MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Consider a function $f(x)=\dfrac{2x^2+x+1}{x^2+1}$, which of the following options is correct?

(1) $f(x)$ is one-one for $x\in(0, 2)$

(2) $f(x)$ is many-one for $x\in(0, 2)$

(3) $f(x)$ is one-one for $x\in(0, \infty)$

(4) $f(x)$ is one-one for $x\in(1, \infty)$

Answer (1)

Sol. $f(x)=2+\dfrac{x-1}{x^2+1}$

$$f'(x)=\frac{-x^2+2x+1}{\left(x^2+1\right)^2}$$

$Q(x) = -x^2 + 2x + 1$ is having positive sign in interval (0, 2) so function is one-one

2. If real part of the product of z_1 and z_2 is zero, *i.e.* $\text{Re}(z_1z_2) = 0$ and $\text{Re}(z_1 + z_2) = 0$, then $\text{Im}(z_1)$ and $\text{Im}(z_2)$ is

(1) > 0, > 0 (2) < 0, < 0

(3) = 0, = 0 (4) > 0, < 0

Answer (4)

Sol. Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

$\text{Re}(z_1z_2) = 0$

$\Rightarrow a_1a_2 = b_1b_2$ ...(i)

$\text{Re}(z_1 + z_2) = 0 \Rightarrow a_2 = -a_1$ ...(ii)

From (i) and (ii), $b_1b_2=-a_1^2<0$

$\therefore$ $\text{Re}(z_1)$ and $\text{Re}(z_2)$ are of opposite sign.

3. Consider $y = f(x)$ passing through (1, 1) satisfying the following differential equation.

$y(x + 1)\,dx + x^2\,dy = 0$, then $y = f(x)$ is given by

(1) $\ln xy=\dfrac{1}{x}-1$ (2) $\ln xy=\dfrac{1}{x}$

(3) $\ln xy=\dfrac{1}{x}+1$ (4) $\ln xy=\dfrac{1}{x^2}$

Answer (1)

Sol. $y(x + 1)\,dx + x^2\,dy = 0$

$$\int\frac{x+1}{x^2}dx=\int-\frac{1}{y}dy$$

$$\ln x-\frac{1}{x}=-\ln y+c$$

$\because$ It passes (1, 1)

So, $\boxed{-1=c}$

$\therefore$ $\ln x-\dfrac{1}{x}=-\ln y-1$

$\Rightarrow$ $\boxed{\ln xy=\dfrac{1}{x}-1}$

4. If $[A]$ is 3 × 3 matrix and $A^2 = 3A + aI$, $A^4 = 21A + bI$, then $a + b$ is

(1) –9 (2) –10

(3) 9 (4) 10

Answer (1)

Sol. $A^4=A^2\cdot A^2$

$=(3A+aI)(3A+aI)$

$=9A^2+6aA+a^2I=21A+bI$

Again using $A^2=3A+aI$ in LHS

$\Rightarrow$ $9(3A+aI)+6aA+a^2I=21A+bI$

$\Rightarrow$ $(27+6a)A+\left(9a+a^2\right)I=21A+bI$

$\therefore$ $27+6a=21$ and $9a+a^2=b$

$\therefore$ $a = -1$, $b = -8$

$\therefore$ $a + b = -9$

5. Find area common to following region $x^2 + y^2 \le 21$, $x \ge 1$ & $y^2 \le 4x$

(1) $8\sqrt{3}-\frac{8}{3}+\frac{21}{2}-\frac{21}{2}\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

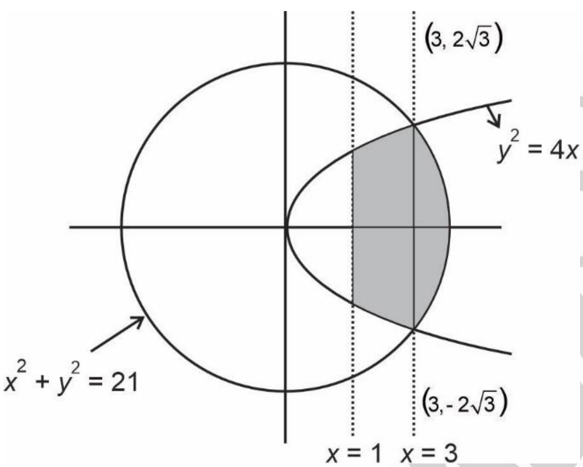
(2) $2\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}-21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

(3) $8\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}$

(4) $8\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}-21\sin^{-1}\left(\sqrt{\frac{3}{11}}\right)$

Answer (2)

Sol. Area of Required Region



$$= 2\left(\int_1^3 2\sqrt{x}\,dx + \int_3^{\sqrt{21}} \sqrt{21-x^2}\,dx\right)$$

$$= 2\left(2\left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_1^3 + \left.\frac{21\sin^{-1}\left(\frac{x}{\sqrt{21}}\right)+x\sqrt{21-x^2}}{2}\right|_3^{\sqrt{21}}\right)$$

$$= 2\left(4\sqrt{3}-\frac{4}{3}\right)+\left(21\sin^{-1}1+0\right)$$

$$-\left(21\sin^{-1}\frac{3}{\sqrt{21}}+3\sqrt{12}\right)$$

$$= 8\sqrt{3}-\frac{8}{3}+\frac{21\pi}{2}-6\sqrt{3}-21\sin^{-1}\sqrt{\frac{3}{7}}$$

$$= 2\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}-21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$$

6. Domain of $f(x) = \dfrac{\log_x(x-1)}{\log_{(x-1)}(x-4)}$ is

(1) (0, 1)

(2) $(4, \infty)$

(3) [1, 4]

(4) $(4, \infty) - \{5\}$

Answer (4)

Sol. For domain

$x > 0$, $x - 1 > 0$, $x - 4 > 0$

And $x - 1 \ne 1$, $x \ne 1$, $x - 4 \ne 1$

$\Rightarrow$ $x \in (4, \infty) - \{5\}$

7. Consider $f(x) = \max\{x^2, 1+[x]\}$ where $[x]$ is greatest integer function. Then the value of $\int_0^2 f(x)dx$ is

(1) $\frac{4\sqrt{2}+5}{3}$ (2) $\frac{6\sqrt{2}+5}{3}$

(3) $\frac{8\sqrt{2}+5}{3}$ (4) $\frac{8\sqrt{2}+3}{5}$

Answer (1)

Sol. $f(x) = \begin{cases} 1+[x], & 0 \le x \le \sqrt{2} \\ x^2 & \sqrt{2} < x \le 2 \end{cases}$

$$\int_0^2 f(x)dx = \int_0^{\sqrt{2}} (1+[x])dx + \int_{\sqrt{2}}^2 x^2 dx$$

$$= \int_0^1 1dx + \int_1^{\sqrt{2}} 2dx + \left.\frac{x^3}{3}\right]_{\sqrt{2}}^2$$

$$= 1+2\left(\sqrt{2}-1\right)+\frac{1}{3}\left(8-2\sqrt{2}\right)$$

$$= \frac{4\sqrt{2}+5}{3}$$

8. In a football club, there are 15 players each player has a T-shirt of their own name. Find the probability that at least thirteen players pick the correct T-shirt of their own name.

(1) 107 (2) 106

(3) 108 (4) 109

Answer (2)

Sol. At least 13 players pick correct T-shirt = exactly 13 T-shirt players pick correct + exactly 14 players pick correct + exactly 15 players pick correct T-shirt.

$= {}^{15}C_2 \times 1 + 0 + 1$

$= 105 + 0 + 1$

$= 106$

9. If 3 bad and 7 good apples are mixed, then find probability of finding 4 good apples, if 4 apples are drawn simultaneously

(1) $\frac{5}{12}$ (2) $\frac{1}{6}$

(3) $\frac{7}{13}$ (4) $\frac{6}{7}$

Answer (2)

Sol. $P(E) = \frac{{}^{7}C_4}{{}^{10}C_4} = \frac{7!}{4!3!} \cdot \frac{4!6!}{10!}$

$= \frac{4 \cdot 5 \cdot 6}{8 \cdot 9 \cdot 10}$

$= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

10. If $x = 2$ is a root of $x^2 + px + q = 0$ and

$$f(x) = \begin{cases} \frac{1 - \cos(x^2 - 4px + q^2 + 8q + 16)}{(x - 2p)^2}, & x \neq 0 \\ 0, & x = 2p \end{cases}$$

then $\lim_{x \to 2p^+} f(x)$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) 0 (4) $-\frac{1}{2}$

Answer (1)

Sol. $2p + q = -4$

OR $(q + 4)^2 = 4p^2$

$$\lim_{x \to 2p^+} \frac{1 - \cos(x^2 - 4px + 4p^2)}{(x - 2p)^2}$$

$$= \lim_{x \to 2p^+} \frac{(x^2 - 4px + 4p^2)^2}{2} \cdot \frac{1}{(x - 2p)^2}$$

$$= \frac{1}{2}$$

11. Incident ray $y = \frac{x}{\sqrt{3}}$ is reflected by surface $x + y = 1$. Find the point of intersection of reflecting ray with x-axis.

(1) $\left(1 - \frac{1}{\sqrt{3}}, 0\right)$

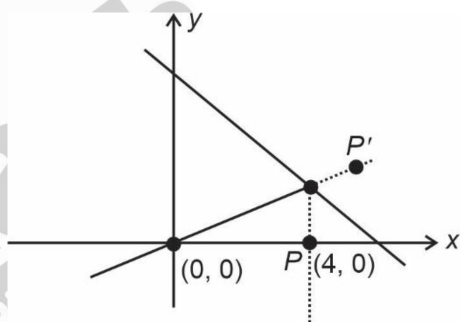
(2) $\left(1 + \frac{1}{\sqrt{3}}, 0\right)$

(3) $\left(\frac{1}{\sqrt{3}}, 0\right)$

(4) $\left(\frac{2}{\sqrt{3}}, 0\right)$

Answer (1)

Sol. Let intersection point is $P(h, 0)$ then it's image with respect to the line $x + y = 1$ will be P' which lies on $y = \frac{x}{\sqrt{3}}$.



$\therefore \frac{x - h}{1} = \frac{y}{1} = \frac{-2(h - 1)}{2}$

$\therefore x = -h + 1 + h = 1,$

$y = -h + 1$

So, $-h + 1 = \frac{1}{\sqrt{3}} \Rightarrow h = 1 - \frac{1}{\sqrt{3}}$

$\therefore P \equiv \left(1 - \frac{1}{\sqrt{3}}, 0\right)$

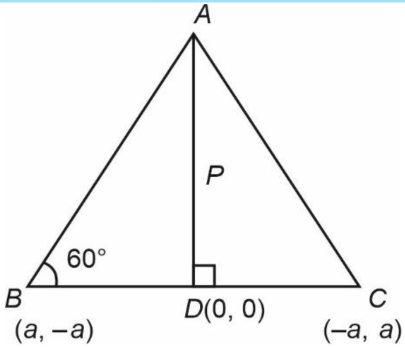
12. In an equilateral triangle ABC, point A lies on line $y - 2x = 2$ and point B and C are lying on line $y + x = 0$. Points B and C are symmetric with respect to origin. Find Area of ΔABC.

(1) $4\sqrt{3}$ sq. units (2) 8 sq. units

(3) $\frac{8}{\sqrt{3}}$ sq. units (4) $8\sqrt{3}$ sq. units

Answer (3)

Sol.



A lies on perpendicular bisector of BC

i.e., $y = x$

A is point of intersection of $y = x$ and $y – 2x = 2$

$\Rightarrow$ $A = (–2, –2)$

$P = AD = 2\sqrt{2}$

Area of $\Delta ABC = \frac{1}{2}\cdot\frac{2P}{\sqrt{3}}\cdot P$

$= \frac{P^2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$ sq. unit

13. It is given that $((p \wedge q) \vee r) \vee (p \wedge r) \rightarrow (\sim q) \vee r$ is fallacy. Then truth values of p, q and r are given by

(1) p : True, q : True, r : False

(2) p : False, q : False, r : False

(3) p : True, q : True, r : True

(4) None of these

Answer (1)

Sol. $s \rightarrow t$ is always false if s is true and t is false

$\therefore$ $(\sim q) \vee r$ is false

$\Rightarrow$ $(\sim q)$ is false and r is false

$\Rightarrow$ q is true and r is false

Also, if p is true, then $(p \wedge q) \vee r) \vee (p \wedge r)$ is true

14. Let region for $x \in [0, 1]$ given by

$A : 2x \le y \le \sqrt{4(x-1)^2}$ with y-axis

$B : y = \min\left\{2x, \sqrt{4(x-1)^2}\right\}$, with x-axis then

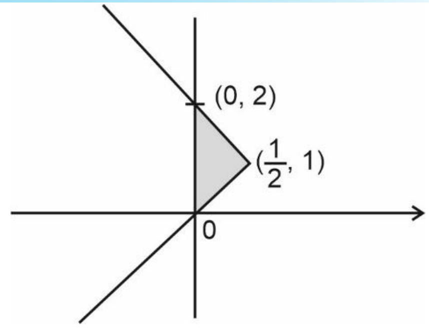
$\frac{A}{B}$ equals

(1) 1 (2) 2

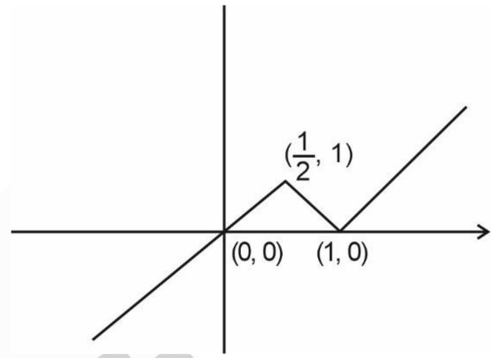
(3) 3 (4) 4

Answer (1)

Sol.



$A = \frac{1}{2}\times 2\times\frac{1}{2} = \frac{1}{2}$



$B = \frac{1}{2}\times 1\times 1 = \frac{1}{2}$

$\frac{A}{B} = 1$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A function $f(x)$ is such that $f(x + y) = f(x) + f(y) – 1$ $\forall$ $x, y \in R$, also $f'(0) = 2$, then $|f(–2)|$ is

Answer (3)

Sol. $f(x + y) = f(x) + f(y) - 1$

Put $x = y = 0$

We get $f(0) = f(0) + f(0) - 1$

$\therefore \quad f(0) = 1$

Now,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x)+f(h)-1-f(x)}{h}$$

$f'(x) = f'(0)$

$f'(x) = 2$

Integrating both side

$f(x) = 2x + c$

Now,

$\because \quad f(0) = 1$

$\Rightarrow \quad 1 = c$

$\therefore \quad f(x) = 2x + 1$

$f(-2) = -3$

$|f(-2)| = 3$

22. If $a_1, a_2, \ldots$ are positive numbers in G.P. such that $a_5 + a_7 = 12$ and $a_4 \cdot a_6 = 9$ then $a_7 + a_9$ equals

Answer (36)

Sol. Let first term of G.P. be a with common ratio r

$\therefore \quad a(r^4 + r^6) = 12 \ \& \ a^2r^8 = 9$

$\Rightarrow \quad ar^4 = 3$

$\Rightarrow \quad a_5 = 3 \ \& \ a_7 = 9$

$\therefore \quad r = \sqrt{3} \quad \Rightarrow a_9 = 27$

$\therefore \quad a_7 + a_9 = 9 + 27 = 36$

23. If $f(x + y) = f(x) + f(y)$, $f(1) = \frac{1}{5}$ and

$$\sum_{n=1}^{n} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12},$$ then the value of n is

Answer (10)

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow \quad f(x) = kx$

$f(1) = \frac{1}{5} \Rightarrow k = \frac{1}{5}$

$\therefore \quad f(x) = \frac{1}{5}x$

$$\sum_{n=1}^{n} \frac{f(n)}{n(n+1)(n+2)} = \sum_{n=1}^{n} \frac{\frac{1}{5}n}{n(n+1)(n+2)}$$

$$= \frac{1}{5}\sum_{n=1}^{n}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$= \frac{1}{5}\left(\frac{1}{2} - \frac{1}{n+2}\right)$$

$$= \frac{n}{10(n+2)} = \frac{1}{12}$$

$\Rightarrow \quad n = 10$

24. Let $S = \{1, 2, 3, 5, 7\}$. The rank of 35773 if all 5 digit number formed by the set S are arranged in a dictionary in ascending order and repetition of digits is allowed.

Answer (1748)

Sol. For rank of 35773

All number starting from 1 and 2 will come first

i.e. 1 _ _ _ _ $\rightarrow 5^4$

2 _ _ _ _ $\rightarrow 5^4$

If first digit is 3 (number of number that comes before 35773)

3 1 _ _ _ $\rightarrow 5^3$

3 2 _ _ _ $\rightarrow 5^3$

3 3 _ _ _ $\rightarrow 5^3$

35 1 _ _ $\rightarrow 5^2$

35 2 _ _ $\rightarrow 5^2$

35 3 _ _ $\rightarrow 5^2$

35 5 _ _ $\rightarrow 5^2$

357 1 _ $\rightarrow 5$

357 2 _ $\rightarrow 5$

357 3 _ $\rightarrow 5$

357 5 _ $\rightarrow 5$

3577 $\underline{1}$ $\to 1$

3577 $\underline{2}$ $\to 1$

3577 $\underline{3}$ $\to 1$

$\therefore$ Rank $= 2\left(5^4\right) + 3\left(5^3\right) + 4\left(5^2\right) + 4(5) + 3$

= 1250 + 375 + 100 + 20 + 3

= 1748

25. If coefficient of 3 consecutive terms in expansion of $(1 + 2x)^n$ is 10 : 35 : 84, then n is equal to

Answer (10.00)

Sol. $\dfrac{{}^nC_r\ 2^r}{{}^nC_{r+1}\ 2^{r+1}} = \dfrac{2}{7}$

or $\dfrac{r+1}{n-r} \cdot \dfrac{1}{2} = \dfrac{2}{7}$ ...(1)

$\dfrac{{}^nC_{r+1}\ \ 2^{r+1}}{{}^nC_{r+2}\ \ 2^{r+2}} = \dfrac{5}{12}$

or $\dfrac{r+2}{n-r-1} \cdot \dfrac{1}{2} = \dfrac{5}{12}$ ...(2)

Solving (1) and (2)

n = 10 and r = 3

26. Consider 3 coplanar vectors

$\vec{a} = 3\hat{i} - 4\hat{j} + \lambda\hat{k}$

$\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ and

$\vec{c} = \hat{i} + 3\hat{j} - 4\hat{k}$

Then 9λ is

Answer (87.00)

Sol. For co-planar vectors

$$\begin{vmatrix} 3 & -4 & \lambda \\ 4 & 3 & -1 \\ 1 & 3 & -4 \end{vmatrix} = 0$$

$3[-12 + 3] + 4[-16 + 1] + \lambda[12-3] = 0$

$\Rightarrow$ $-27 - 60 + 9\lambda = 0$

$\Rightarrow$ $9\lambda = 87$

27.

28.

29.

30.

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