.
- 5. Mark A. Pinsky, Partial Differential Equations and Boundary value Problems with Applications.
- 6. Sandro Salsa. Partial Differential Equations in Action: From Modelling to Theory, 2015.
- 7. Lawrence C. Evans Partial Differential Equations , 2nd edition, 2010.

## MTH316

## Measure and Probability 3-0-0-3

## Contents:

- Lebesgue Measure on  $\mathbb{R}^d$ : Outer Measure and its properties, Measurable sets, Lebesgue measure,  $\sigma$ -algebra of Borel sets.
- Probability space, Events, Conditioning, independence.
- Measurable functions, random variables, probability distribution, Independence of random variables.
- Littlewood's three Principles.
- The Lebesgue Integral, Convergence Theorems, Fubini's Theorem. Integration with respect to probability distributions, Expectation of a random variable, Characteristic function.
- $L^p$  Spaces, Holder and Minkowski Inequalities. Completeness of  $L^p$  Spaces.
- The Hilbert space  $L^2$ ; Orthogonal Projections; Moments, Conditional Expectation and Independence.

## Suggested Textbooks:

1. M. Capinsky and E. Kopp. Measure Integral and Probability. Springer UG Texts. Second Edition.

- 1. E. Stein and R. Shakarchi. Real Analysis.
- 2. E. Stein and R. Shakarchi. Functional Analysis.

## Topology 3-0-0-3

## Contents:

- Topological Motivation from Metric Spaces, surfaces etc. Topological spaces; Definition, Diverse Examples. Open sets, closed sets, closure, interior, boundary. Continuous maps. Existence of continuous functions. For metric spaces there are 'sufficiently many'. Convergence of sequences of functions.
- Subspace Topology, Product Topology, and Identification Topology; Examples of identification Spaces.
- Compactness and Connectedness. Compact sets and finite sets. Compactness and Continuity. Tychonoff 's Theorem (without proof ). Connectedness, path connectedness. Examples.
- Homotopy of paths. The Fundamental group. Some Constructions. Homotopy type. Covering Spaces. Triangulations, Barycentric Subdivision, Simplicial Complexes. Simplicial Approximation Theorem.

## Suggested Textbooks:

- 1. G. Simmons. Introduction to Topology and ModernAnalysis.
- 2. James R. Munkres. Topology.
- 3. M.A. Armstrong. Basic Topology.
- 4. Singer and Thorpe. Lecture Notes on Elementary Topology and Geometry.
- 5. <u>https://people.maths.ox.ac.uk/ritter/masterclasses/ritterlectures-on-geomery-and-topology.pdf</u>

## CS321

## Theory of Computation 3-0-0-3

*Objective:* The student should be able to appreciate the fundamental models of computation for regular, context-free, recursive and recursively enumerable languages. The student should be able tell apart computable problems from those that are not.

## Prerequisite: CS221 Discrete Structures

#### Contents:

- Notion of a formal language, regular languages, finite state automata, DFA, NFA, regular expressions, rquivalence of all the notions, Myhill-Nerode theorem, pumping lemma, closure and decision properties of regular languages, equivalence and minimization of DFA
- Notion of a grammar, context free grammars and languages, derivation and parsing, PDAs, PDAs and CFGs capture same language class, Chomsky normal form CFGs, pumping lemma, closure and decision properties of CFLs.
- Turing Machine, historical motivation, robustness of the TM, universal Turing machine, recursive and

r.e. languages, separation of the two classes, undecidable problems, Rice's theorem.

## Suggested Textbooks:

- 1. Introduction to Automata Theory, Languages, and Computation, J. E. Hopcroft, R. Motwani, J. D. Ullman, 2008
- 2. Introduction to the Theory of Computation, M. Sipser, 2014

## **Reference Texts:**

1. Automata and Computability, D. Kozen, 2007

## CS331

## Machine Learning 3-0-3-4

*Objective*: To equip students to apply machine learning methods to real-world applications such as recommender systems, computer vision, bioinformatics, and text mining.

Prerequisite: CS230 Probability and Statistics for CS

## Contents:

- Data science basics, how to wrangle, visualize, and analyze data, using models to explore your data
- Supervised learning, linear and logistic regression, generative learning, maximum likelihood estimation (MLE), maximum a posteriori (MAP) estimation, support vector machines, artificial neural networks and deep learning.
- Unsupervised learning, mixture of Gaussians, EM algorithm, autoencoders
- Bias-variance tradeoff, regularization and model selection
- Dimensionality reduction, principal component analysis, singular value decomposition

## Suggested Textbooks:

1. Machine Learning, T. Mitchell, 2009.

#### **Reference Texts:**

- 1. Elements of Statistical Learning, T. Hastie, R. Tibshirani, and J. Friedman, 2017.
- 2. Pattern Recognition and Machine Learning, C. Bishop, 2010.
- 3. Foundations of Data Science, M. Blum, J. Hopcroft, and R. Kannan, 2018.
- 4. R for Data Science: Import, Tidy, Transform, Visualize, and Model Data, H. Wickham and G. Grolemund, 2016.