QUESTION PAPER CODE 65/3/MT

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

		Marks
1.	Order : 2, degree : 2, Product : 4	½ + ½ m
2.	$\frac{dy}{dx} = -\alpha A \sin \alpha x + \alpha B \cos \alpha x$	½ m
	$\frac{d^2y}{dx^2} = -\alpha^2 (A \cos \alpha x + B \sin \alpha x)$ $\frac{d^2y}{dx^2} + \alpha^2 y = 0$	½ m
3.	$\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$ or any other correct example	½ + ½ m
4.	Value $= 3$	1 m
5.	Writing dr's correctly	½ m
	D.C'S $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$	½ m
6.	Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{\lvert \vec{b} \rvert}$	½ m
	Projection $= \frac{5}{\sqrt{2}}$	½ m

SECTION - B

7. $\int x \sin^{-1}x \, dx$

$\frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}} \, dx$ 1 m

$\frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}} \, dx$ ½ m

$\Rightarrow \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int \sqrt{1-x^2} \, dx - \frac{1}{2}\int \frac{dx}{\sqrt{1-x^2}} \, dx$ 1 m

$\frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right) - \frac{1}{2}\sin^{-1}x + c$ 1½

or $\frac{x^2}{2}\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1}x + c$

8. $\int_0^2 \left(x^2 + e^{2x+1}\right) dx$

$h = \frac{2}{n}$ ½ m

$\int_0^2 \left(x^2 + e^{2x+1}\right) dx = \lim_{h\to 0} h \left[f(0) + f(0+h) + f(0+2h) + \ldots\ldots\right.$

$\left. + \ldots\ldots\ldots f(0+n-1)h \right]$ 1 m

$= \lim_{h\to 0} h \left[h^2 \left(1^2 + 2^2 + \ldots\ldots + (n-1)^2\right)\right.$

$\left. + e\left(1 + e^{2h} + e^{4h} + \ldots\ldots e^{2(n-1)h}\right)\right]$ 1 m

$$= \lim_{h\to 0} \frac{(nh)(nh-h)(2nh-h)}{6}$$ ½ m

$$+ \lim_{h\to 0} e.h.\left(\frac{e^{2nh}-1}{e^{2h}-1}\right)$$ ½ m

$$= \frac{8}{3} + \frac{(e^4-1)e}{2} = \frac{8}{3} + \frac{e^5-e}{2}$$ ½ m

OR

$$\int_0^{\pi} \frac{x \tan x \, dx}{\sec x \operatorname{cosec} x}$$

$$\int_0^{\pi} x \sin^2 x \, dx$$ 1 m

$$\text{Let } I = \int_0^{\pi} x \sin^2 x \, dx$$

$$= \int_0^{\pi} (\pi - x) \sin^2 (\pi - x) \, dx$$ ½ m

$$= \int_0^{\pi} (\pi - x) \sin^2 x \, dx$$ ½ m

$$2I = \pi \int_0^{\pi} \sin^2 x \, dx = \pi \int_0^{\pi} \frac{1-\cos 2x}{2} \, dx$$ ½ m

$$= \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi}$$ 1 m

$$= \frac{\pi^2}{2}$$

$$I = \frac{\pi^2}{4}$$ ½ m

9. $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$ ½ m

$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu$ ½ m

$x = 3\lambda + 1, y = -\lambda + 1, z = -1$ 1 m

$x = 2\mu + 4, y = 0, z = 3\mu - 1$

At the point of intersection

$\lambda = 1, \mu = 0$ 1 m

so $3\lambda + 1 = 4 = 2\mu + 4$ ½

Hence the lines are intersecting

Point of intersection is (4, 0, –1) ½ m

10. Coordinats of Q are $-3\mu + 1, \mu - 1, 5\mu + 2$ ½ m

D.R's of $\vec{PQ}$ $-3\mu - 2, \mu - 3, 5\mu - 4$ 1 m

as $\vec{PQ}$ is parallel to the plane $x - 4y + 3z = 1$

$1(-3\mu - 2) - 4(\mu - 3) + 3(5\mu - 4) = 0$ 1½ m

$\mu = \frac{1}{4}$ 1 m

OR

The D.R's of the line are 2, –6, 4 1 m

mid point of the line 2, 1, –1 1 m

The plane passes through (2, 1, –1) and is perpendicular to the

plane

eqn. : $2(x-2) - 6(y-1) + 4(z+1) = 0$

$x - 3y + 2z + 3 = 0$ 1 m

Vector from: $\vec{r} \cdot \left(\hat{i} - 3\hat{j} + 2\hat{k}\right) + 3 = 0$ 1 m

11. No's divisible by 6 .... 16 1m

No's divisible by 8 .... 12 1m

No's not divisible by 24 .... 20 1m

Required probabilty $= \frac{20}{100} = \frac{1}{5}$ 1 m

12. Let $x = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1 m

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$

$$\begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$ 1½ m

$a + 4b = -7,\ c + 4d = 2,\ 2a + 5b = -8,\ 2c + 5d = 4$

Solving $a = 1,\ b = -2,\ c = 2,\ d = 0$ 1 m

$\therefore\ x = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ ½ m

OR

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$|A| = 1 \neq 0$, A^{-1} will exist ½ m

$$\text{adj } A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$ (Any four correct Cofactors : 1 mark) 2 m

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$ ½ m

$$A^{-1} A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ 1 m

13. $f(x) = |x-3| + |x-4|$

$$= \begin{cases} 7-2x, & x < 3 \\ 1, & 3 \le x < 4 \\ 2x-7, & x \ge 4 \end{cases}$$ 1 m

L. H. D at x = 3 $\lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$

$$\lim_{x \to 3^-} \frac{6-2x}{x-3} = -2$$

R. H. D at x = 3 $\lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$

$$= \frac{1-1}{x-3} = 0$$

$L.H.D \neq R.H.D \;\therefore\; f(x)$ is not diffrentiable at $x = 3$ 1½ m

L. H. D at $x = 4$ $\lim_{x \to 4^-} \frac{f(x) - f(4)}{x - 4}$

$= \frac{1-1}{x-4} = 0$

R. H. D at $x = 4$ $\lim_{x \to 4^+} \frac{f(x) - f(4)}{x - 4}$

$\lim_{x \to 4^+} \frac{2x - 7 - 1}{x - 4} = 2$

L. H. D at $x = 4 \neq$ R.H.D at $x = 4$

$f(x)$ is not differentiable at $x = 4$ 1½ m

14. $y = x^{e^{-x^2}}$

$\log y = e^{-x^2} \log x$ 1 m

Diff. w. r. t x

$\frac{1}{y}\frac{dy}{dx} = \frac{e^{-x^2}}{x} + \log x \, e^{-x^2}(-2x)$ 2 m

$\frac{dy}{dx} = y\left(\frac{e^{-x^2}}{x} - 2x \log x \, e^{-x^2}\right)$ ½ m

$= x^{e^{-x^2}} e^{-x^2}\left(\frac{1}{x} - 2x \log x\right)$ ½ m

OR

$\log\sqrt{x^2 + y^2} = \tan^{-1}\frac{x}{y}$

Diff. w. r. t. x

$$\frac{1}{2\left(x^2+y^2\right)}\left(2x+2y\frac{dy}{dx}\right) = \frac{1}{1+\frac{x^2}{y^2}}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)$$ 2 m

$$\frac{x+y\frac{dy}{dx}}{x^2+y^2} = \frac{y^2}{x^2+y^2}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)$$ 1 m

$$\frac{dy}{dx}\left(y+x\right) = y-x$$ ½ m

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$ ½ m

15. $y = \sqrt{x+1} - \sqrt{x-1}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x-1}}$$ 1 m

$$= \frac{\sqrt{x-1}-\sqrt{x+1}}{2\sqrt{x^2-1}}$$ ½ m

$$4\left(x^2-1\right)\left(\frac{dy}{dx}\right)^2 = y^2$$ ½ m

$$4\left(x^2-1\right)2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + 8x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx}$$ 1 m

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{y}{4}$$ ½ m

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{y}{4} = 0$$ ½ m

16. $\int \frac{1-\cos x}{\cos x\,(1+\cos x)}\,dx$

$= \int \frac{1+\cos x - 2\cos x}{\cos x\,(1+\cos x)}\,dx$ 1½ m

$\int \frac{dx}{\cos x} - 2\int \frac{dx}{1+\cos x}$ ½ m

$\int \sec x\,dx - \int \sec^2 \frac{x}{2}\,dx$ 1 m

$\log|\sec x + \tan x| - 2\tan\frac{x}{2} + c$ 1 m

17.

	M W C	Expenses		Family expenses
Family A Family B Family C	$\begin{pmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{pmatrix}$	$\begin{pmatrix} 200 \\ 150 \\ 200 \end{pmatrix}$	=	$\begin{pmatrix} 1050 \\ 1150 \\ 2300 \end{pmatrix}$

2 m

Expenses for family A = ₹ 1050

Expenses for family B = ₹ 1150 1 m

Expenses for family C = ₹ 2300

Any relevant impact 1 m

18. $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2} - \tan^{-1}z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \cot^{-1}z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}\left(\frac{1}{z}\right)$ as $z>0$ 1 m

$$\frac{x+y}{1-xy} = \frac{1}{z}$$ ½ m

$xy + yz + zx = 1$ ½ m

19. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$R_1 \rightarrow R_1 + R_2 + R_3$

$(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ 1 m

$C_1 \rightarrow C_1 - C_2,\ C_2 \rightarrow C_2 - C_3$

$(a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix} = 0$ 2 m

$(a + b + c)\ (ab + bc + ca - a^2 - b^2 - c^2) = 0$

given $a \neq b \neq c$, so $ab + bc + ca - a^2 - b^2 - c^2 \neq 0$ ½ m

$\Rightarrow\ (a+b+c) = 0$ ½ m

SECTION - C

20. Let E_1 be the event of following course of

meditation and yoga and E_2 be the event of following

course of drugs 1 m

$P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{2}$ 1 m

$P(A|E_1) = \frac{70 \times 40}{100 \times 100}$ $\quad P(A|E_2) = \frac{75}{100} \times \frac{40}{100}$ 1 m

Formula 1 m

$$P(E_1|A) = \frac{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100}\right)}{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100} + \frac{1}{2} \times \frac{75}{100}\right)}$$

$$= \frac{70}{145} = \frac{14}{29}$$ 2 m

21. Let the no. of items in the item A = x

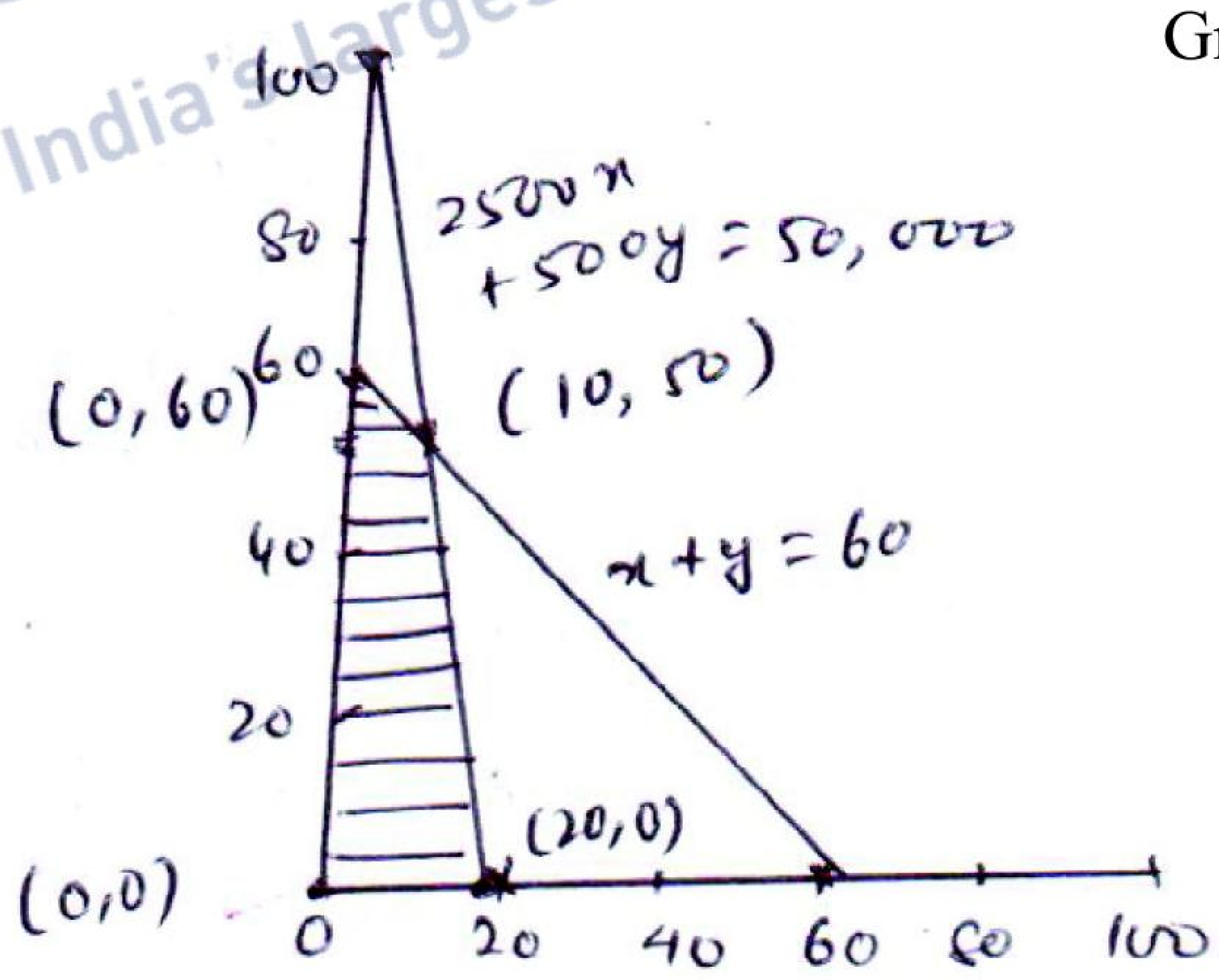
Let the no. of items in the item B = y

(Maximize) $z = 500x + 150y$ 1 m

$x + y \le 60$

$2500x + 500y \le 50{,}000$ Graph 2 m

$x, y \ge 0$



$z(0,0) = 0$

$z(10,50) = 12{,}500$

$z(20,0) = 10{,}000$ 2 m

$z(0,60) = 9{,}000$

Max. Profit = Rs. 12,500 1 m

OR

Let the no. of packets of food X = x

Let the no. of packets of food Y = y

(minimize) $P = (6x + 3y)$ 1 m

subject to

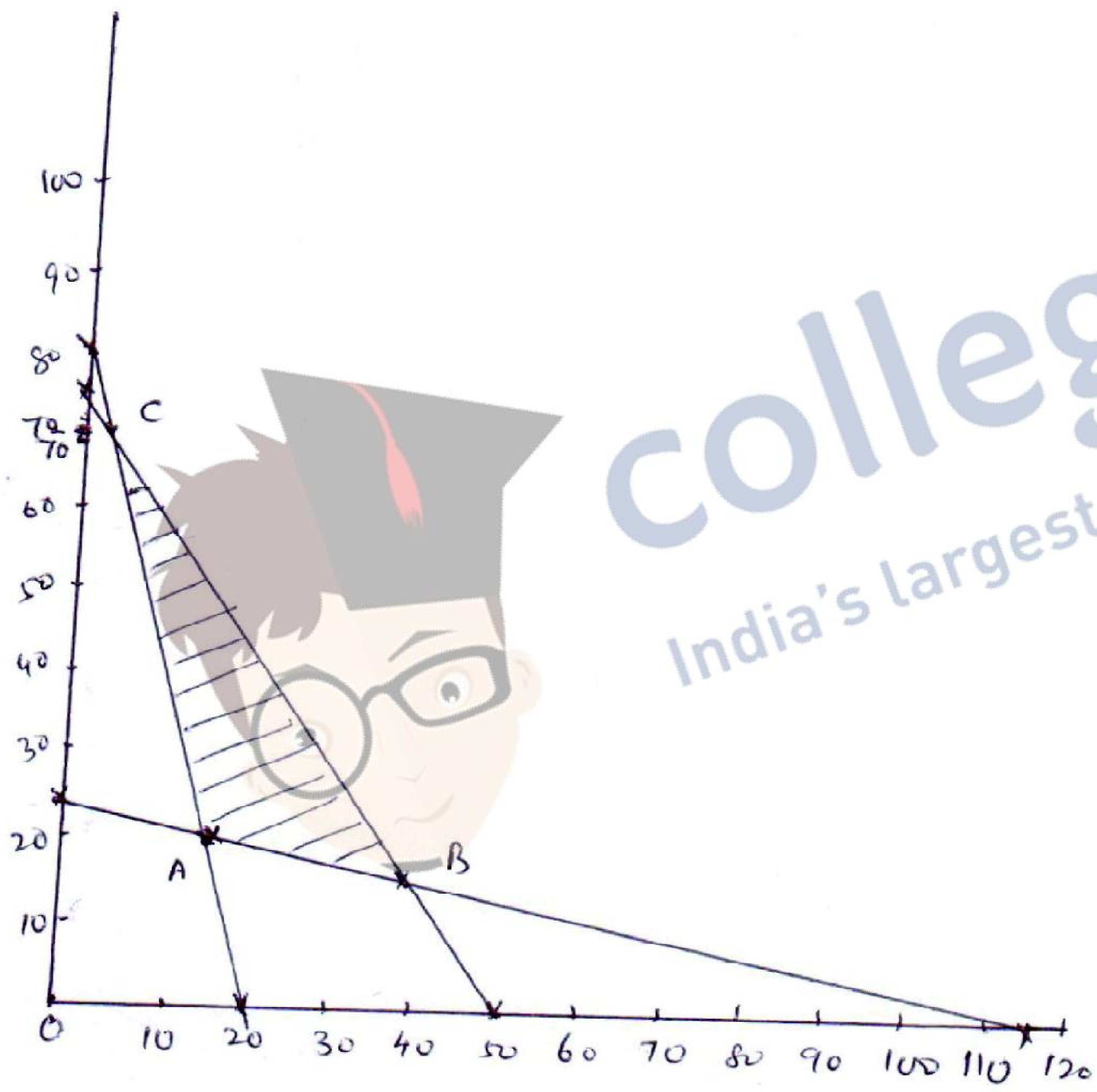
$12x + 3y \geq 240$

$4x + 20y \geq 460$

$6x + 4y \leq 300, \; x, y \geq 0$

or

$$\left.\begin{array}{l} 4x + y \geq 80 \\ x + 5y \geq 115 \\ 3x + 2y \leq 150 \\ x, y \geq 0 \end{array}\right\} \quad 2\text{ m}$$



Correct points of feasible region

A (15, 20), B (40, 15), C (2, 72)

So P (15, 20) = 150

P (40, 15) = 285

P (2, 72) = 228

Graph 2 m

minimum amount of vitamin A = 150 units when 15 packets of food X and 20 packets of food Y are used 1 m

22. For every $a \in A$, $(a, a) \in R$

$\because \; |a - a| = 0$ is divisible by 2

$\therefore$ R is reflexive 1 m

For all $a, b \in A$

$(a, b) \in R \Rightarrow |a-b|$ is divisible by 2

$\Rightarrow |b-a|$ is divisible by 2

$\therefore$ $(b, a) \in R$ $\therefore$ R is symmetric 1 m

For all $a, b, c \in A$

$(a, b) \in R \Rightarrow |a-b|$ is divisible by 2

$(b, c) \in R \Rightarrow |b-c|$ is divisible by 2

So, $a-b = \pm 2k$ 1 m

$$\begin{array}{l} b-c = \pm 2\ell \\ \hline a-c = \pm 2m \end{array}$$

$\Rightarrow |a-c|$ is divisible by 2

$\Rightarrow (a, c) \in R$

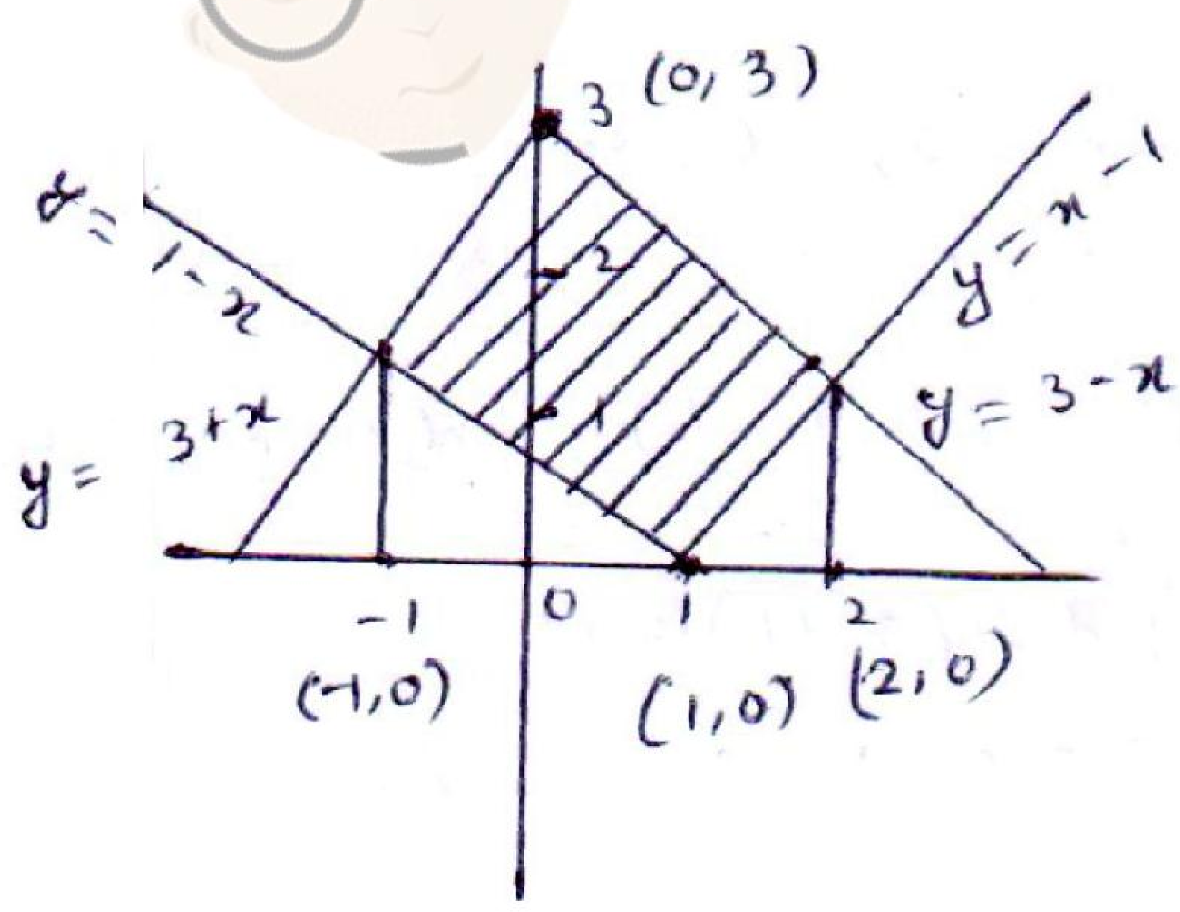
$\Rightarrow$ R is transitive 1 m

Showing elements of $\{1, 3, 5\}$ and 1 m

$\{2, 4\}$ are related to each other

and $\{1, 3, 5\}$ and $\{2, 4\}$ are not related to each other 1 m

23. Graph 2 + 2 m



Area of shaded reigon

$$= \int_{-1}^{0}(3+x+x-1)\,dx + \int_{0}^{2}(3-x)\,dx - 2\int_{1}^{2}(x-1)\,dx$$ 1 m

$$= 2\frac{(x+1)^2}{2}\Bigg]_{-1}^{0} - \frac{(3-x)^2}{2}\Bigg]_{0}^{2} - 2\frac{(x-1)^2}{2}\Bigg]_{1}^{2}$$

$$= 1 - \frac{1}{2}(1-9) - 1 = 4 \text{ sq. units}$$ 1 m

24. $$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$$

Let $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$ ½ m

$$v + x\frac{dv}{dx} = \frac{v^2}{v-1}$$ 1½ m

$$x\frac{dv}{dx} = \frac{v}{v-1}$$

$$\frac{dx}{x} = \left(\frac{v-1}{v}\right) dv$$ 1½ m

$$\int\frac{dx}{x} = \int\left(1 - \frac{1}{v}\right) dv$$

$$\log x = v - \log v + c$$ 1 m

$$\log y = \frac{y}{x} + c \text{ or } x \log y - y = c\,x$$ 1½ m

OR

$$\sin 2x\frac{dy}{dx} - y = \tan x$$

$$\frac{dy}{dx} - \frac{y}{\sin 2x} = \frac{\tan x}{\sin 2x}$$ 1 m

$$\frac{dy}{dx} - y(\text{cosec } 2x) = \frac{\sec^2 x}{2}$$

$P = -\operatorname{cosec} 2x,\ Q = \frac{1}{2}\sec^2 x$

$\int P\,dx = -\int \operatorname{cosec} 2x\,dx$

$= -\frac{1}{2}\log|\tan x|$

So $e^{\int P\,dx} = \frac{1}{\sqrt{\tan x}}$ 1½ m

Solution is

$\frac{y}{\sqrt{\tan x}} = \frac{1}{2}\int \frac{\sec^2 x\,dx}{\sqrt{\tan x}} \quad \left(\begin{array}{c}\sqrt{\tan x} = t \\ \Rightarrow \frac{1}{2}\frac{\sec^2 x\,dx}{\sqrt{\tan x}} = dt\end{array}\right)$ 1½ m

$\frac{y}{\sqrt{\tan x}} = \sqrt{\tan x} + c$ 1 m

Getting $c = 1$ ½ m

$\Rightarrow y = \tan x - \sqrt{\tan x}$ ½ m

25. Eqn. of plane

$(x+y+z-6)+\lambda(2x+3y+4z+5) = 0$ 2 m

it passes through $(1,1,1)$

$-3+14\lambda = 0 \Rightarrow \lambda = \frac{3}{14}$ 2 m

Eqn. of plane will be

$20x + 23y + 26z - 69 = 0$ 1 m

vector from : $\vec{r} \cdot \left(20\hat{i} + 23\hat{j} + 26\hat{k}\right) = 69$ 1 m

26. $y = \dfrac{x}{1+x^2}$

$\dfrac{dy}{dx} = \dfrac{1-x^2}{\left(1+x^2\right)^2}$ 2 m

Let $f(x) = \dfrac{1-x^2}{\left(1+x^2\right)^2}$

$f'(x) = 0 \Rightarrow \dfrac{-2x\left(3-x^2\right)}{\left(1+x^2\right)^3} = 0$

For max or min $x\left(3-x^2\right) = 0 \Rightarrow x = 0$ or $x = \pm\sqrt{3}$ 2 m

Calculating $\dfrac{d^2f(x)}{dx^2}$ at $x = 0 < 0$

1 m

at $x = \pm\sqrt{3} > 0$

$\Rightarrow$ $x = 0$ is the point of local maxima

1 m

$\Rightarrow$ the required pt is (0, 0)