

GGSIPO mathematics 2014

1. For integers $m, n, s \geq 0$ $\sum_k {}^{n-r+8}C_k {}^{n+r-s}C_{n-k} {}^{r+k}C_{m+n}$ is equal to
 - a 0
 - b ${}^nC_m {}^sC_n$
 - (c) ${}^rC_m {}^sC_n$
 - d ${}^sC_n {}^mC_r$

2. $\lim_{x \rightarrow \infty} \sin x$ is equal to
 - a 0
 - b ∞
 - c exists is finite and non-zero
 - d Does not exist

3. If $x = a+b, y = a\omega + b\omega^2, z = a\omega^2 + b\omega$, then xyz equals to where, ω is the cube root of unity
 - a $a+b$
 - b $a^2 + b^2$
 - c $a^3 + b^3$
 - d $a^4 + b^4$

4. $\lim_{n \rightarrow \infty} \left(\frac{2n^3}{2n^2+3} + \frac{1-5n^2}{5n+1} \right)$ is equal to
 - a 0
 - b 1
 - (c) $.5$
 - d ∞

5. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin(x - \frac{\pi}{6})}{\sqrt{3} - 2 \cos x}$ is equal to
 - a 0
 - b $\frac{1}{\sqrt{3}-2}$
 - (c) 1
 - d ∞

6. $\lim_{x \rightarrow \infty} \left(\frac{2x^2+3}{2x^2+5} \right)^{8x^2+3}$ is equal to
 - a 0
 - b 1
 - (c) e^8
 - d e^{-8}

7. For $y = \frac{x}{x^2-1}$, $\frac{d^n y}{dx^n}$ is equal to

- a $\frac{n!}{2} \left[\frac{1}{(x-1)^n} + \frac{1}{(x-1)^n} \right]$
- b $\frac{(-1)^n n!}{2} \left[\frac{1}{(x+1)^n} - \frac{1}{(x-1)^n} \right]$
- c $\frac{n!}{2} \left[\frac{1}{(x+1)^{n+1}} - \frac{1}{(x-1)^{n+1}} \right]$
- d $\frac{(-1)^n n!}{2} \left[\frac{1}{(x+1)^{n+1}} - \frac{1}{(x-1)^{n+1}} \right]$

8. Find the slope of the normal to the curve $4x^3 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at the point $(-2, 3)$.

- a ∞ b 11
- (c) $\frac{9}{2}$ d $\frac{2}{9}$

9. $\lim_{x \rightarrow 0} \frac{\sin 3x^2}{\ln \cos (2x^2 - x)}$ is equal to

- a 0 b -6
- (c) 1 (d) ∞

10. $\int_{-\pi/2}^{\pi/2} \cos x \ln \left(\frac{1+x}{1-x} \right) dx$ is equal to

- a 0 b $\frac{\pi^2}{4} \left(-1 + \frac{\pi}{2} \right)$
- (c) 1 d $\frac{\pi^2}{2}$

11. $\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{n!}}{n} \right)$ is equal to

- a 0 b 1
- (c) -1 d e^{-1}

12. $\int_0^{\pi} \sqrt{\frac{1 + \cos 2x}{2}} dx$ equals to

- a 0 b 2 c 4 d -2

13. The quadrangle with the vertices A $(-3, 5, 6)$, B $(1, -5, 7)$, C $(-3, -1)$ and D $(4, 7, -2)$ is a

- a square b rectangle
- c parallelogram d trapezoid

14. $|a| = |b| = 5$ and the angle between a and b is $\frac{\pi}{4}$, The area of the triangle constructed on the vectors $a-2b$ and $3a+2b$ is

- a 50 b $50\sqrt{2}$
 c $\frac{50}{\sqrt{2}}$ d 100

15. In the triangle with vertices $A(1, -1)$, $B(5, -6)$ and $C(3, -1)$ find the altitude $n = |BD|$.

- a 5 b 10 c $5\sqrt{2}$ d $\frac{10}{\sqrt{2}}$

16. If $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$, then a, b and c are in

- a AP b HP
 c GP d Both b and c

17. Given lines

$$L_1: \frac{x}{-2} = \frac{y-1}{0} = \frac{z+2}{1}$$

$$L_2: \frac{x+1}{0} = \frac{y+1}{2} = \frac{z-2}{-1}$$

Find the distance between the given straight lines.

- a 12 b $\frac{\sqrt{21}}{12}$ c $\frac{21}{\sqrt{12}}$ d $\frac{12}{\sqrt{21}}$

18. Compute the shortest distance between the circle $x^2 + y^2 - 10x - 14y - 151 = 0$ and the point $(-7, 2)$.

- a 0 b 1 c 2 d 4

19. On the ellipse $9x^2 + 25y^2 = 225$, find the point the distance from which to the our focus F_1 is four times the distance to the other focus F_2 ,

- a $(-15, \sqrt{63})$ b $(\frac{-15}{4}, \frac{\sqrt{63}}{2})$
 c $(\frac{-15}{4}, \frac{\sqrt{63}}{4})$ d $(\frac{-15}{2}, \frac{\sqrt{63}}{2})$

20. On the parabola $y^2 = 64x$, find the point nearest to the straight line $4x + 3y - 14 = 0$.

- a -24, 9 b 9, 12
 c -9, 24 d 9, -24

21. The determinant $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$ is divisible by

- a $x - y$ b $x^2 - y^2 + xy$
 c $x^2 + xy + y^2$ d $x^2 - xy + y^2$

22. The curve $5x^2 + 12xy - 22x - 12y - 19 = 0$ is

- a ellipse b parabola
 c hyperbola d parallel straight lines

23. The derivative of $y = x^{2^x}$ w.r.t. x is

- a $x^{2^x} 2^x \left(\frac{1}{x} + \ln x \ln 2 \right)$ (b) $x^{2^x} \left(\frac{1}{x} + \ln x \ln 2 \right)$
 (c) $x^{2^x} 2^x \left(\frac{1}{x} + \ln x \right)$ d $x^{2^x} 2^x \left(\frac{1}{x} + \frac{\ln x}{\ln 2} \right)$

24. $\lim_{x \rightarrow \frac{\pi}{2}} (\pi - 2x)^{\cos x}$ is equal to

- a 0 b 1 c e d e^{-1}

25. $\int_0^1 x \tan^{-1} x \, dx$ is equal to

- a $\frac{\pi}{4}$ b $\frac{\pi}{4} + \frac{1}{2}$
 c $\frac{\pi}{4} - \frac{1}{2}$ d $\frac{1}{2}$

26. $\int_0^{\pi/3} \frac{\cos \theta}{5 - 4 \sin \theta} \, d\theta$ equal to

- a $\frac{1}{4} \log \left(\frac{5}{5 + 2\sqrt{3}} \right)$ (b) $\frac{1}{4} \log \left(\frac{5}{5 - 2\sqrt{3}} \right)$
 c $\frac{1}{4} \log \left(\frac{5 + 2\sqrt{3}}{5} \right)$ (d) $\frac{1}{4} \log \left(\frac{5 - 2\sqrt{3}}{5} \right)$

27. $\int \frac{x \, dx}{(1+x)^{3/2}}$ is equals to

- a $2 \left(\frac{2+x}{\sqrt{1+x}} \right) + C$ b $\frac{2+x}{\sqrt{1+x}} + C$
 c $\frac{3-x}{2\sqrt{1+x}} + C$ d $\frac{3-2+x}{2\sqrt{1+x}} + C$

28. $\int a^x \, dx$ is equal to

a $\frac{a^x}{x \log a} + C$ b $a^x \log a + C$

(c) $\frac{a^x}{\log a} + C$ d $\frac{xa^x}{\log a} + C$

29. $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2 dx$, where p and q are integers, is equal to

a $-\pi$ b 0

c π d 2π

30. The solution of the differential equation $x^2 - y^2 dx + 2xy dy = 0$, is

a $x^2 - y^2 = Cx$ b $x^2 - y^2 = Cy$

c $x^2 + y^2 = Cx$ d $x^2 - y^2 = Cy$

31. The solution of the differential equation $\frac{d^2y}{dx^2} + 3y = -2x$ is

a $c_1 \cos \sqrt{3x} + c_2 \sin \sqrt{3x} - \frac{2}{3}x^2$

b $c_1 \cos \sqrt{3x} + c_2 \sin \sqrt{3x} - \frac{4}{5}$

c $c_1 \cos \sqrt{3x} + c_2 \sin \sqrt{3x} - 2x^2 + \frac{4}{9}$

d $c_1 \cos \sqrt{3x} + c_2 \sin \sqrt{3x} - \frac{2}{3}x^2 + \frac{4}{9}$

32. Angles A, B, C of a $\triangle ABC$ are in AP and $b:c = \sqrt{3} + \sqrt{2}$, then the $\angle A$ is given by

a 45° b 60°

c 75° d 90°

33. The angle between the vectors $a = \hat{i} + 2\hat{j} + 2\hat{k}$ and $b = \hat{i} - 2\hat{j} + 2\hat{k}$ is

a $\sin^{-1} \frac{1}{9}$ b $\cos^{-1} \frac{8}{9}$

c $\sin^{-1} \frac{8}{9}$ d $\cos^{-1} \frac{1}{9}$

34. The straight line $r = \hat{i} - \hat{j} + \hat{k} + \lambda (2\hat{i} + \hat{j} - \hat{k}) = 4$ are

a perpendicular to each other

b parallel

c inclined at an angle 60°

d inclined at an angle 45°

35. If two cards are drawn simultaneously from the same set, the probability that atleast one of them will be the ace of hearts is

a $\frac{1}{13}$ b $\frac{1}{26}$ c $\frac{1}{52}$ d $\frac{3}{13}$

36. In a class there are 10 boys and 8 girls. When 3 students are selected at random, the probability that 2 girls and 1 boy are selected is

a $\frac{35}{102}$ b $\frac{15}{102}$

c $\frac{55}{102}$ d $\frac{25}{102}$

37. If M and N are any two events, the probability that exactly one of them occurs is for an event set A, the complement is A^c

a $P(M + N) - 2P(M \cap N)$

b $P(M + N) - P(M \cap N)$

c $P(M^c + N^c) - 2P(M^c \cap N^c)$

d $P(M \cap N^c) + P(M^c \cap N)$

38. If three squares are chosen on a chess board, the chance that they should be in a diagonal line is

a $\frac{7}{144}$ b $\frac{5}{744}$

(c) $\frac{7}{544}$ d $\frac{11}{744}$

39. Let $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, then

a $A^2 + 7A - 5I = 0$ b $A^2 - 7A + 5I = 0$

c $A^2 + 5A - 7I = 0$ d $A^2 - 5A + 7I = 0$

40. $\int_0^1 \frac{dx}{1+x+x^2}$ is equal to

a $\frac{\pi}{\sqrt{3}}$ b $\frac{\pi}{2\sqrt{3}}$ c $\frac{2\pi}{3\sqrt{3}}$ d $\frac{\pi}{3\sqrt{3}}$

41. A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 420 consumers like product B. Then, the least number of consumers that must have liked both the products is

a 170 b 180 c 210 d 190

42. The polar number $z =$

X	5	2	1	4	3
Y	5	8	4	2	10

form of complex

$$\frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$$
 is

- a $\frac{1}{\sqrt{2}} \left(\cos \frac{3\pi}{12} + i \sin \frac{3\pi}{12} \right)$
 b $\sqrt{2} \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right)$
 c $\sqrt{2} \left(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12} \right)$
 d $\frac{1}{\sqrt{2}} \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right)$

43. The equation of the plane passing through the points 2,2,1, 9,3,6 and perpendicular to the plane $2x+6y+6z = 1$ is

- a $2x-4y+5z-9 = 0$ b $3x+4y -z-5 = 0$
 c $3x+4y -5z-9 = 0$ d $x+4y -9z-3 = 0$

44. The line of regression of y on x for the following data

Is given by

- a $Y+0.4x = 1$ b $y+ 0.5x = 5$
 c $y+0.4x = 7$ d $y+1.4x = 7$

45. The measure of the chord intercepted by circle $x^2+y^2 = 9$ and the line $x-y+2 = 0$ is

- a $\sqrt{28}$ b $2\sqrt{5}$
 c 7 d 5

46. $\tan^{-1} \sqrt{3} - \cot^{-1} -\sqrt{3}$ equals to

- a 0 b $2\sqrt{3}$ c $-\frac{\pi}{2}$ d π

47. The sum of the deviations of the variates from the arithmetic mean is always

- a $+1$ b 0
 c -1 d real number

48. A single letter is selected at random from the word "PROBABILITY". The probability that it is a vowel is

- a $\frac{8}{11}$ b $\frac{4}{11}$
c $\frac{2}{11}$ d $\frac{3}{11}$

49. An object is observed from three points A, B and C in the same horizontal line passing through the base of the object. The angle of elevation at B is twice and at C thrice that at A. If $AB = a$, $BC = b$, then the height of the object is

- a $\frac{a}{2b} \sqrt{(a+b)(3b-a)}$
b $\frac{a}{2b} \sqrt{(a-b)(3b-a)}$
c $\frac{a}{2b} \sqrt{(a-b)(3b+a)}$
d $\frac{a}{2b} \sqrt{(a+b)(3b+a)}$

50. The angle between the lines whose direction ratios are $1, 1, 2$, $\sqrt{3}-1, -\sqrt{3}-1, 4$ is

- a $\cos^{-1}\left(\frac{1}{65}\right)$ b $\frac{\pi}{6}$
c $\frac{\pi}{3}$ d $\frac{\pi}{2}$