ALGEBRAIC EQUATIONS

Algebraic equations are polynomial equations. In examination, generally equations of 1 degree, 2 degree or 3 degrees are asked.

Linear Equation
Polynomial equations with degree 1 i.e., $ax + c = 0$ are called as linear equations. Some examples of linear equations are as follows –

$2x + 3y = 4$

$x + y + z = 10$

Q1. In this question two equations numbered I and II are given. You have to solve both the equations and find out the relation between $x$ and $y$.

I. $5x = 7y + 21$

II. $11x + 4y + 109 = 0$

Solution:

I. $2x + 3y = 13$  \hspace{1cm} (1)

II. $3x + 2y = 12$ \hspace{1cm} (2)

$(3 \times \text{Equation 2}) - (2 \times \text{Equation 1})$ gives us

$\Rightarrow 5x = 10$

$\Rightarrow x = 2$

Putting value of $x$ in equation 1, we get $y$

$= 3$

Hence, $x < y$.

Q2. In the given question, two equations numbered I and II are given. You have to solve both the equations and mark the appropriate answer-

I. $4x + 5y = 14$

II. $2x + 3y = 5$

Solution:

$4x + 5y = 14$ \hspace{1cm} (1)

$2x + 3y = 5$ \hspace{1cm} (2)

On multiplying equation (2) by 2.

$4x + 6y = 10$ \hspace{1cm} (3)

Subtracting equation (1) from equation (3),
\[ y = -4 \]
\[ x = 1 \text{ (on putting value of } y \text{ in the above equation)} \]
\[ \therefore x > y. \]

**Quadratic Equation**

Polynomial equations with degree 2 i.e., \( ax^2 + bx + c = 0 \) are called quadratic equations. Some examples of quadratic equations are as follows –

\[ x^2 + 2x + 3 = 0 \]
\[ y^2 - 3y + 4 = 0 \]

**Methods to solve quadratic equation**

1) **Factorisation method**

In it quadratic equation \( ax^2 + bx + c = 0 \) is factorized as \((x - \alpha)(x - \beta) = 0\) and then equation is solved to get \( x = \alpha \) or \( x = \beta \).

Q3. **Solve quadratic equation**

\[ x^2 - 2x - 15 = 0 \]
**Solution:**

\[ x^2 - 2x - 15 = 0 \]
\[ \Rightarrow x^2 - 5x + 3x - 15 = 0 \]
\[ \Rightarrow x(x - 5) + 3(x - 5) = 0 \]
\[ \Rightarrow (x + 3)(x - 5) = 0 \]
\[ \Rightarrow x + 3 = 0 \text{ or } x - 5 = 0 \]
\[ \Rightarrow x = -3 \text{ or } x = 5 \]

2) **Sridharachrya’s method**

In it quadratic equation \( ax^2 + bx + c = 0 \) is solved by using formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Which gives us \( x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \) or \( x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \)
Q4. Solve quadratic equation \( x^2 - 2x - 15 = 0 \)

Solution:
\[
X1= \frac{-b + \sqrt{b^2-4ac}}{2a} = 5
\]
\[
X2= \frac{-b - \sqrt{b^2-4ac}}{2a} = -3
\]

Q5. In the following question two equations are given. You have to solve both the equations and find the relation between \( x \) and \( y \).

I. \( x^2 = 625 \)

II. \( y = \sqrt{625} \)

Solution:
We will solve both the equations separately. \( x^2 = 625 \)
\[ \Rightarrow x = +25 \text{ or } -25 \text{ (we will consider two values of } x \text{ because of } x^2) \]
\[ y = \sqrt{625} \]
\[ \Rightarrow y = 25 \text{ (The square root is used to refer to only the positive square root i.e. } \sqrt{x^2} = |x| \).
\[ \therefore x \leq y \]

Q6. In the given question, two equations numbered I and II are given. You have to solve both the equations and find the relation between \( m \) and \( n \).

I) \( m = \sqrt{324} \)

II) \( n^2 - 16n - 36 = 0 \)

Solution:

<table>
<thead>
<tr>
<th>Value of ( m )</th>
<th>Value of ( n )</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>18</td>
<td>( m = n )</td>
</tr>
<tr>
<td>18</td>
<td>-2</td>
<td>( m &gt; n )</td>
</tr>
</tbody>
</table>

\[ m = \sqrt{324} \]
\[ m = 18 \]
\[ n^2 - 16n - 36 = 0 \]
\[ \Rightarrow n^2 - 18n + 2n - 36 = 0 \]
\( \Rightarrow n (n - 18) + 2(n - 18) = 0 \)
\( \Rightarrow (n - 18) (n + 2) = 0 \)
\( \Rightarrow n = (18, -2) \)
Hence, \( m \geq n \).

**Cubic Equation**

Polynomial equations with degree 3 i.e., \( ax^3 + bx^2 + cx + d = 0 \) are called as cubic equations.

Some examples of cubic equations are as follows –

\[ x^3 + 2x^2 + 3x + 4 = 0 \]
\[ 2x^3 + 12x^2 + 30x + 48 = 0 \]
\[ X = \sqrt[3]{625} \]

**Q7.** In the given question, two equations numbered I and II are given. You have to solve both the equations and mark the appropriate answer

\[ X = \sqrt[3]{15625} \]
\[ y^2 = 625 \]

**Solution:**

\[ X = \sqrt[3]{15625} = 25 \]
\[ Y = 625 \]
\[ Y = (+25, -25) \]
\[ Y \leq X \]