Sample Paper 8

ANSWERKEY																			
1	(c)	2	(d)	3	(d)	4	(d)	5	(b)	6	(d)	7	(c)	8	(a)	9	(c)	10	(c)
11	(d)	12	(d)	13	(c)	14	(a)	15	(a)	16	(b)	17	(b)	18	(b)	19	(c)	20	(d)
21	(b)	22	(a)	23	(b)	24	(d)	25	(a)	26	(b)	27	(d)	28	(b)	29	(b)	30	(a)
31	(a)	32	(d)	33	(b)	34	(d)	35	(a)	36	(a)	37	(b)	38	(c)	39	(d)	40	(b)
41	(d)	42	(a)	43	(d)	44	(c)	45	(b)	46	(c)	47	(a)	48	(d)	49	(a)	50	(b)

SOLUTIONS

1. **(c)** Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr then speed of boat in downstream is (x + y) km/hr and the speed of boat upstream is (x – y) km/hr.

Ist case : Distance covered upstream = 12 km

$\therefore$ time = $\frac{12}{x-y}$hr

Distance covered downstream = 40 km

$\therefore$ time = $\frac{40}{x+y}$hr

Total time is 8 hr $\therefore$ $\frac{12}{x-y}+\frac{40}{x+y}=8$ ...(i)

IInd case :

Distance covered upstream = 16 km

$\therefore$ time = $\frac{16}{x-y}$hr

Distance covered downstream

= 32 km $\therefore$ time = $\frac{32}{x+y}$hr

Total time taken = 8 hr

$\therefore$ $\frac{16}{x-y}+\frac{32}{x+y}=8$ ...(ii)

Solving (i) and (ii), we get,

x = speed of boat in still water = 6 km/hr,

y = speed of stream = 2 km/hr

2. **(d)** $A\left(\sqrt{3}+1,\sqrt{2}-1\right), B\left(\sqrt{3}-1,\sqrt{2}+1\right)$

$$AB=\sqrt{\left(\sqrt{3}-1-\sqrt{3}-1\right)^2+\left(\sqrt{2}+1-\sqrt{2}+1\right)^2}$$

$$=\sqrt{(-2)^2+(2)^2}$$

$$=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

3. **(d)** isosceles and similar

4. **(d)** Let us first find the H.C.F. of 210 and 55.

Applying Euclid's division lemma on 210 and 55, we get

210 = 55 × 3 + 45 ..... (i)

Since, the remainder 45 $\neq$ 0. So, we now apply division lemma on the divisor 55 and the remainder 45 to get

55 = 45 × 1 + 10 ..... (ii)

We consider the divisor 45 and the remainder 10 and apply division lemma to get

45 = 4 × 10 + 5 ..... (iii)

We consider the divisor 10 and the remainder 5 and apply division lemma to get

10 = 5 × 2 + 0 ..... (iv)

We observe that the remainder at this stage is zero. So, the last divisor i.e., 5 is the H.C.F of 210 and 55.

$\therefore$ 5 = 210 × 5 + 55y $\Rightarrow$ $y=\frac{-1045}{55}=-19$

5. **(b)** There are a total of six digits (1, 2, 2, 3, 4, 6) out of which four are even (2, 2, 4, 6)

So, required probility = $\frac{4}{6}=\frac{2}{3}$

6. **(d)**

(a) It is quadratic polynomial

[$\because$ the graph meets the x-axis in two points]

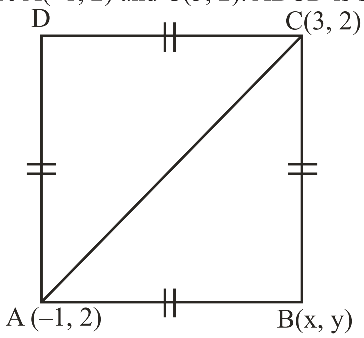
(b) It is a quadratic polynomial

[$\because$ the graph meets the x-axis in two points]

(c) It is a quadratic polynomial
[$\because$ the graph meets the x-axis in two points]

(d) It is a not quadratic polynomial
[$\because$ the graph meets the x-axis in one point]

7. **(c)** Let ABCD be a square and two opposite vertices of it are A(–1, 2) and C(3, 2). ABCD is square.



$\Rightarrow$ AB = BC

$\Rightarrow$ $AB^2 = BC^2$

$\Rightarrow$ $(x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$\Rightarrow$ $x^2 + 2x + 1 = x^2 - 6x + 9$

$\Rightarrow$ $2x + 6x = 9 - 1 = 8$

$\Rightarrow$ $8x = 8 \Rightarrow x = 1$

ABC is right Δ at B, then

$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow$ $(3+1)^2 + (2-2)^2$

$= (x+1)^2 + (y-2)^2 + (x-3)^2 + (y-2)^2$

$\Rightarrow$ $16 = 2(y-2)^2 + (1+1)^2 + (1-3)^2$

$\Rightarrow$ $16 = 2(y-2)^2 + 4 + 4$

$\Rightarrow$ $(y-2)^2 = 4 \Rightarrow y - 2 = \pm 2$

$\Rightarrow$ y = 4 and 0

i.e., when x = 1 then y = 4 and 0

Coordinates of the opposite vertices are :

B(1, 0) or D(1, 4)

8. **(a)**

9. **(c)** In right angled triangle POR

$PR^2 = PO^2 + OR^2 = (6)^2 + (8)^2 = 36 + 64 = 100$

$\therefore$ PR = 10 cm

Again in right angled triangle

PQR, $QR^2 = (26)^2 = 676$

$PQ^2 + PR^2 = (24)^2 + (10)^2 = 576 + 100 = 676$

$\therefore$ $QR^2 = PQ^2 + PR^2$

$\therefore$ ΔPQR is a right angled triangle with right angle at P.

i.e., $\angle QPR = 90°$

10. **(c)** $\dfrac{\pi r_1^2}{\pi r_2^2} = \dfrac{25}{16}$

$\Rightarrow \dfrac{r_1}{r_2} = \dfrac{5}{4}$

$\Rightarrow \dfrac{2\pi r_1}{2\pi r_2} = \dfrac{5}{4} = \dfrac{5\times125}{4\times125} = \dfrac{625}{500}$

11. **(d)** For any rational number $\dfrac{p}{q}$, where prime factorization of q is of the form $2^n.5^m$, where n and m are non-negative integers, the decimal representation is terminating.

12. **(d)** Let the y-axis divides the line segment joining (4, 5) and (– 10, 2) in the ratio k : 1.

x coordinate will be zero on y-axis.

We know that the coordinates of the point dividing the line segment joining (x_1, y_1) and (x_2, y_2) in the ratio m : n are given by $\left(\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right)$

Here, x coordinate of the point dividing the line segment joining (4, 5) and (– 10, 2) is equal to zero.

So, $\dfrac{k\times(-10)+1\times4}{k+1} = 0$

$\Rightarrow -10k = -4 \Rightarrow k = \dfrac{2}{5}$

Therefore, the line segment joining (4, 5) and (– 10, 2) is cut by the y-axis in the ratio 2 : 5.

13. **(c)** There are 4 cards of king and 4 cards of Jack n(S) = 52, n(E) = 4 + 4 = 8

$P(E) = \dfrac{n(E)}{n(S)} = \dfrac{8}{52} = \dfrac{2}{13}$

14. **(a)** The graph of $y = ax^2 + bx + c$ is a parabola open upward if a > 0. So, for $y = x^2 - 6x + 9$, a = 1 > 0, the graph is a parabola open upward.

15. **(a)** If 1, 1 and 2 are sides of a right triangle then sum of squares of any two sides is equal to square of third side.

Case 1 $(1)^2 + (1)^2 = 2 \neq (2)^2$

Case 2 $(1)^2 + (2)^2 = 1 + 4 = 5 \neq (1)^2$

Case 3 $(2)^2 + (1)^2 = 5 \neq (1)^2$

16. **(b)** $n(S) = 6 \times 6 = 36$

E = {(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4), (5, 6), (6, 5)}

n(E) = 10

$P(E) = \dfrac{n(E)}{n(S)} = \dfrac{10}{36} = \dfrac{5}{18}$

17. **(a)** $\sqrt{2}$ is not a rational number. It can't be expressed in the fractional form.

18. **(b)** Area $= \dfrac{\theta}{360°}\times\pi r^2 = \dfrac{30°}{360°}\times\pi(7)^2 = \dfrac{49\pi}{12}$

19. **(c)** Inconsistent system

20. **(d)** S = {1, 2, 3, 4......, 25}

n(S) = 25

E = {2, 3, 5, 7, 11, 13, 17, 19, 23}

n(E) = 9

$\therefore \quad P(E) = \dfrac{9}{25}$

21. **(b)** $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}=\frac{\frac{3abc-b^3}{a^3}}{\left(\frac{c}{a}\right)^3}$

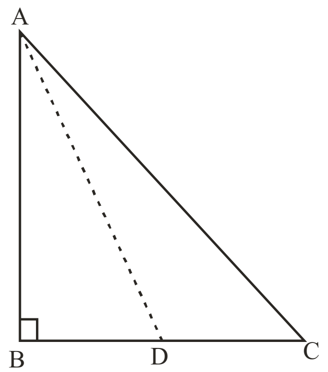
$\Rightarrow \frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{3abc-b^3}{c^3}$

22. **(a)** S = {S, M, T, W, Th, F, Sa}
n(S) = 7
A non-leap year contains 365 days,
i.e., 52 weeks + 1 day.
E = {Sa}
n(E) = 1
$\therefore$ $P(E)=\frac{n(E)}{n(S)}=\frac{1}{7}$

23. **(b)** Let the given points be A(4, 3) and B(x, 5)
Since A and B lies on the circumference of a circle with centre O(2, 3), we have
OA = OB
$\Rightarrow$ $OA^2 = OB^2$
$\Rightarrow$ $(4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$
$\Rightarrow$ $4 + 0 = (x-2)^2 + 4$
$\Rightarrow$ $(x-2)^2 = 0 \Rightarrow x = 2$

24. **(d)** Since
$\frac{13}{125}=\frac{13}{53}=\frac{13\ 2^3}{(2)^3(5)^3}=\frac{104}{1000}=0.104$

25. **(a)**



Given : A ΔABC in which $\angle$B = 90° and D is the midpoint of BC.
Join AD.
In ΔABC, $\angle$B = 90°.
$\therefore$ $AC^2 = AB^2 + BC^2$ ....(i)
[by Pythagoras' theorem]
In ΔABD, $\angle$B = 90°
$\therefore$ $AD^2 = AB^2 + BD^2$ ....(ii)
[by Pythagoras' theorem]
$\Rightarrow$ $AB^2 = (AD^2 - BD^2)$.
$\therefore$ $AC^2 = (AD^2 - BD^2) + BC^2$ [using (i)]
$\Rightarrow$ $AC^2 = AD^2 - CD^2 + (2CD)^2$
[$\because$ BD = CD and BC = 2CD]
$\Rightarrow$ $AC^2 = AD^2 + 3CD^2$
Hence, $AC^2 = AD^2 + 3CD^2$

26. **(b)** $\frac{3}{x}+\frac{4}{y}=1$ ...(i) $\frac{4}{x}+\frac{2}{y}=\frac{11}{12}$ ...(ii)
Multiplying (ii) by 2
$\Rightarrow \frac{8}{x}+\frac{4}{y}=\frac{22}{12}$ ...(iii)
Subtracting (i) from (iii) $\Rightarrow \frac{5}{x}=\frac{10}{12}$
$\therefore$ $x=\frac{5\times 12}{10}=6$
Substituting x = 6 in (i)
$\Rightarrow \frac{3}{6}+\frac{4}{y}=1 \Rightarrow \frac{4}{y}=1-\frac{1}{2}=\frac{1}{2}$ $\therefore$ y = 8.
Hence, x = 6 and y = 8

27. **(d)**

28. **(b)** n(S) = 6× 6 = 36, E = {(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)}
n(E) = 12
$P(E)=\frac{n(E)}{n(S)}=\frac{12}{36}=\frac{1}{3}$

29. **(b)** Put x + 1 = 0 or x = – 1 and x + 2 = 0 or x = – 2 in p (x)
Then, p (–1) = 0 and p (–2) = 0
$\Rightarrow$ p (–1) =
$\Rightarrow$ $-1+3+2\alpha+\beta=0 \Rightarrow \beta=-2\alpha-2$ .... (i)
$p(-2)=(-2)^3+3(-2)^2-2\alpha(-2)+\beta=0$
$\Rightarrow$ $-8+12+4\alpha+\beta=0 \Rightarrow \beta=-4\alpha-4$ .... (ii)
By equalising both of the above equations, we get
$-2\alpha-2=-4\alpha-4$
$\Rightarrow$ $2\alpha=-2$ $\Rightarrow$ $\alpha=-1$
put α in eq. (i)
$\Rightarrow$ $\beta=-2(-1)-2=2-2=0$
Hence, $\alpha=-1,\ \beta=0$

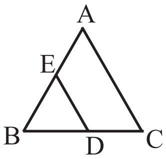
30. **(a)** Let D be the window at a height of 9m on one side of the street and E be the another window at a height of 12 m on the other side.
In ΔADC
$AC^2 = 152 - 92$
= 225 – 81
AC = 12 m
In ΔECB
$CB^2 = 15^2 - 12^2$
= 225 – 144
CB = 9 m
Width of the street = (12 + 9)m = 21 m

31. **(a)**

32. **(d)** All equilateral triangles are similar
$\therefore$ Δ ABC~ Δ EBD
$\Rightarrow \frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta BDE}=\frac{BC^2}{BD^2}$

$$= \frac{2BD^2}{BD^2} = \frac{4}{1}$$

$\Rightarrow$ Area (DABC) : Area (DBBE) = 4 : 1



33. **(b)** As A lies on x-axis and B lies on y-axis, so their coordinates are (x, 0) and (0, y), respectively. Then,

$\frac{x+0}{2} = 4$ and $\frac{0+y}{2} = -3 \Rightarrow$ x = 8 and y = – 6

Hence, the points A and B are (8, 0) and (0, –6).

34. **(d)** If 6^x ends with 5, then 6^x would contain the prime 5. But $6^x = (2 \times 3)^x = 2^x \times 3^x$.

$\Rightarrow$ The only prime numbers in the factorization of 6x are 2 and 3.

$\therefore$ By uniqueness of fundamental theorem, there are no primes other than 2 & 3 in 6^x. So, 6^x will never end with 5.

35. **(a)** (a) Area $= \frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{22}{7} \times (6)^2 = \frac{132}{7}$ cm^2

(b) Area of minor sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{60°}{360°} \times \frac{22}{7} \times 14 \times 14 = 102.57$ cm^2

Area of major sector

= Area of circle – Area of minor sector

$= \frac{22}{7}(14)^2 - 102.57$

= 615.44 – 102.57 = 512.87 cm^2

(c) $\frac{C}{A} = \frac{2\pi(5)}{\pi(5)^2} = \frac{2}{5}$

(d) Given, $\left(\frac{\theta}{360°}\right) 2\pi r = 22$

$\therefore$ Area of sector $= \left(\frac{\theta}{360°}\right)\pi r^2 = \left(\frac{\theta}{360°}\right)\frac{\pi r}{2}(2r)$

$= \left(\frac{\theta}{360°}\right) 2\pi r \left(\frac{r}{2}\right) = \frac{22 \times 6}{2} = 66$ cm^2

36. **(a)** Let AD = 5x cm and DB = 4x cm.

Then,

AB = (AD + DB) = (5x + 4x) cm = 9x cm.

In ΔADE and ΔABC, we have

$\angle$ADE = $\angle$ABC (corres. $\angle$s)

$\angle$AED = $\angle$ACB (corres, $\angle$s)

$\therefore$ $\Delta ADE \sim \Delta ABC$ [by AA-similarity]

$\Rightarrow \frac{DE}{BC} = \frac{AD}{AB} = \frac{5x}{9x} = \frac{5}{9}$

$\Rightarrow \frac{DE}{BC} = \frac{5}{9}$ ...(i)

In ΔDFE and ΔCFB, we have

$\angle$EDF = $\angle$BCF (alt. int. $\angle$s)

$\angle$DEF = $\angle$CBF (alt. int. $\angle$s)

$\therefore$ $\Delta DFE \sim \Delta CBF$

$\Rightarrow \frac{ar(\Delta DFE)}{ar(\Delta CFB)} = \frac{DE^2}{CB^2} = \frac{DE^2}{BC^2}$

$= \left(\frac{DE}{BC}\right)^2 = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$

$\Rightarrow$ ar (ΔDFE) : ar (ΔCFB) = 25 : 81

37. **(b)** Let f (x) = $6x^3 - 11x^2 + kx - 20$

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$$

$\Rightarrow 6.\frac{64}{27} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$

$\Rightarrow$ 128 – 176 + 12k – 180 = 0

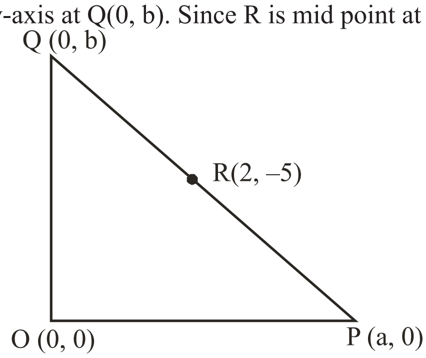
$\Rightarrow$ 12k + 128 – 356 = 0 12 k = 228

$\Rightarrow$ k = 19

38. **(c)** For coincident lines, $\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$

$\frac{1}{2} = \frac{1}{k} \Rightarrow k = 2$

39. **(d)** Let the line $\frac{x}{a} + \frac{y}{b} = 1$ meet x-axis at P(a, 0) and y-axis at Q(0, b). Since R is mid point at PQ.



$\therefore \frac{a+0}{2} = 2, \frac{0+b}{2} = -5$

$\therefore$ a = 4, b = –10

$\therefore$ P is (4, 0), Q is (0, –10)

40. **(b)** $\frac{21}{45} = \frac{21}{9 \times 5} = \frac{21}{3^2 \times 5}$

Clearly, 45 is not of the form 2m × 5n. So the decimal expansion of $\frac{21}{45}$ is non-terminating and repeating.

41. **(d)** Sample space = {HH, HT, TH, TT}

Total number of elementary events = 4

Favourable event E = HH

n (E) = 1

P(E) = $\frac{1}{4}$

42. **(a)** Favourable event E = {TH, HT}

n(E) = 2

P(E) = $\frac{2}{4} = \frac{1}{2}$

43. **(d)** Favourable event E = {TT}

n(E) = 1

$P(E) = \frac{1}{4}$

44. **(c)** At most one head

= {HT, TH, TT}

$P = \frac{3}{4}$

45. **(b)** At least one head

{HH, HT, TH}

$P = \frac{3}{4}$

46. **(c)** Area of minor sector OAPB

$$= \frac{\theta}{360} \times \pi r^2 = \frac{90}{360} \times 3.14 \times (10)^2$$

= 78.5 cm^2

47. **(a)** Area of minor segment APB

$$= \left(\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) r^2$$

$$= \left(3.14 \times \frac{90}{360} - \sin 45^\circ \cos 45^\circ\right) (10)^2$$

= 28.5 cm^2

48. **(d)** Area of the major sector OAQB

= Area of circle – Area of minor sector OAPB.

= (314 – 78.5)cm^2 = 235.5 cm^2

49. **(a)** Area of major segment AQB

= Area of the circle – Area of the minor segment APB

= (3.14 × 10 × 10 – 28.5) cm^2

= 285. 5 cm^2

50. **(b)** Length of arc APB

$$= \frac{90}{360} \times 2 \times \frac{22}{7} \times 10$$

= 15.71 cm