CBSE Class 12 Maths Question Paper Solution 2020 Set 65/1/1

QUESTION PAPER CODE 65/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions. Select the correct option.

Q.No.		Marks
1.	If A is a square matrix of order 3 and $\|A\| = 5$, then the value of $\|2A'\|$ is **(A)** –10 **(B)** 10 **(C)** –40 **(D)** 40 **Ans:** (D) 40	1
2.	If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to **(A)** I **(B)** 0 **(C)** I – A **(D)** I + A **Ans:** (A) I	1
3.	The principal value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ **(A)** $\frac{2\pi}{5}$ **(B)** $\frac{-2\pi}{5}$ **(C)** $\frac{3\pi}{5}$ **(D)** $\frac{-3\pi}{5}$ **Ans:** (B) $-\frac{2\pi}{5}$	1
4.	If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$, is zero, then the value of λ is **(A)** 0 **(B)** 1 **(C)** $\frac{-2}{3}$ **(D)** $\frac{-3}{2}$ **Ans:** (C) $-\frac{2}{3}$	1
5.	The vector equation of the line passing through the point (–1, 5, 4) and perpendicular to the plane z = 0 is **(A)** $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$ **(B)** $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$ **(C)** $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$ **(D)** $\vec{r} = \lambda\hat{k}$ **Ans:** (B) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$	1
6.	The number of arbitrary constants in the particular solution of a differential equation of second order is (are) **(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **Ans:** (A) 0	1

7. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x dx$

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2

Ans: (D) 2 — 1

8. The length of the perpendicular drawn from the point (4, –7, 3) on the y-axis is

(A) 3 units **(B)** 4 units **(C)** 5 units **(D)** 7 units

Ans: (C) 5 units — 1

9. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is equal to

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{4}$ **(D)** 1

Ans: (C) $\frac{3}{4}$ — 1

10. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4,0), (2, 4) and (0, 5). If the maximum value of $z = ax + by$, where a, b > 0 occurs at both (2, 4) and (4,0), then

(A) a = 2b **(B)** 2a = b **(C)** a = b **(D)** 3a = b

Ans: (A) $a = 2b$ — 1

Fill in the blanks in questions numbers 11 to 15

11. A relation R in a set A is called ______________, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

Ans: Symmetric — 1

12. The greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at x = ___________.

Ans: 1

13. If A is a matrix of order 3 × 2, then the order of the matrix A′ is _________.

Ans: 2 × 3 — 1

OR

A square matrix A is said to be skew-symmetric, if ________

Ans: $A = -A'$ (or, $A' = -A$) — 1

14. The equation of the normal to the curve $y^2 = 8x$ at the origin is __________

Ans: $y = 0$ — 1

OR

The radius of a circle is increasing at the uniform rate of 3 cm/s. At the instant when the radius of the circle is 2 cm, its area increases at the rate of ____________ cm^2/s.

Ans: 12π **1**

15. The position vectors of two points A and B are $\overrightarrow{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\overrightarrow{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is _______

Ans: $2\hat{i} - \hat{j} + \hat{k}$ **1**

Question numbers 16 to 20 are very short answer type questions

16. If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find A (adj A).

Ans: $A \cdot adj(A) = |A|\, I$ **1/2**

$\therefore\ A \cdot adj\,(A) = 2I$ or $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ **1/2**

17. Find $\int x^4 \log x\,dx$

Ans: $\int x^4 \cdot \log x\,dx = \log x \cdot \frac{x^5}{5} - \int \frac{1}{x} \cdot \frac{x^5}{5} dx$ **1/2**

$= \frac{x^5 \cdot \log x}{5} - \frac{x^5}{25} + c$ **1/2**

OR

Find $\int \frac{2x}{\sqrt[3]{x^2+1}} dx$

Ans: Let, $x^2 + 1 = t$ $\therefore$ $2x\,dx = dt$ **1/2**

$\int \frac{2x}{\sqrt[3]{x^2+1}} dx = \int \frac{1}{\sqrt[3]{t}} dt = \int t^{-1/3} dt = \frac{3}{2} t^{2/3} + c$

$= \frac{3}{2}\left(x^2+1\right)^{2/3} + c$ **1/2**

18. Evaluate $\int_1^3 |2x-1|\,dx$.

Ans: $\int_1^3 |2x-1|dx = \int_1^3 (2x-1)\,dx = \left[\frac{1}{4}(2x-1)^2\right]_1^3$ **1/2**

$= 6$ **1/2**

19. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

Ans: $\frac{^{26}C_1 \times {}^{26}C_1}{^{52}C_2} = \frac{26}{51}$ **1/2+1/2**

20. Find $\int \frac{dx}{\sqrt{9-4x^2}}$.

Ans: $\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{3^2-(2x)^2}}$ **1/2**

$= \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + c$ **1/2**

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. Prove that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \le x \le 1$

Ans: Put $x = \cos\theta \Leftrightarrow \theta = \cos^{-1}x$ **1/2**

L.H.S. $= \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$= \sin^{-1}(2\cos\theta \sin\theta) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\cos^{-1}x =$ R.H.S. **$1\frac{1}{2}$**

OR

Consider a bijective function $f : R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R_+ is the set of all positive real numbers. Find the inverse function of f.

Ans: Let $y = f(x) = 16x^2 + 24x + 7 = (4x+3)^2 - 2$ **1**

$\Rightarrow f^{-1}(y) = x = \frac{\sqrt{y+2}-3}{4}$ **1**

22. If $x = at^2, y = 2at$, then find $\frac{d^2y}{dx^2}$.

Ans: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$ **1**

$\frac{d^2y}{dx^2} = -\frac{1}{t^2} \cdot \frac{dt}{dx} = -\frac{1}{t^2} \cdot \frac{1}{2at} = -\frac{1}{2a\,t^3}$ **1**

23. Find the points on the curve $y = x^3 - 3x^2 - 4x$ at which the tangent lines are parallel to the line $4x + y - 3 = 0$.

Ans: $\frac{dy}{dx} = -4 \Rightarrow 3x^2 - 6x - 4 = -4$ **1**

$\Rightarrow 3x(x - 2) = 0 \quad \therefore \quad x = 0 \,;\, x = 2$ **1/2**

Points on the curve are (0, 0), (2, –12) **1/2**

24. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

Ans: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix} = 24\hat{i} - 24\hat{j} - 12\hat{k}$ **1**

Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$ **1**

OR

Find the volume of the parallelopiped whose adjacent edges are represented by $2\vec{a}, -\vec{b}$ and $3\vec{c}$, where $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$

Ans: Volume of the parallelopiped = $\begin{vmatrix} 2 & -2 & 4 \\ -3 & -4 & 5 \\ 6 & -3 & 9 \end{vmatrix}$ **1**

$= |-24| = 24$ **1**

25. Find the value of k so that the lines x = –y = kz and x – 2 = 2y + 1 = –z + 1 are perpendicular to each other.

Ans: The lines, $\frac{x}{1} = \frac{y}{-1} = \frac{z}{\frac{1}{k}}$ and $\frac{x-2}{1} = \frac{y+\frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$ **1**

are perpendicular $\therefore \quad 1 - \frac{1}{2} - \frac{1}{k} = 0 \Rightarrow k = 2$ **1**

26. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

Ans: Probability of green signal on crossing $X = \frac{30}{100} = \frac{3}{10}$

Probability of not a green signal on crossing $X = 1 - \frac{3}{10} = \frac{7}{10}$ **1**

Probability of a green signal on X on two concecutative days out of three

$= \frac{3}{10} \times \frac{3}{10} \times \frac{7}{10} + \frac{7}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{63}{500}$ **1**

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) iff ad = bc for all a, b, c, d ∈ N. Show that R is an equivalence relation.

Ans: Reflexive: For any (a, b) ∈ N × N

$a \cdot b = b \cdot a$

∴ (a, b) R (a, b) thus R is reflexive **1**

Symmetric: For (a, b), (c, d) ∈ N × N

(a, b) R (c, d) $\Rightarrow a \cdot d = b \cdot c$

$\Rightarrow c \cdot b = d \cdot a$

⇒ (c, d) R (a, b) ∴ R is symmetric **$1\frac{1}{2}$**

Transitive : For any (a, b), (c, d), (e, f), ∈ N × N

(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow a \cdot d = b \cdot c$ and $c \cdot f = d \cdot e$

$\Rightarrow a \cdot d \cdot c \cdot f = b \cdot c \cdot d \cdot e \Rightarrow a \cdot f = b \cdot e$

∴ (a, b) R (e, f), ∴ R is transitive **$1\frac{1}{2}$**

∴ R is an equivalance Relation

28. If $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\frac{dy}{dx}$.

Ans. Let $u = (\cos x)^x \Rightarrow y = e^{x^2 \cdot \cos x} + u$

$\therefore \frac{dy}{dx} = e^{x^2 \cdot \cos x}(2x \cdot \cos x - x^2 \cdot \sin x) + \frac{du}{dx}$ **$1\frac{1}{2}$**

$\log u = \log (\cos x)^x \Rightarrow \log u = x \cdot \log(\cos x)$

Differentiate w.r.t. "x"

$$\frac{1}{u}\frac{du}{dx} = \log(\cos x) - x\tan x \Rightarrow \frac{du}{dx} = (\cos x)^x\{\log(\cos x) - x\tan x\}$$ **2**

Therefore,

$$\frac{dy}{dx} = e^{x^2\cdot\cos x}(2x\cdot\cos x - x^2\cdot\sin x) + (\cos x)^x\{\log(\cos x) - x\tan x\}$$ **1/2**

29. Find $\int \sec^3 x dx$.

Ans. $\int \sec^3 x dx = \int \sec x\cdot\sec^2 x dx = \int\sqrt{1+\tan^2 x}\cdot\sec^2 x dx$ **$1\frac{1}{2}$**

(Put $\tan x = t$; $\sec^2 x\, dx = dt$) **1/2**

$$= \int\sqrt{1+t^2}\,dt$$

$$= \frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\log\left|t+\sqrt{1+t^2}\right| + c$$ **$1\frac{1}{2}$**

$$= \frac{\sec x\cdot\tan x}{2} + \frac{1}{2}\log|\tan x + \sec x| + c$$ **1/2**

30. Find the general solution of the differential equation $ye^y dx = (y^3 + 2xe^y)dy$.

Ans. $y\cdot e^y dx = \left(y^3 + 2xe^y\right)dy \Rightarrow y\cdot e^y\frac{dy}{dx} = y^3 + 2xe^y$

$$\therefore \frac{dx}{dy} - \frac{2}{y}x = y^2\cdot e^{-y}$$ **1**

I.F. (Integrating factor) $= e^{-2\int\frac{1}{y}dy} = e^{-2\log y} = e^{\log\frac{1}{y^2}} = \frac{1}{y^2}$ **1**

$\therefore$ Solution is

$$x\cdot\frac{1}{y^2} = \int y^2\cdot e^{-y}\cdot\frac{1}{y^2}dy + c = \int e^{-y}dy + c$$ **1**

$$\Rightarrow \frac{x}{y^2} = -e^{-y} + c \quad \text{or} \quad x = -y^2e^{-y} + cy^2$$ **1**

OR

Find the particular solution of the differential equation

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right),\ \text{given that } y = \frac{\pi}{4} \text{ at } x = 1.$$

Ans. The differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x},\ \text{let } y = vx\ \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$ **1**

$$\Rightarrow \quad v + x\frac{dv}{dx} = v - \tan v \;\Rightarrow\; \cot v \, dv = -\frac{1}{x}dx$$

Integrate both sides

$$\log \sin v = -\log|x| + \log c \Rightarrow \log \sin\frac{y}{x} = \log\frac{c}{x}$$ 2

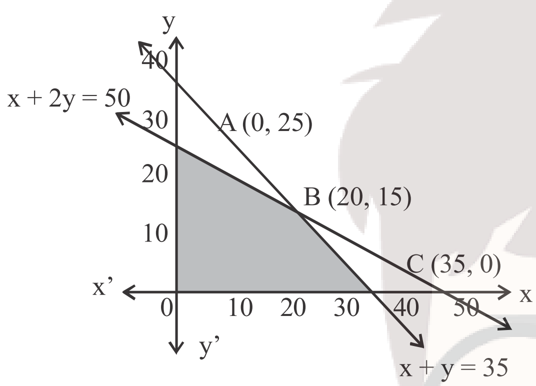
$$\Rightarrow \quad x \cdot \sin\frac{y}{x} = c \text{, Put } y = \frac{\pi}{4} \text{ and } x = 1$$

$$\Rightarrow \quad \sin\frac{\pi}{4} = c \text{ or } c = \frac{1}{\sqrt{2}}$$ 1/2

$\therefore$ Particular solution is $x \cdot \sin\left(\frac{y}{x}\right) = \frac{1}{\sqrt{2}}$ 1/2

31. A furniture trader deals in only two items – chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1000 and a table costs him ₹ 2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

Ans.



Let No. of chairs = x, No. of tables = y

Then L.P. P. is:

Maximize (Profit) : $Z = 150x + 250y$ 1

Subject to : $x + y \leq 35$

$1000x + 2000y \leq 50000 \Rightarrow x + 2y \leq 50$ 1

$x, y \geq 0$

Correct graph $1\frac{1}{2}$

Corner:	Value of Z
A(0, 25)	₹ 6250
B(20, 15)	₹ 6750 (Max)
C(35, 0)	₹ 5250

$\therefore$ Max (z) = ₹ 6750

Number of chairs = 20, Tables = 15 1/2

32. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

Ans. E_1 = Event that the ball transfered from Bag I is Black

E_2 = Event that the ball transfered from Bag I is Red 1/2

A = Event that the ball drawn from Bag II is Black

$$P(E_1) = \frac{5}{8};\ P(E_2) = \frac{3}{8};\ P\left(\frac{A}{E_1}\right) = \frac{4}{8} = \frac{1}{2};\ P\left(\frac{A}{E_2}\right) = \frac{3}{8}$$ 2

Required Probability:

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)}=\frac{\frac{5}{8}\cdot\frac{1}{2}}{\frac{5}{8}\cdot\frac{1}{2}+\frac{3}{8}\cdot\frac{3}{8}}=\frac{20}{29}$$ $1\frac{1}{2}$

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

Ans. Let X = No. of white balls = 0, 1, 2

X :	0	1	2	1/2
P(X) :	$\frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}=\frac{7}{15}$	$3\times\frac{8}{10}\times\frac{7}{9}\times\frac{2}{8}=\frac{7}{15}$	$3\times\frac{2}{10}\times\frac{1}{9}\times\frac{8}{8}=\frac{1}{15}$	$1\frac{1}{2}$
$X\cdot P(X)$:	0	$\frac{7}{15}$	$\frac{2}{15}$	1/2
$X^2P(X)$:	0	$\frac{7}{15}$	$\frac{4}{15}$	

Mean = $\sum XP(X)=\frac{9}{15}=\frac{3}{5}$ 1/2

Variance = $\sum X^2P(x)-\left[\sum X\,P(X)^2\right]=\frac{11}{15}-\left[\frac{3}{5}\right]^2=\frac{28}{75}$ 1

SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33. If $A=\begin{bmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations:

$x+2y-3z=6$

$3x+2y-2z=3$

$2x-y+z=2$

Ans. $|A|=7$; $\text{adj}(A)=\begin{bmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{bmatrix}$; $A^{-1}=\frac{1}{|A|}\text{adj}\,A=\frac{1}{7}\begin{bmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{bmatrix}$ $1+1\frac{1}{2}+\frac{1}{2}$

The system of equations in Matrix form can be written as :

$$A \cdot X = B, \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$ **1**

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$ **1**

$\therefore$ x = 1, y = –5, z = –5 **1**

OR

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

Ans. $$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ca \\ a^2+b^2 & c^2 & ab \end{vmatrix} \quad (C_1 \to C_1 - 2C_3)$$ **1**

$$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix} \quad (C_1 \to C_1 + C_2)$$ **1**

$$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b^2-a^2 & ca-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix} \quad (R_2 \to R_2 - R_1, R_3 \to R_3 - R_1)$$ **2**

$$= (b-a)(c-a)\begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b+a & -c \\ 0 & c+a & -b \end{vmatrix}$$ **1**

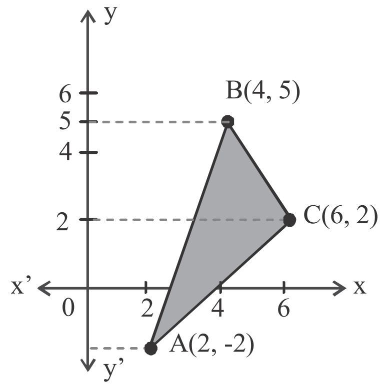
Expand along C_1

$$= \left(a^2+b^2+c^2\right)(b-a)(c-a)\left(-b^2-ab+c^2+ac\right)$$

$$= (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$ **1**

34. Using integration, find the area of the region bounded by the triangle whose vertices are (2, –2), (4,5) and (6,2).

Ans.



Let A(2, –2) ; B(4, 5) ; C(6, 2)

Equations of the lines

$$AB : x = \frac{2}{7}(y+9)$$

$$BC : x = -\frac{2}{3}(y-11)$$

$$AC : x = y + 4$$

$1\frac{1}{2}$

Correct graph 1/2

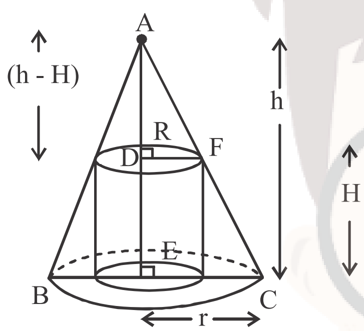
$$ar(\Delta ABC) = \int_{-2}^{2}(y+4)dy + \left(\frac{-2}{3}\right)\int_{2}^{5}(y-11)dy - \int_{-2}^{5}\frac{2}{7}(y+9)\,dy$$ 2

$$= \frac{1}{2}\left[(y+4)^2\right]_{-2}^{2} - \frac{1}{3}\left[(y-11)^2\right]_{2}^{5} - \frac{1}{7}\left[(y+9)^2\right]_{-2}^{5}$$ $1\frac{1}{2}$

$$= 16 + 15 - 21 = 10$$ 1/2

35. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

Ans.



Let H = Height of cylinder

R = Radius of cylinder

Volume of cone $= \frac{\pi}{3}r^2h$ 1/2

V = Volume of cylinder $= \pi R^2 H$ 1/2

$$\Delta ADF \sim \Delta AEC \Rightarrow \frac{h-H}{h} = \frac{R}{r} \Rightarrow R = \frac{r}{R}(h-H)$$ 1

$$\therefore V = \pi \cdot H \cdot \frac{r^2}{h^2}(h-H)^2 = \frac{\pi r^2}{h^2}\left(H^3 - 2hH^2 + Hh^2\right)$$ 1

$$V'(H) = \frac{\pi r^2}{h^2}\left(3H^2 - 4hH + h^2\right),\ V'(h) = 0 \Rightarrow H = \frac{h}{3}$$ 1+1

$$V''(H) = \frac{\pi r^2}{h^2}(6H - 4h),\ V''\left(H = \frac{h}{3}\right) = \frac{\pi r^2}{h^2}(-2h) < 0$$ 1/2

$$\therefore\ V \text{ is max iff } H = \frac{h}{3} \text{ and } R = \frac{2r}{3}$$

$$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{3\pi R^2 H}{\pi r^2 h} = 3 \cdot \frac{4r^2}{9} \cdot \frac{h}{3} \cdot \frac{1}{r^2 h} = \frac{4}{9}$$ **1/2**

36. Find the equation of the plane that contains the point A(2,1,–1) and is perpendicular to the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 2$. Also find the angle between the plane thus obtained and the y-axis.

Ans. Let equation of the required plane be:

$a(x - 2) + b(y - 1) + c(z + 1) = 0$ **$1\frac{1}{2}$**

Also : $2a + b - c = 0$

$a + 2b + c = 0$

Solving: $\frac{a}{3} = \frac{b}{-3} = \frac{c}{3} = k \Rightarrow a = 3k, b = -3k, c = 3k$ **$1\frac{1}{2}$**

$\therefore$ Equation of plane is : $3k(x - 2) - 3k(y - 1) + 3k(z + 1) = 0$

$\Rightarrow x - y + z = 0$ **$1\frac{1}{2}$**

Let angle between y-axis and plane = θ

then, $\sin\theta = \left|\frac{0-1+0}{\sqrt{1+1+1}}\right| = \left|\frac{-1}{\sqrt{3}}\right| \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **$1\frac{1}{2}$**

OR

Find the distance of the point P(–2, –4, 7) from the point of intersection Q of the line $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$. Also write the vector equation of the line PQ.

Ans. General point on line is: $\vec{r} = (3 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ **1**

For the point of intersection:

$\left[(3 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (6 + 2\lambda)\hat{k}\right] \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 6$ **1**

$\Rightarrow 3 + 2\lambda + 2 + \lambda + 6 + 2\lambda = 6 \Rightarrow \lambda = -1$ **1**

$\therefore Q(\hat{i} - \hat{j} + 4\hat{k}) = Q(1, -1, 4)$ **1**

$PQ = 3\sqrt{3}$, equation of the line PQ : $\vec{r} = -2\hat{i} - 4\hat{j} + 7\hat{k} + \mu\left(3\hat{i} + 3\hat{j} - 3\hat{k}\right)$ **1+1**