Inverse Trigonometric Functions

$1+2+2+3(+3)$

	0°	30°	45°	60°	90°	$15^\circ \left(\frac{\pi}{12}\right)$	75° $\frac{5\pi}{12}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (or) $2-\sqrt{3}$	$\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (or) $2+\sqrt{3}$

$\cos\pi = -1$; $\cos(\text{odd }\pi) = -1$

$\cos(\text{even }\pi) = +1$

$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

$\cos 36^\circ = \frac{\sqrt{5}+1}{4}$

$\sin 54^\circ = \frac{\sqrt{5}+1}{4}$

$\cos 72^\circ = \frac{\sqrt{5}-1}{4}$

$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$

$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$

$\tan\frac{A}{2} = \sqrt{\frac{1-\cos A}{1+\cos A}}$

$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$1 - \cos 2\theta = 2\sin^2\theta$; $1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

$1 + \cos 2\theta = 2\cos^2\theta$; $1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$

$\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \tan^2\left(\frac{\theta}{2}\right)$

$\sqrt{1 + \sin 2x} = \cos x + \sin x$

$\left[\sqrt{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \sqrt{(\cos x + \sin x)^2} = \cos x + \sin x\right]$

** $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \tan\left(\frac{\pi}{4} - \theta\right)$

$\sqrt{1 - \sin x} = \cos\frac{x}{2} - \sin\frac{x}{2}$

$\left[\text{LHS} = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1 - \tan\theta}{1 + \tan\theta} = \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}\right.$

$\left. = \tan\left(\frac{\pi}{4} - \theta\right) = \text{RHS}\right]$

$\frac{\cos 3\theta - \cancel{\cos}\sin 3\theta}{\cos 3\theta + \sin 3\theta} = \tan\left(\frac{\pi}{4} - 3\theta\right)$

$\frac{\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

→ Trigonometric functions cannot have their inverses in the entire range of real numbers.

→ ITF will be one-one fun^n and onto fun^n only in certain intervals and hence they have their inverses only in their corresponding domain and range.

These values are called principal values.

Eg: $y = \sin x \Rightarrow x = \sin^{-1} y$

$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ – Range

$y \in [-1, 1]$ – Domain

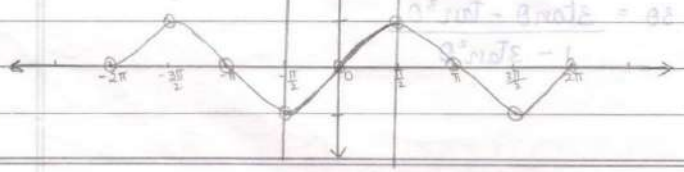


Table of (principal value branches) domain and range of ITF's

	Domain	Range
$\sin^{-1} x$	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1,1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\operatorname{cosec}^{-1} x$	$R-(-1,1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$
$\sec^{-1} x$	$R-(-1,1)$	$[0, \pi]-\left\{\frac{\pi}{2}\right\}$
$\cot^{-1} x$	R	$(0, \pi)$

Theorems:

1] (a) $\sin^{-1}(-x) = -\sin^{-1} x$; $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

(b) $\cos^{-1}(-x) = \pi - \cos^{-1} x$; $\sec^{-1}(-x) = \pi - \sec^{-1} x$

(c) $\tan^{-1}(-x) = -\tan^{-1} x$; $\cot^{-1}(-x) = \pi - \cot^{-1} x$

2] (a) $\sin^{-1} x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$; $\operatorname{cosec}^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$

(b) $\cos^{-1} x = \sec^{-1}\left(\frac{1}{x}\right)$; $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$

(c) $\tan^{-1} x = \cot^{-1}\left(\frac{1}{x}\right)$; $\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$

3] (a) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

(b) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

(c) $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$

4] (a) $\sin(\sin^{-1} x) = x$

(b) $\cos(\cos^{-1} x) = x$

(c) $\tan(\tan^{-1} x) = x$

5] (a) $\sin^{-1}(\sin\theta) = \theta$ iff $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(b) $\cos^{-1}(\cos\theta) = \theta$ iff $0 \le \theta \le \pi$

(c) $\tan^{-1}(\tan\theta) = \theta$ iff $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

6] (a) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy < 1$

(b) $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy > 1$

(c) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(d) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

(e) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right]$

EXERCISE 2.1

Eg 1] $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

** Eg 2] $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(a) $\cot^{-1}(-1) = \frac{3\pi}{4} = \left(\pi - \frac{\pi}{4}\right)$

(b) $\cot^{-1}(-\sqrt{3}) = \frac{5\pi}{6}$

1] $\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\frac{1}{2} = -\frac{\pi}{6}$

2] $\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

3] $\mathrm{cosec}^{-1}(2) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

4] $\tan^{-1}(-\sqrt{3}) = -\tan^{-1}\sqrt{3} = -\frac{\pi}{3}$

5] $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

6] $\tan^{-1}(-1) = -\tan^{-1} 1 = -\frac{\pi}{4}$

7] $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

8] $\cot^{-1}(\sqrt{3}) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

9] $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

10] $\operatorname{cosec}^{-1}(-\sqrt{2}) = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\sin^{-1}\frac{1}{\sqrt{2}} = -\frac{\pi}{4}$

Find the values of the following:

** 11] $\tan^{-1}(1) + \underbrace{\cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)}$

$= \frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi + 2\pi}{4} = \frac{3\pi}{4}$

12] $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

$= \underbrace{\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)} + \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{2} + \frac{\pi}{6} = \frac{3\pi + \pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

13] If $\sin^{-1} x = y$ then

(a) $0 \le y \le \pi$ (c) $0 < y < \pi$

(b) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

** 14] $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \frac{\pi}{3} - \left[\pi - \sec^{-1}(2)\right] = \frac{\pi}{3} - \pi + \frac{\pi}{3} = \frac{\pi - 3\pi + \pi}{3} = \frac{-\pi}{3}$

EXERCISE 2.2

Prove the following:

Eg 3] (a) $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$, $\boxed{\frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}}$

LHS $= \sin^{-1}(2x\sqrt{1-x^2})$, put $x = \sin\theta$

$\theta = \sin^{-1}x$

$= \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\sin^{-1}x$ = RHS

(b) $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x$, $\boxed{\frac{1}{\sqrt{2}} \le x \le 1}$

LHS $= \sin^{-1}(2x\sqrt{1-x^2})$, put $x = \cos\theta$

$= \sin^{-1}(2\cos\theta\sqrt{1-\cos^2\theta})$ $\qquad \theta = \cos^{-1}x$

$= \sin^{-1}(2\cos\theta\sin\theta)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\cos^{-1}x$ = RHS

Eg 7] $\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$

LHS $= \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}\left\{\frac{x + \frac{2x}{1-x^2}}{1 - \frac{2x^2}{1-x^2}}\right\}$

$= \tan^{-1}\left\{\frac{x - x^3 + 2x}{1 - x^2 - 2x^2}\right\} = \tan^{-1}\left\{\frac{3x - x^3}{1-3x^2}\right\}$ = RHS

Eg 8] Find the value of $\cos(\sec^{-1}x + \csc^{-1}x)$

$= \cos\frac{\pi}{2} = 0$

1] Prove:

$3\sin^{-1}x = \sin^{-1}(3x-4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

RHS $= \sin^{-1}(3x-4x^3)$, put $x = \sin\theta$

$= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$ $\quad \theta = \sin^{-1}x$

$= \sin^{-1}(\sin 3\theta)$

$= 3\theta = 3\sin^{-1}x =$ RHS

2] $3\cos^{-1}x = \cos^{-1}(4x^3-3x)$, $x \in \left[\frac{1}{2}, 1\right]$

RHS $= \cos^{-1}(4x^3-3x)$, put $x = \cos\theta$

$= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$ $\quad \theta = \cos^{-1}x$

$= \cos^{-1}(\cos 3\theta)$

$= 3\theta = 3\cos^{-1}x =$ LHS

3] $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

LHS $= \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$ $\qquad \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$= \tan^{-1}\left[\frac{\frac{2}{11}+\frac{7}{24}}{1-\left(\frac{2}{11}\right)\left(\frac{7}{24}\right)}\right] = \tan^{-1}\left[\frac{\frac{48+77}{264}}{\frac{264-14}{264}}\right]$

$= \tan^{-1}\left[\frac{48+77}{264-14}\right] = \tan^{-1}\left(\frac{125}{150}\right) = \tan^{-1}\frac{1}{2}$ = RHS

Eg 4] $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}$

LHS $= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11}$

$= \tan^{-1}\left[\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{2}{22}}\right] = \tan^{-1}\left[\frac{11+4}{22-2}\right] = \tan^{-1}\left(\frac{15}{20}\right)$

$= \tan^{-1}\left(\frac{3}{4}\right) =$ RHS

*] $\tan^{-1}2 + \tan^{-1}3 = \tan^{-1}\left(\frac{5}{1-6}\right) \cancel{+\tan^{-1}(-1)}$ $[2\times 3 = 6 > 1]$

$= \pi + \tan^{-1}\left[\frac{2+3}{1-6}\right]$

$= \pi + \tan^{-1}(-1)$

$= \pi - \tan^{-1}1$

$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Prove the following(or) Simplify

1] $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ put $x = \sin\theta$; $\theta = \sin^{-1}x$

$= \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\sin^{-1}x$

2] $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1}x$ put $x = \cos\theta$; $\theta = \cos^{-1}x$

$= \sin^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right)$

$= \sin^{-1}(2\cos\theta\sin\theta)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\cos^{-1}x$

3] $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $x = \tan\theta$; $\theta = \tan^{-1}x$

$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\tan^{-1}x$

4] $\cos^{-1}(1-2x^2)$ $x = \sin\theta$; $\theta = \sin^{-1}x$

$= \cos^{-1}(1-2\sin^2 x)$

$= \cos^{-1}(\cos 2\theta)$

$= 2\theta = 2\sin^{-1}x$

5] $\cos^{-1}\left(\frac{2x}{1+x^2}\right)$ $x = \tan\theta$; $\theta = \tan^{-1}x$

$= \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \cos^{-1}(\sin 2\theta)$

$= \cos^{-1}\left(\cos \frac{\pi}{2} - 2\theta\right)$

$= \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\tan^{-1}x$

6] $\sin^{-1}(2x^2-1)$ $x = \cos\theta$; $\theta = \cos^{-1}x$

$= \sin^{-1}(2\cos^2\theta - 1)$

$= \sin^{-1}(\cos 2\theta)$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{2} - 2\theta\right)\right)$

$= \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\cos^{-1}x$

7] $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ $x = \tan\theta$; $\theta = \tan^{-1}x$

$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \cos^{-1}(\cos 2\theta)$

$= 2\theta = 2\tan^{-1}x$

8] $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ $x = \tan\theta$; $\theta = \tan^{-1}x$

$= \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$= \tan^{-1}(\tan 2\theta)$

$= 2\theta = 2\tan^{-1}x$

9] Prove: $\sin^{-1}(3x-4x^3) = 3\sin^{-1}x$

LHS $= \sin^{-1}(3x-4x^3)$ $\quad x = \sin\theta$; $\theta = \sin^{-1}x$

$= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$

$= 3\theta = 3\sin^{-1}x$

10] Simplify : $\cos^{-1}(4x^3-3x)$ $\quad x = \cos\theta$; $\theta = \cos^{-1}x$

$= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta)$

$= 3\theta = 3\cos^{-1}x$

11] $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ $\quad x = \tan\theta$; $\theta = \tan^{-1}x$

$= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$

$= \tan^{-1}(\tan 3\theta)$

$= 3\theta = 3\tan^{-1}x$

NOTE: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

s	b	h
3	4	5
8	15	17
~~13~~	~~84~~	~~85~~
5	12	13
56	33	65

EXERCISE 2.2

** 4] $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

$LHS = 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} \cancel{+ \tan^{-1}\frac{31}{17}}$

$= \tan^{-1}\left[\frac{2(\frac{1}{2})}{1-(\frac{1}{2})^2}\right] + \tan^{-1}\frac{1}{7}$

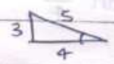
$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

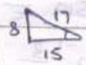
$= \tan^{-1}\left[\frac{\frac{4}{3}+\frac{1}{7}}{1-(\frac{4}{3})(\frac{1}{7})}\right]$

$= \tan^{-1}\left(\frac{28+3}{21-4}\right) = \tan^{-1}\left(\frac{31}{17}\right) = RHS$

Mis Eg 10] Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

$LHS = \sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17}$



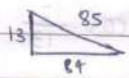


$\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$ | $\sin^{-1}\frac{8}{17} = \tan^{-1}\frac{8}{15}$

$= \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{8}{15}$

$= \tan^{-1}\left(\frac{\frac{3}{4}-\frac{8}{15}}{1+\frac{24}{60}}\right) = \tan^{-1}\left(\frac{45-32}{60+24}\right)$

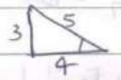
$= \tan^{-1}\left(\frac{13}{84}\right)$



$= \cos^{-1}\left(\frac{84}{85}\right)$

Mis Ex 3] PT: $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

LHS $= 2\sin^{-1}\frac{3}{5}$



$= 2\tan^{-1}\frac{3}{4}$

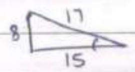
$= \tan^{-1}\left(\dfrac{2\left(\frac{3}{4}\right)}{1-\frac{9}{16}}\right)$

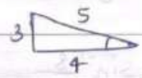
$= \tan^{-1}\left(\frac{3}{\cancel{2}}\cdot\frac{\cancel{16}^{8}}{7}\right)$

$= \tan^{-1}\left(\frac{24}{7}\right)$ = RHS

Mis Ex 4] PT: $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

LHS $= \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$





$\sin^{-1}\frac{8}{17} = \tan^{-1}\frac{8}{15}$ | $\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$

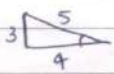
$= \tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4}$

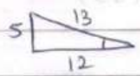
$= \tan^{-1}\left(\dfrac{\frac{8}{15}+\frac{3}{4}}{1-\frac{24}{60}}\right) = \tan^{-1}\left(\dfrac{32+45}{60-24}\right)$

$= \tan^{-1} \cancel{85}\left(\frac{77}{36}\right)$ = RHS

Mis Ex 5] PT: $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

LHS $= \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$



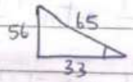


$\cos^{-1}\frac{4}{5} = \tan^{-1}\frac{3}{4}$ | $\cos^{-1}\frac{12}{13} = \tan^{-1}\frac{5}{12}$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{5}{12}$

$= \tan^{-1}\left(\dfrac{\frac{3}{4}+\frac{5}{12}}{1-\frac{15}{48}}\right) = \tan^{-1}\left(\dfrac{36+20}{48-15}\right)$

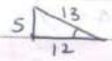
$= \tan^{-1}\left(\dfrac{56}{33}\right)$

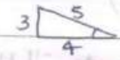


$= \cos^{-1}\left(\dfrac{33}{65}\right) = RHS$

MisEx6] PT : $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

LHS $= \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5}$

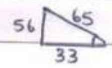




$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$

$= \tan^{-1}\left(\dfrac{\frac{5}{12}+\frac{3}{4}}{1-\frac{15}{48}}\right)$

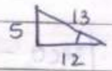
$= \tan^{-1}\left(\dfrac{20+36}{48-15}\right) = \tan^{-1}\left(\dfrac{56}{33}\right)$

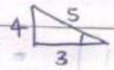


$= \sin^{-1}\left(\dfrac{56}{65}\right) = RHS$

MisEx7] $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

RHS $= \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$





$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$

$= \tan^{-1}\left(\dfrac{\frac{5}{12}+\frac{4}{3}}{1-\frac{20}{36}}\right) = \tan^{-1}\left(\dfrac{15+48}{36-20}\right)$

~~Xgx~~

$= \tan^{-1}\left(\frac{63}{16}\right) = LHS$

Mis Ex 8] $\tan^{-1}\frac{1}{5} + \tan\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

$LHS = \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$

$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{35}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{24}}\right)$

$= \tan^{-1}\left(\frac{7+5}{35-1}\right) + \tan^{-1}\left(\frac{8+3}{24-1}\right)$

$= \tan^{-1}\left(\frac{12}{34}\right) + \tan^{-1}\left(\frac{11}{23}\right)$

$= \tan^{-1}\left(\frac{6}{17}\right) + \tan^{-1}\left(\frac{11}{23}\right)$

$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{66}{391}}\right) = \tan^{-1}\left(\frac{138+187}{391-66}\right)$

$= \tan^{-1}\left(\frac{325}{325}\right) = \tan^{-1}(1) = \frac{\pi}{4} = RHS$

Write the following funⁿ in the simplest form:

** 5] $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ $x = \tan\theta$; $\theta = \tan^{-1}x$

$= \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right] = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$$= \tan^{-1}\left[\frac{\cancel{2}\sin^{\cancel{2}}\theta/2}{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right] = \tan^{-1}\left[\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\theta}{2}\right)\right] = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$

*** Mis Ex 11] $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ put $x = \cos 2\theta$

$\theta = \frac{1}{2}\cos^{-1}x$

$$= \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\cancel{\sqrt{2}}(\cos\theta-\sin\theta)}{\cancel{\sqrt{2}}(\cos\theta+\sin\theta)}\right]$$ ÷ Num & Deno by '$\cos\theta$'

$$= \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]$$

$$= \frac{\pi}{4}-\theta = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

*] $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, put $x^2 = \cos 2\theta$

$\theta = \frac{1}{2}\cos^{-1}(x^2)$

$$= \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\cancel{\sqrt{2}}(\cos\theta+\sin\theta)}{\cancel{\sqrt{2}}(\cos\theta-\sin\theta)}\right]$$

$$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$$

$$= \frac{\pi}{4}+\theta = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$$

*] $\tan^{-1}\left[\frac{\sqrt{1+x^3}-\sqrt{1-x^3}}{\sqrt{1+x^3}+\sqrt{1-x^3}}\right]$ put, $x^3 = \cos 2\theta$

$\theta = \frac{1}{2}\cos^{-1}x^3$

$$= \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{2}\,(\cos\theta-\sin\theta)}{\sqrt{2}\,(\cos\theta+\sin\theta)}\right]$$

$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]$$

$$= \frac{\pi}{4}-\theta = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^3$$

5] Simplify : $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ put $x = \tan\theta$

$\theta = \tan^{-1}x$

$$= \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right]$$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}{\cancel{2}\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right) = \cancel{\tan^{-1}}\left(\cancel{\tan}\frac{\theta}{2}\right)$

$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

5] Simplify : $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ put $x = \tan\theta$

$\theta = \tan^{-1}x$

$= \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$

$= \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right] = \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}{\cancel{2}\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right) = \cancel{\tan^{-1}}\left(\cancel{\tan}\frac{\theta}{2}\right)$

$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

Eg5] Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $\frac{-3\pi}{2} < x < \frac{\pi}{2}$ in simplest form:

$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$

$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$

$= \tan^{-1}\left[\frac{\cancel{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^{\cancel{2}}}\right]$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right] \quad \div \text{ Nom \& Den by } \cos\frac{x}{2}$$

$$= \tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$$

$$= \frac{\pi}{4}+\frac{x}{2}$$

Eg 5] Simplyfy : $\tan^{-1}\left[\frac{\cos x}{1-\sin x}\right]$

$$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\cancel{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^{\cancel{2}}}\right]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right] \quad \div \text{ Num \& Deno by } \cos\frac{x}{2}$$

$$= \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$$

$$= \frac{\pi}{4}+\frac{x}{2}$$

Eg 5(a)] Simplify $\tan^{-1}\left(\frac{\sin x}{1-\cos x}\right)$

$$= \tan^{-1}\left[\frac{\cancel{2}\sin\frac{x}{2}\cos\frac{x}{2}}{\cancel{2}\sin^2\frac{x}{2}}\right] = \tan^{-1}\left[\left(\cot\frac{x}{2}\right)\right] =$$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right] = \frac{\pi}{2}-\frac{x}{2}$

Eg 5] Simplify $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$

$= \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$

$= \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$

$= \tan^{-1}\left[\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] = \tan^{-1}\left[\tan\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}\right]$

$= \frac{\pi}{4}+\frac{x}{2}$

6] $\tan^{-1}\frac{1}{\sqrt{x^2-1}}, \ |x|>1$

put $x = \operatorname{cosec}\theta$; $\theta = \operatorname{cosec}^{-1}x$

$= \tan^{-1}\left(\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}\right) = \tan^{-1}\left(\frac{1}{\cot\theta}\right)$

$= \tan^{-1}(\tan\theta) = \theta = \operatorname{cosec}^{-1}x$

** Eg 6] Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), \ x>1$ in simplest form.

put $x = \sec\theta$; $\theta = \sec^{-1}x$

$= \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta-1}}\right) = \cot^{-1}\left(\frac{1}{\tan\theta}\right)$

$= \cot^{-1}(\cot\theta) = \theta = \sec^{-1}x$

7] $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), 0 < x < \pi$

$= \tan^{-1}\left(\sqrt{\frac{\cancel{2}\sin^2\frac{x}{2}}{\cancel{2}\cos^2\frac{x}{2}}}\right) = \tan^{-1}\left(\sqrt{\tan^2\frac{x}{2}}\right)$

$= \tan^{-1}\left(\tan\left(\frac{x}{2}\right)\right) = \frac{x}{2}$

*] $\tan^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

$= \tan^{-1}\left(\sqrt{\frac{\cancel{2}\cos^2\frac{x}{2}}{\cancel{2}\sin^2\frac{x}{2}}}\right) = \tan^{-1}\left(\sqrt{\cot^2\frac{x}{2}}\right)$

$= \tan^{-1}\left(\cot\left(\frac{x}{2}\right)\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right]$

$= \frac{\pi}{2} - \frac{x}{2}$

9] $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right), |x| < a$

put $x = a\sin\theta$; $\theta = \sin^{-1}\left(\frac{x}{a}\right)$

$= \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right) = \tan^{-1}\left(\frac{\cancel{a}\sin\theta}{\cancel{a}\cos\theta}\right)$

$= \tan^{-1}(\tan\theta) = \theta = \sin^{-1}\left(\frac{x}{a}\right)$

10] $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a > 0; \frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

put $x = a\tan\theta$, $\theta = \tan^{-1}\left(\frac{x}{a}\right)$

$= \tan^{-1}\left(\frac{3a^3\tan\theta - a^3\tan^3\theta}{a^3 - 3a^3\tan^2\theta}\right)$

$\div$ Num & Den by a^3

$= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$

$= \tan^{-1}(\tan 3\theta) = 3\theta = 3\tan^{-1}\frac{x}{a}$

Find the values of following:

11] $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$= \tan^{-1}\left[2\cos\left(\cancel{2}\cdot\frac{\pi}{\cancel{6}3}\right)\right] = \tan^{-1}\left[2\cos\left(\frac{\pi}{3}\right)\right]$

$= \tan^{-1}\left(2\cdot\frac{1}{2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

12] $\cot(\tan^{-1}a + \cot^{-1}a)$

$= \cot\frac{\pi}{2} = 0$

**13] $\tan\frac{1}{2}$ $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$, $|x|<1, y>0, xy<1$

$x = \tan A$	$y = \tan B$
$A = \tan^{-1}x$	$B = \tan^{-1}y$

$= \tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2\tan A}{1+\tan^2 A}\right) + \cos^{-1}\left(\frac{1-\tan^2 B}{1+\tan^2 B}\right)\right]$

$= \tan\frac{1}{2}\left[\cancel{\sin^{-1}}(\cancel{\sin}\, 2A) + \cancel{\cos^{-1}}(\cancel{\cos}\, 2B)\right]$

$= \cancel{\tan\frac{1}{2}(2\tan^{-1}x + 2\tan^{-1}y)} = \tan\frac{1}{2}(2A + 2B)$

$= \tan(A+B)$

$= \dfrac{\tan A + \tan B}{1 - \tan A \tan B} \quad = \dfrac{x+y}{1-xy}$

14] $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, find x.

$= \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}1$

$= \sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$

By theorem, $\sin^{-1}\frac{1}{5} + \cos^{-1}\frac{1}{5} = \frac{\pi}{2}$

comparing, $x = \frac{1}{5}$

15] $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$, find x.

$$\Rightarrow \tan^{-1}\left\{\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{(x^2-1)}{x^2-4}}\right\} = \frac{\pi}{4}$$

$$\Rightarrow \frac{x^2 - x + 2x - 2 + x^2 + x - 2x - 2}{x^2 - 4 - x^2 + 1} = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2 - 4}{-3} = 1$$

$\Rightarrow 2x^2 = -3 + 4$

$2x^2 = 1$

$x = \pm\frac{1}{\sqrt{2}}$

Find the values of :

16] $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$

$= \sin^{-1}(\sin 120^\circ)$

$= \sin^{-1}[\sin(180-60)]$

$= \sin^{-1}[\sin(60^\circ)] \quad = 60^\circ = \frac{\pi}{3}$

17] $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$

$= \tan^{-1}(\tan 135^\circ)$

$= \tan^{-1}\{\tan(180-45)\}$

$= \tan^{-1}[-\tan(45)]$

$= \tan^{-1}(-1)$

$= -\frac{\pi}{4}$

18] $\tan\left[\underbrace{\sin^{-1}\frac{3}{5}}_{A} + \underbrace{\cot^{-1}\frac{3}{2}}_{B}\right]$

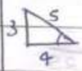
$= \tan(A+B)$

$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{6}{12}}$

$= \frac{9+8}{12-6} = \frac{17}{6}$

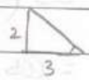
$\sin^{-1}\frac{3}{5} = A$



$\sin A = \frac{3}{5}$

$\tan A = \frac{3}{4}$

$\cot^{-1}\frac{3}{2} = B$

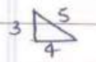


$\cot B = \frac{3}{2}$

$\tan B = \frac{2}{3}$

*] $\sin\left[\tan^{-1}\frac{3}{4} - \cos^{-1}\frac{5}{13}\right]$

$\tan^{-1}\frac{3}{4} = A$

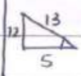


$\frac{3}{4} = \tan A$

$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$

$\cos^{-1}\frac{5}{13} = B$



$\frac{5}{13} = \cos B$

$\cancel{\cos}\ \sin B = \frac{12}{13}$

$= \sin(A - B)$

$= \sin A \cos B - \cos A \sin B$

$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{15}{65} - \frac{48}{65} = -\frac{33}{65}$

*] $\cos\left(2\cos^{-1}\frac{3}{5}\right)$

$\cos^{-1}\frac{3}{5} = A$

$\frac{3}{5} = \cos A$

$= 2\cos^2 A - 1$

$= 2\left(\frac{9}{25}\right) - 1$

$= \frac{18-25}{25} = \frac{-7}{25}$

*] $\cos\left(2\tan^{-1}\frac{5}{12}\right)$

$\tan^{-1}\frac{5}{12} = A$

$\frac{5}{12} = \tan A$

$= \cos 2A$

$= \frac{1-\tan^2 A}{1+\tan^2 A}$

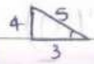
$= \frac{1-\frac{25}{144}}{1+\frac{25}{144}} = \frac{144-25}{144+25} = \frac{119}{169}$

*] $\cos\left(\frac{1}{2}\cos^{-1}\frac{3}{5}\right)$

$= \cos\left(\frac{A}{2}\right) = \sqrt{\frac{1+\cos A}{2}}$

$= \sqrt{\frac{1+\frac{3}{2}}{2}} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

*] $\sin\left[\frac{1}{2}\overbrace{\tan^{-1}\frac{4}{3}}^{\theta}\right]$



$\cos\theta = \frac{3}{5}$

$= \sin\frac{\theta}{2}$

$= \sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{2}{10}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$

Simplify: $\tan\left[\frac{1}{2}\overbrace{\left(\cos^{-1}\frac{12}{13}\right)}^{\theta}\right]$

$\cos^{-1}\frac{12}{13} = \theta$

$\frac{12}{13} = \cos\theta$

$= \tan\left(\frac{\theta}{2}\right)$

$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{1-\frac{12}{13}}{1+\frac{12}{13}}} = \sqrt{\frac{13-12}{13+12}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$

19] Find the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

$= \cos^{-1}(\cos 210^\circ)$

$= \cos^{-1}(\cos 180+30)$

$= \cos^{-1}(-\cos 30)$

$= \cancel{\cos^{-1}}\ \pi - \cos^{-1}(\cos 30^\circ)$

$= \pi - 30^\circ$

$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

**20] $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

$= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right]$

$= \sin\left(\frac{\pi}{2}\right) = 1$

21] $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$= \tan^{-1}\sqrt{3} - \left(\pi - \cot^{-1}(\sqrt{3})\right)$

$= \underline{\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}} - \pi$

$= \frac{\pi}{2} - \pi$

$= -\frac{\pi}{2}$

Mis Eg 9] $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

$= \sin^{-1}\left[\sin\left[\pi - \frac{2\pi}{5}\right]\right]$ $\pi - ? = \frac{3\pi}{5}$ $\pi - \frac{3\pi}{5} = \frac{2\pi}{5}$

$= \sin^{-1}\left[\sin\left(\frac{2\pi}{5}\right)\right]$ (less than 90°)

$= \frac{2\pi}{5}$

Mis Eg 12] Simplify $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right]$, if $\frac{a}{b}\tan x > -1$

÷ Num & Den by $b\cos x$

$= \tan^{-1}\left[\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right]$ $\left[\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$= \tan^{-1}\frac{a}{b} - \tan^{-1}(\tan x)$

$= \tan^{-1}\frac{a}{b} - x$

Mis Eg13] Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4} \longrightarrow (1)$

$\Rightarrow \tan^{-1}\left[\frac{2x+3x}{1-6x^2}\right] = \frac{\pi}{4}$

$\Rightarrow \frac{2x+3x}{1-6x^2} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 5x = 1-6x^2$

$\Rightarrow 6x^2+5x-1 = 0$

$\Rightarrow 6x^2+6x-x-1 = 0$

$\Rightarrow 6x(x+1)-1(x+1) = 0$

$\Rightarrow (6x-1)(x+1) = 0$

$\Rightarrow x = -1, \frac{1}{6}$

$x=-1$ does not satisfy (1)

$\therefore \boxed{x = \frac{1}{6}}$

Mis Ex1] $\cos^{-1}\left[\cos\left(\frac{13\pi}{6}\right)\right]$

$= \cos^{-1}(\cos 390^\circ)$

$= \cos^{-1}(\cos 360+30)$

$= \cancel{\cos^{-1}}(\cancel{\cos} 30^\circ)$

$= 30^\circ = \frac{\pi}{6}$

Mis Ex2] $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

$= \tan^{-1}(\tan 210^\circ)$

$= \tan^{-1}(\tan(180^\circ+30^\circ))$

$= \cancel{\tan^{-1}}(\cancel{\tan} 30^\circ) = 30^\circ = \frac{\pi}{6}$

Mis Ex 9] $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in [0,1]$, Prove

~~TPT: $2\tan^{-1}\sqrt{x} = \cos^{-1}\left(\frac{1-x}{1+x}\right)$~~ $x = (\tan\theta)^2$

Put $x = \tan^2\theta$; $\theta = \tan^{-1}\sqrt{x}$

$$RHS = \frac{1}{2}\cos\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \frac{1}{2}\cos(\cos 2\theta)$$

$$= \frac{1}{2}2\theta = 2\tan^{-1}\sqrt{x} = LHS$$

Mis Ex 10] $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] = \frac{x}{2}$, $x \in \left(0, \frac{\pi}{4}\right)$

$$\sqrt{1+\sin x} = \sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2} = \cos\frac{x}{2}+\sin\frac{x}{2}$$

Ily $\sqrt{1-\sin x} = \cos\frac{x}{2} - \sin\frac{x}{2}$

$$LHS = \cot^{-1}\left[\frac{\cos\frac{x}{2}+\cancel{\sin\frac{x}{2}}+\cos\frac{x}{2}-\cancel{\sin\frac{x}{2}}}{\cancel{\cos\frac{x}{2}}+\sin\frac{x}{2}-\cancel{\cos\frac{x}{2}}+\sin\frac{x}{2}}\right]$$

$$= \cot^{-1}\left[\frac{\cancel{2}\cos\frac{x}{2}}{\cancel{2}\sin\frac{x}{2}}\right] = \cot^{-1}\left[\cot\left(\frac{x}{2}\right)\right] = \frac{x}{2} = RHS$$

*] $\cot^{-1}\left[\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right]$

$$= \cot^{-1}\left[\frac{\cancel{\cos\frac{x}{2}}+\sin\frac{x}{2}-\cancel{\cos\frac{x}{2}}+\sin\frac{x}{2}}{\cos\frac{x}{2}+\cancel{\sin\frac{x}{2}}+\cos\frac{x}{2}-\cancel{\sin\frac{x}{2}}}\right]$$

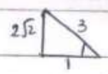
$$= \cot^{-1}\left[\frac{\cancel{2}\sin\frac{x}{2}}{\cancel{2}\cos\frac{x}{2}}\right] = \cot^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \cot^{-1}\left[\cot\left(\frac{\pi}{2}-\frac{x}{2}\right)\right] = \frac{\pi}{2}-\frac{x}{2}$$

Mis Ex 12] $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$; Prove.

Let it be $\frac{9\pi}{8} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3} + \frac{9}{4}\sin^{-1}\frac{1}{3}$

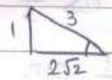
$$\text{RHS} = \frac{9}{4}\left[\sin^{-1}\frac{2\sqrt{2}}{3} + \sin^{-1}\frac{1}{3}\right]$$



$\sin A = \frac{2\sqrt{2}}{3}$

$\tan A = \frac{2\sqrt{2}}{1}$

$A = \tan^{-1} 2\sqrt{2}$



$\sin A = \frac{1}{3}$

$\tan A = \frac{1}{2\sqrt{2}}$

$A = \tan^{-1}\frac{1}{2\sqrt{2}}$

$$= \frac{9}{4}\left[\tan^{-1}(2\sqrt{2}) + \tan^{-1}\left(\frac{1}{2\sqrt{2}}\right)\right]$$

$$= \frac{9}{4}\tan^{-1}\left[\frac{2\sqrt{2}+\frac{1}{2\sqrt{2}}}{1-2\sqrt{2}\left(\frac{1}{2\sqrt{2}}\right)}\right]$$

$$= \frac{9}{4}\tan^{-1}\left[\frac{2\sqrt{2}+\frac{1}{2\sqrt{2}}}{0}\right]$$

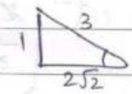
$$= \frac{9}{4}\left[\tan^{-1}(\infty)\right]$$

$$= \frac{9}{4}\left(\frac{\pi}{2}\right)$$

$$= \frac{9\pi}{8} = \text{LHS}$$

(or)

$$RHS = \frac{9}{4}\left[\sin^{-1}\frac{2\sqrt{2}}{3} + \sin^{-1}\frac{1}{3}\right]$$



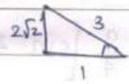
$$= \frac{9}{4}\left[\sin^{-1}\frac{2\sqrt{2}}{3} + \cos^{-1}\frac{2\sqrt{2}}{3}\right] \qquad \sin^{-1}\frac{1}{3} = \cos^{-1}\frac{2\sqrt{2}}{3}$$

$$= \frac{9}{4}\left(\frac{\pi}{2}\right) = \frac{9\pi}{8} = LHS$$

(or)

$$PT: \quad \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$$

$$LHS = \frac{9}{4}\left[\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right]$$



$$= \frac{9}{4}\cos^{-1}\frac{1}{3}$$

$$= \frac{9}{4}\cdot\sin^{-1}\frac{2\sqrt{2}}{3}$$

$$= RHS.$$

Mis Ex 13] $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\mathrm{cosec}\, x)$; Solve : $x = ?$

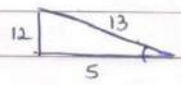
$$\cancel{LHS} \Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\,\mathrm{cosec}\, x)$$

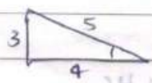
$$\Rightarrow \frac{\cancel{2}\cos x}{\sin^{\cancel{2}} x} = \frac{\cancel{2}}{\cancel{\sin x}}$$

$$= \cot x = 1$$

$$x = \frac{\pi}{4}$$

Eg 11] Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$





$$\sin^{-1}\frac{12}{13} = \tan^{-1}\frac{12}{5} \qquad \cos^{-1}\frac{4}{5} = \tan^{-1}\frac{3}{4}$$

$\Rightarrow \tan^{-1}\frac{12}{5} + \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{63}{16}$

$= \tan^{-1}\left[\dfrac{\frac{12}{5}+\frac{3}{4}}{1-\frac{36}{20}}\right] + \tan^{-1}\frac{63}{16}$

$= \tan^{-1}\left[\dfrac{48+15}{20-36}\right] + \tan^{-1}\frac{63}{16}$

$= \tan^{-1}\left(\frac{63}{-16}\right) + \tan^{-1}\frac{63}{16}$ $(xy > 1)$

$= \pi - \cancel{\tan^{-1}\left(\frac{63}{16}\right)} + \cancel{\tan^{-1}\frac{63}{16}}$

$= \pi = RHS$

Proofs:

(a) $\sin^{-1}(-x) = -\sin^{-1}x$

let $\sin^{-1}(-x) = \theta \longrightarrow ①$

$-x = \sin\theta$

$x = -\sin\theta = \sin(-\theta)$

$\sin^{-1}x = -\theta$

$-\sin^{-1}x = \theta \longrightarrow ②$

① & ② $\Rightarrow \sin^{-1}(-x) = -\sin^{-1}x$

(c) $\tan^{-1}(-x) = -\tan^{-1}x$

let $\tan^{-1}(-x) = \theta \rightarrow ①$

$-x = \tan\theta$

$x = -\tan\theta = \tan(-\theta)$

$\tan^{-1}x = -\theta$

$-\tan^{-1}x = \theta \rightarrow ②$

① & ② $\Rightarrow \tan^{-1}(-x) = -\tan^{-1}x$

b) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

let $\cos^{-1}(-x) = \theta \rightarrow ①$

$-x = \cos\theta$

$x = -\cos\theta = \cos(\pi - \theta)$

$\cos^{-1}x = \pi - \theta$

$\theta = \pi - \cos^{-1}x \rightarrow ②$

① & ② $\Rightarrow \cos^{-1}(-x) = \pi - \cos^{-1}x$

d) $\sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}$

let $\sin^{-1}x = \theta \rightarrow ①$

$x = \sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\frac{1}{x} = \operatorname{cosec}\theta$

$\operatorname{cosec}^{-1}\frac{1}{x} = \theta \rightarrow ②$

① & ② $\Rightarrow \sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}$

e) $\cos^{-1}x = \sec^{-1}\frac{1}{x}$

let $\cos^{-1}x = \theta \rightarrow ①$

$x = \cos\theta = \frac{1}{\sec\theta}$

$\frac{1}{x} = \sec\theta$

$\sec^{-1}\frac{1}{x} = \theta \rightarrow ②$

① & ② $\Rightarrow \cos^{-1}x = \sec^{-1}\frac{1}{x}$

f) $\tan^{-1}x = \cot^{-1}\frac{1}{x}$

let $\tan^{-1}x = \theta \rightarrow ①$

$x = \tan\theta = \frac{1}{\cot\theta}$

$\frac{1}{x} = \cot\theta$

$\cot^{-1}\frac{1}{x} = \theta \rightarrow ②$

① & ② $\Rightarrow \tan^{-1}x = \cot^{-1}\frac{1}{x}$

(2m) ** g) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

let $\sin^{-1}x = \theta \rightarrow ①$

$x = \sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$

$\cos^{-1}x = \frac{\pi}{2} - \theta$

$\theta + \cos^{-1}x = \frac{\pi}{2}$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ [using ①]

h] $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

let $\sec^{-1}x = \theta \rightarrow ①$

$x = \sec\theta = \operatorname{cosec}\left(\frac{\pi}{2} - \theta\right)$

$\operatorname{cosec}^{-1}x = \frac{\pi}{2} - \theta$

$\theta + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

i] $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

let $\tan^{-1}x = \theta \rightarrow ①$

$x = \tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$

$\cot^{-1}x = \frac{\pi}{2} - \theta$

$\theta + \cot^{-1}x = \frac{\pi}{2}$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

**j) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$

Consider $\tan^{-1}x = A$, $x = \tan A$ | $\tan^{-1}y = B$, $y = \tan B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) = \frac{x+y}{1-xy}$$

$$A + B = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

k) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

Consider $\tan^{-1}x = A$, $x = \tan A$ | $\tan^{-1}y = B$, $y = \tan B$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(A-B) = \frac{x-y}{1+xy}$$

$$A - B = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

l) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

consider, $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

put $y = x$

$\tan^{-1}x + \tan^{-1}x = \tan^{-1}\left[\frac{x+x}{1-x(x)}\right]$

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

m) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right]$

LHS $= \tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$= \tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z$

$= \tan^{-1}\left\{\frac{\frac{x+y}{1-xy} + z}{1 - \frac{xz+yz}{1-xy}}\right\}$

$= \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-xz-yz}\right]$ = RHS

n) (i) $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$; $|x| \leq 1$

(ii) $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$; $x \geq 0$

(iii) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$; $-1 < x < 1$

(i) RHS $= \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

put, $x = \tan\theta$

$\theta = \tan^{-1}x$

$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\tan^{-1}x = \text{LHS}$

(ii) RHS $= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

put, $x = \tan\theta$; $\theta = \tan^{-1}x$

$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \cos^{-1}(\cos 2\theta)$

$= 2\theta = 2\tan^{-1}x = \text{LHS}$

Mis Ex(14) Solve for x:

$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$

$2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}x$

$\tan^{-1}\left[\frac{\frac{2-2x}{1+x}}{1-\left(\frac{1+x^2-2x}{1+x^2+2x}\right)}\right] = \tan^{-1}x$

$\frac{2(1-x)}{\cancel{(1+x)}} \cdot \frac{(1+x)^{\cancel{2}}}{(1+x)^2-(1-x)^2} = x$

$2-2x\left[\frac{(1+x)}{4x}\right] = x$

$(2-2x)(1+x) = 4x^2$

$$2+\cancel{2x}-\cancel{2x}-2x^2 = 4x^2$$

$$2-2x^2 = 4x^2$$

$$\frac{2(1-x)^2}{4x} = x$$

$$2-2x^2 = 4x^2$$

$$2 = {}^{3}\cancel{6}x^2$$

$$x^2 = \frac{1}{3}$$

$$x = \pm\frac{1}{\sqrt{3}}$$

(or)

put $x = \tan\theta$, $\theta = \tan^{-1}x$

$$\tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right] = \frac{1}{2}\theta$$

$$\cancel{\tan^{-1}}\left[\cancel{\tan}\left(\frac{\pi}{4}-\theta\right)\right] = \frac{1}{2}\theta$$

$$\frac{\pi}{4} = \frac{\theta}{2}+\theta$$

$$\frac{\pi}{{}_2\cancel{4}} = \frac{3\theta}{\cancel{2}}$$

$$\frac{\pi}{6} = \tan^{-1}x$$

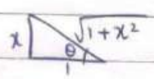
$$x = \tan\frac{\pi}{6}$$

$$x = \frac{1}{\sqrt{3}}$$

Mis Ex 15] $\sin(\tan^{-1}x)$, $|x|<1$ =

A) $\frac{x}{\sqrt{1-x^2}}$ B) $\frac{1}{\sqrt{1-x^2}}$ C) $\frac{1}{\sqrt{1+x^2}}$ D) $\frac{x}{\sqrt{1+x^2}}$

$\tan^{-1}x = \theta$



$\Rightarrow \frac{x}{1} = \tan\theta$

$$\rightarrow \sin\theta = \frac{x}{\sqrt{1+x^2}}$$

Mis Ex 16] $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$; $x = ?$

A) $0, \frac{1}{2}$ B) $1, \frac{1}{2}$ ~~C)~~ 0 D) $\frac{1}{2}$

only $x = 0$ satisfies the equation.

Mis Ex 17] $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y} =$

A) $\frac{\pi}{2}$ B) $\frac{\pi}{3}$ ~~C)~~ $\frac{\pi}{4}$ D) $\frac{-3\pi}{4}$

$\Rightarrow \tan^{-1}\frac{x}{y} - \tan^{-1}\left[\dfrac{\frac{x}{x} - \frac{y}{x}}{\frac{x}{x} + \frac{y}{x}}\right]$

$\Rightarrow \tan^{-1}\frac{x}{y} - \tan^{-1}\left[\dfrac{1 - \frac{y}{x}}{1 + \frac{y}{x}}\right]$

$\Rightarrow \tan^{-1}\frac{x}{y} - \left[\tan^{-1}1 - \tan^{-1}\frac{y}{x}\right]$

$\Rightarrow \tan^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x} - \tan^{-1}1$

$\Rightarrow \tan^{-1}\frac{x}{y} + \cot^{-1}\frac{x}{y} - \tan^{-1}1$

$\Rightarrow \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$