QUESTION PAPER CODE 65/C/1

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions. Select the correct option.

Q.No. **Marks**

1. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the question independently, the probability that the question is solved is

(A) $\frac{7}{15}$ **(B)** $\frac{8}{15}$ **(C)** $\frac{2}{15}$ **(D)** $\frac{14}{15}$

Ans: (A) $\frac{7}{15}$ 1

2. The objective function of an LPP is

(A) a constant **(B)** a linear function to be optimised

(C) an inequality **(D)** a quadratic expression

Ans: (B) a linear function to be optimised 1

3. If the two lines

$$L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : x = 2, \frac{y}{-1} = \frac{z}{2-\alpha}$$

are perpendicular, then the value of α is

(A) $\frac{2}{3}$ **(B)** 3 **(C)** 4 **(D)** $\frac{7}{3}$

Ans: (D) $\frac{7}{3}$ 1

4. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of the points A(2, 3, –4), B(3, –4, –5) and C(3, 2, –3) respectively, then $\left|\vec{a}+\vec{b}+\vec{c}\right|$ is equal

(A) $\sqrt{113}$ **(B)** $\sqrt{185}$ **(C)** $\sqrt{203}$ **(D)** $\sqrt{209}$

Ans: (D) $\sqrt{209}$ 1

5. The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is

(A) 1, 1 **(B)** 1, 2 **(C)** 2, 1 **(D)** 2, 2

Ans: (A) 1, 1 — 1

6. $\int_0^1 \tan\left(\sin^{-1} x\right) dx$ equals

(A) 2 **(B)** 0 **(C)** –1 **(D)** 1

Ans: (D) 1 — 1

7. $\int \frac{e^x}{x+1}\left[1+(x+1)\log(x+1)\right]dx$ equals

(A) $\frac{e^x}{x+1}+c$ **(B)** $e^x \frac{x}{x+1}+c$

(C) $e^x \log(x+1) + e^x + c$ **(D)** $e^x \log(x+1) + c$

Ans: (D) $e^x \log(x+1) + c$ — 1

8. If $\sec^{-1}\left(\frac{1+x}{1-y}\right)=a$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x-1}{y-1}$ **(B)** $\frac{x-1}{y+1}$ **(C)** $\frac{y-1}{x+1}$ **(D)** $\frac{y+1}{x-1}$

Ans: (C) $\frac{y-1}{x+1}$ — 1

9. If $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, then A^2 equals

(A) $\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$ **(B)** $\begin{bmatrix}2 & -2\\ -2 & -2\end{bmatrix}$ **(C)** $\begin{bmatrix}-2 & -2\\ -2 & 2\end{bmatrix}$ **(D)** $\begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}$

Ans: (A) $\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$ — 1

10. $\begin{vmatrix}43 & 44 & 45\\ 44 & 45 & 46\\ 45 & 46 & 47\end{vmatrix}$ equals

(A) 0 **(B)** –1 **(C)** 1 **(D)** 2

Ans: (A) 0 — 1

Fill in the blanks in questions numbers 11 to 15

11. Two angles of a triangle are $\cot^{-1}2$ and $\cot^{-1}3$. The third angle of the triangle is __________.

Ans: $\frac{3\pi}{4}$ or $135°$ **1**

12. A square matrix A is said to be singular if ____________.

Ans: $|A| = 0$

OR

If $A = \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB|$ = ____________.

Ans: $|AB| = -100$

13. If $y = \log x$, then $\frac{d^2y}{dx^2}$ = ________.

Ans: $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$ **1**

14. The integrating factor of the differential equation $x\frac{dy}{dx} - y = \log x$ is _________

Ans: $\frac{1}{x}$ **1**

15. From a pack of 52 cards, 3 cards are drawn at random (without replacement). The probability that they are two red cards and one black card, is _________.

Ans: $\frac{13}{34}$ **1**

Question numbers 16 to 20 are very short answer type questions

16. Find the distance of the point (a, b, c) from the x-axis.

Ans: $\sqrt{b^2 + c^2}$ **1**

17. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$, then find the ratio $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}$.

Ans: $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}} = \frac{|\vec{a}|}{|\vec{b}|}$ **1/2**

$= \frac{3}{5\sqrt{2}}$ **1/2**

OR

Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

Ans: $\vec{c} \perp \vec{d} \Rightarrow \vec{c} \cdot \vec{d} = 0$ 1/2

$$\Rightarrow \hat{a} \cdot \hat{b} = \frac{1}{2}$$

$\Rightarrow$ Angle between vectors $\hat{a}$ & $\hat{b} = \frac{\pi}{3}$ or $60°$ 1/2

18. Solve the differential equation $\left(e^x + 1\right) y\, dy = e^x (y+1)\, dx$.

Ans: $\int \frac{e^x}{e^x + 1} dx = \int \frac{y}{y+1} dy$ 1/2

$\Rightarrow \log(e^x + 1) = y - \log(y+1) + c$ 1/2

19. If $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, then find the value of x.

Ans: $2x - 3 = x + 2$ 1/2

$\Rightarrow x = 5$ 1/2

OR

If A is a square matrix such that $A^2 = A$, then find $(2 + A)^3 - 19A$.

Ans: $(2 + A)^3 - 19A = A^3 + 8 + 12A + 6A^2 - 19A$ 1/2

$= 8$ 1/2

20. If $f(x) = \frac{1-x}{1+x}$, then find $(f \circ f)(x)$.

Ans: $f \circ f(x) = f(f(x)) = \frac{1 - f(x)}{1 + f(x)}$ 1/2

$= x$ 1/2

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. Let W denote the set of words in the English dictionary. Define the relation R by R = {(x , y) ∈ W × W such that x and y have at least one letter in common}

Show that this relation R is reflexive and symmetric, but not transitive.

Ans: For any word $x \in W$

x and x have atleast one (all) letter in common

$\therefore (x, x) \in R, \forall x \in W \;\therefore$ R is reflexive **1**

Symmetric : Let $(x, y) \in R$, $x, y \in W$

$\Rightarrow$ x and y have atleast one letter in common

$\Rightarrow$ y and x have atleast one letter in common

$\Rightarrow (y, x) \in R \;\therefore$ R is symmetric **1/2**

Transitive : Taking example of three English dictionary words $x, y, z, \in W$ such that $(x, y), (y, z) \in R$ but $(x, z) \notin R$ **1/2**

$\therefore$ R is not transitive

OR

Find the inverse of the function $f(x) = \left(\frac{4x}{3x+4}\right)$.

Ans: Let $y = f(x) = \frac{4x}{3x+4} \Rightarrow x = \frac{4y}{4-3y}$ $1\frac{1}{2}$

$\therefore \; f^{-1}(y) = \frac{4y}{4-3y}\left(\text{or } f^{-1}(x) = \frac{4x}{4-3x}\right)$ **1/2**

22. For the matrix $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$, verify the following:

A(adj A) = (adj A) A = |A| I

Ans: $|A| = -12 + 12 = 0$, adj $A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$ $\frac{1}{2}+1$

$A \cdot (\text{adj } A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = (\text{adj } A) \cdot A = |A|\, I$ $\frac{1}{2}$

23. If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$

Ans: $\frac{dy}{dx} = e^x - e^{-x}$ **1**

$= \sqrt{\left(e^x + e^{-x}\right)^2 - 4}$ **1/2**

$= \sqrt{y^2 - 4}$ **1/2**

24. Solve the following homogeneous differential equation: $x\frac{dy}{dx} = x + y$

Ans: Let $y = vx \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}+\frac{1}{2}$

$\therefore x\left(v + x\frac{dv}{dx}\right) = x + vx \Rightarrow x\frac{dv}{dx} = 1$ $\frac{1}{2}$

$\therefore \int dv = \int \frac{1}{x}dx \Rightarrow v = \log|x| + c$

$\Rightarrow y = x(\log|x| + c)$ $\frac{1}{2}$

25. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

Ans: $\left(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}\right) \cdot \left(|\vec{a}|\vec{b} - |\vec{b}|\vec{a}\right)$

$= \left(|\vec{a}|\vec{b}\right)^2 - \left(|\vec{b}|\vec{a}\right)^2$ 1

$= |\vec{a}|^2|\vec{b}|^2 - |\vec{b}|^2|\vec{a}|^2 = 0$

$\therefore \left(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}\right) \perp \left(|\vec{a}|\vec{b} - |\vec{b}|\vec{a}\right)$ 1

26. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

Ans: Let X = No. of even tickets drawn

X	0	1	2
P(X)	$\frac{^{10}C_2}{^{19}C_2} = \frac{5}{19}$	$\frac{^{10}C_1 \cdot {^9C_1}}{^{19}C_2} = \frac{10}{19}$	$\frac{^9C_2}{^{19}C_2} = \frac{4}{19}$

$\frac{1}{2}$

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

OR

Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

Ans: X = No. of success = No. of times getting a number greater than 5

X	0	1	2
P(X)	$\frac{5}{5} \cdot \frac{5}{6} = \frac{25}{36}$	$2 \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{10}{36}$	$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$\frac{1}{2}$

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$.

Ans: LHS $= 2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$ **2**

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}}\right)=\tan^{-1}\frac{31}{17}$$ **2**

$= \text{RHS}$

28. Using properties of determinants, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

Ans. LHS $= \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

(Applying $C_1 \to C_1 - bC_3$, $C_2 \to C_2 + aC_3$)

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b\left(1+a^2+b^2\right) & -a\left(1+a^2+b^2\right) & 1-a^2-b^2 \end{vmatrix}$$ **2**

(Taking $\left(1+a^2+b^2\right)$ common from C_1 & C_2)

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$ **1/2**

(Applying $C_3 \to C_3 + 2bC_1 - 2aC_2$)

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & -a & 1+a^2+b^2 \end{vmatrix}$$ **1**

(Expand along C_3)

$= \left(1+a^2+b^2\right)^3 = \text{RHS}$ **1/2**

OR

Find the equation of the line joining A(1, 3) and B(0, 0), using determinants. Also, find k if D(k, 0) is a point such that the area of the ΔABD is 3 square units.

Ans: Equation of the line through A(1, 3) and B(0, 0) is

$$\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow 3x - y = 0$$ **2**

$$\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3 \Rightarrow k = \pm 2$$ **2**

29. Prove that three points A, B and C with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively are collinear if and only if $\left(\vec{b} \times \vec{c}\right) + \left(\vec{c} \times \vec{a}\right) + \left(\vec{a} \times \vec{b}\right) = \vec{0}$

Ans. Points $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ are collinear

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = \vec{0}$ **1**

$\Rightarrow \left(\vec{b} - \vec{a}\right) \times \left(\vec{c} - \vec{a}\right) = \vec{0}$ **1**

$\Rightarrow \vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{a} \times \vec{c} + \vec{a} \times \vec{a} = \vec{0}$ **1**

$\Rightarrow \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

Similarly, converse can be proved **1**

30. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y-1}{3} = z, \quad \frac{x+1}{5} = \frac{y-2}{1}, \quad z = 2$$

Ans. Let $\left.\begin{array}{l} \vec{a}_1 = \hat{i} - \hat{j}\ ;\ \vec{a}_2 = -\hat{i} + 2\hat{j} + 2\hat{k} \\ \vec{b}_1 = 2\hat{i} + 3\hat{j} + \hat{k}\ ;\ \vec{b}_2 = 5\hat{i} + \hat{j} \end{array}\right\}$ **1**

then, $\vec{a}_2 - \vec{a}_1 = -2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = -\hat{i} + 5\hat{j} - 13\hat{k}$ **1/2+1$\frac{1}{2}$**

$\therefore$ Shortest distance $= \left|\dfrac{\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \dfrac{9}{\sqrt{195}} \neq 0$ **1/2**

$\therefore$ lines are not intersecting **1/2**

OR

Find the equation of the plane through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j})+6=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}-4\hat{k})=0$, which is at a unit distance from the origin.

Ans: Equation of plane through the line of intersection of the two given planes is

$$\vec{r}\cdot\left[(1+3\lambda)\hat{i}+(3-\lambda)\hat{j}-4\lambda\hat{k}\right]=-6$$ $1\frac{1}{2}$

As per the given condition

$$\left|\frac{-6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}\right|=1 \Rightarrow \lambda=\pm 1$$ $1\frac{1}{2}$

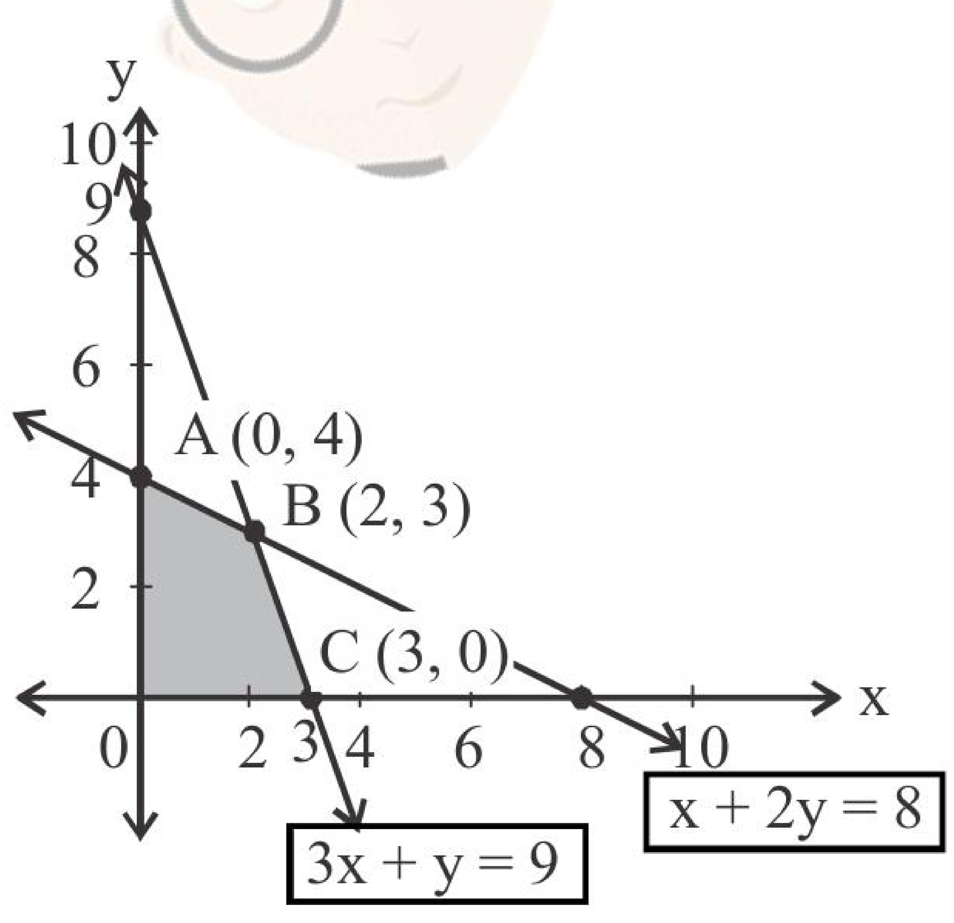
$\therefore$ Equation of plane is : $\vec{r}\cdot(4\hat{i}+2\hat{j}-4\hat{k})+6=0$ **1/2**

or $\vec{r}\cdot(-2\hat{i}+4\hat{j}+4\hat{k})+6=0$ **1/2**

31. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3g of silver and 1g of gold, while that of type B requires 1g of silver and 2g of gold. The company can use at the most 9g of silver and 8 g of gold. If each unit of type A brings a profit of ₹ 120 and that of type B ₹ 150, then find the number of units of each type that the company should produce to maximise profit.

Formulate the above LPP and solve it graphically. Also, find the maximum profit.

Ans.



Let No. of goods type A = x,

Number of goods type B = y.

Then the L.P. P. is:

Maximize (Profit) : Z = 120x + 150y **1**

Subject to constraints :

$3x + y \leq 9$

$x + 2y \leq 8$

$x, y \geq 0$ **1**

Correct figure $1\frac{1}{2}$

Corner:	Value of Z (in ₹)
0(0, 0)	0
A(0, 4)	600
B(2, 3)	690 (Max)
C(3, 0)	360

1/2

$\therefore$ Max. profit = ₹ 690

when Good Type A = 2 units, Type B = 3 units

32. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement), both of which are found to be red. Find the probability that these two balls are drawn from the second bag.

Ans. Let E_1 : Bag I is selected

E_2 : Bag II is selected

A : Two balls drawn at random both are red. **1/2**

$$P(E_1) = P(E_2) = \frac{1}{2}, P\left(\frac{A}{E_1}\right) = \frac{^5C_2}{^9C_2} = \frac{5}{18}, P\left(\frac{A}{E_2}\right) = \frac{^3C_2}{^9C_2} = \frac{1}{12}$$ **1/2+1+1**

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{2}\cdot\frac{1}{12}}{\frac{1}{2}\cdot\frac{5}{18}+\frac{1}{2}\cdot\frac{1}{12}} = \frac{3}{13}$$ **1**

SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33. If $y = x^{\sin x} + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

Ans. Let $u = x^{\sin x}$ $\therefore$ $y = u + \sin^{-1}\sqrt{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{2\sqrt{x}\sqrt{1-x}} \quad \ldots (i)$$ **$1\frac{1}{2}$**

Also, $\log u = \sin x \cdot \log x$ **1**

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x \cdot \log x + \frac{\sin x}{x}$$ **2**

$$\Rightarrow \frac{du}{dx} = x^{\sin x} \cdot \left(\cos x \cdot \log x + \frac{\sin x}{x}\right) \ldots (ii)$$ **$\frac{1}{2}$**

Putting (ii) in (i) we get

$$\frac{dy}{dx} = x^{\sin x}\left(\cos x \cdot \log x + \frac{\sin x}{x}\right) + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$ **1**

34. Evaluate : $\displaystyle\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

Ans. Let $\sin x - \cos x = t \quad \therefore \quad (\sin x + \cos x)\, dx = dt$ $\quad 1+\frac{1}{2}$

Squaring we get, $\sin 2x = 1 - t^2$ $\quad 1\frac{1}{2}$

$$\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx = \int \frac{1}{\sqrt{1-t^2}}\, dt = \sin^{-1} t$$

$$= \sin^{-1}(\sin x - \cos x) \quad 1\frac{1}{2}$$

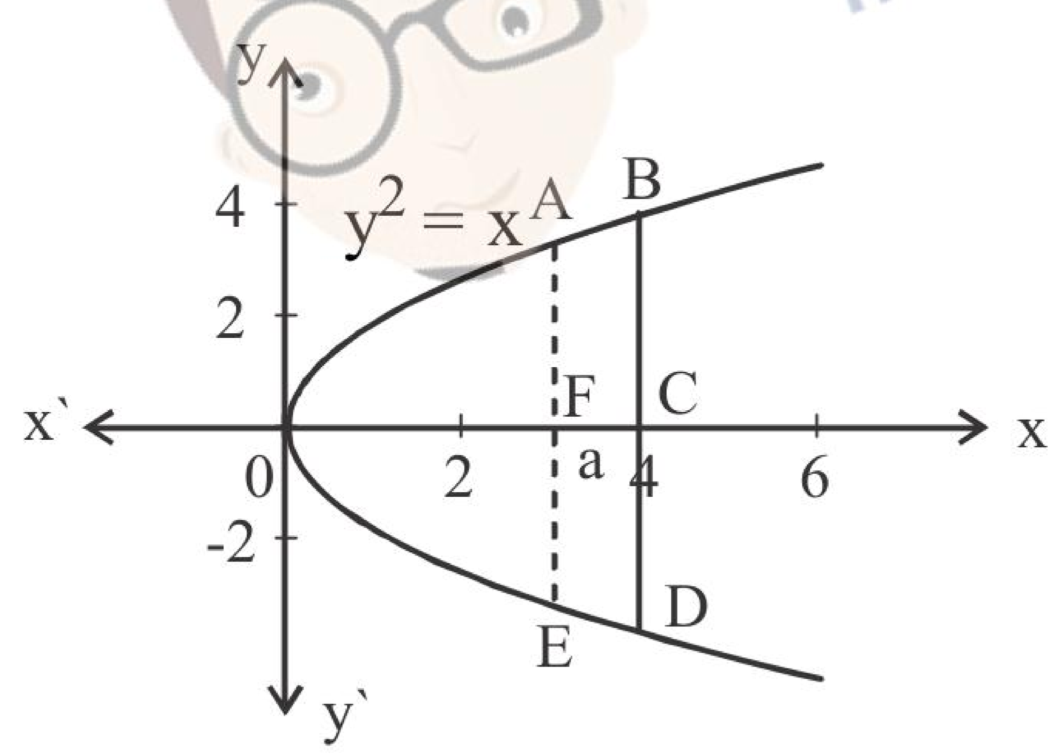
$$\therefore \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx = \sin^{-1}(\sin x - \cos x) \Big]_{\pi/6}^{\pi/3}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) - \sin^{-1}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$$

$$= 2 \cdot \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) \quad 1\frac{1}{2}$$

35. If the area between the curves $x = y^2$ and $x = 4$ divided into two equal parts by the line $x = a$, then find the value of a using integration.

Ans. Correct graph $\quad 1\frac{1}{2}$



$ar(OAEO) = ar(ABDEA)$

$\Rightarrow 2 \cdot ar(OAFO) = 2 \cdot ar(ABCFA)$

$$\int_0^a \sqrt{x}\, dx = \int_a^4 \sqrt{x}\, dx \quad 1\frac{1}{2}$$

$$\frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right) \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right) \quad 1\frac{1}{2}$$

$$\Rightarrow a^{3/2} = 4, \quad \therefore a = 4^{2/3} \quad 1\frac{1}{2}$$

OR

Find: $\int \frac{x}{(x-1)^2(x+2)}dx$

Ans: $\int \frac{x}{(x-1)^2(x+2)}dx \quad = \quad \frac{2}{9}\int \frac{1}{(x-1)}dx + \frac{1}{3}\int \frac{1}{(x-1)^2}dx - \frac{2}{9}\int \frac{1}{4+2}dx$ **4**

$= \frac{2}{9}\log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}\log|x+2| + C$ **2**

36. Find the intervals in which the function f defined as f(x) = sin x + cos x, $0 \le x \le 2\pi$ is strictly increasing or decreasing.

Ans. $f'(x) = \cos x - \sin x,\ 0 \le x \le 2\pi$ **1**

$\therefore\ f'(x) = 0 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$ **2**

Sign of $f'(x)$: $+ve$ on $\left(0, \frac{\pi}{4}\right)$, $-ve$ on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$, $+ve$ on $\left(\frac{5\pi}{4}, 2\pi\right)$ (points $0, \frac{\pi}{4}, \frac{5\pi}{4}, 2\pi$) **$1\frac{1}{2}$**

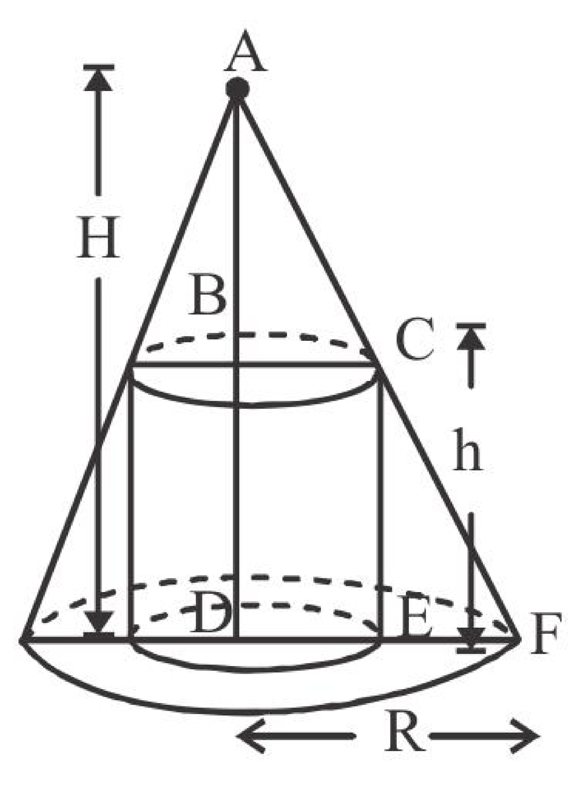
$\therefore$ f(x) is strictly increasing on $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$ **1**

and f(x) is strictly decreasing on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ **$\frac{1}{2}$**

OR

Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

Ans.



Let h = Height of cylinder

r = Radius of cylinder

H = Height of cone

R = Radius of cone

where, H, R are constants

Correct figure **1**

$\frac{H-h}{H} = \frac{r}{R}\ (\because \Delta ABC \sim \Delta ADE)$ ...(i) **$\frac{1}{2}$**

C = curved surface area = $2\pi rh$ **1**

$\therefore\ C = 2\pi rh \cdot H \cdot \left(\frac{R-r}{R}\right)$ (Using (i))

$$= \frac{2\pi H}{R}\left(rR - r^2\right)$$ **1**

$$C'(r) = \frac{2\pi H}{R}(R - 2r),\ C''(r) = \frac{-4\pi H}{R} < 0$$ **1**

$$C'(r) = 0 \Rightarrow r = \frac{R}{2},\ C''\left(r = \frac{R}{2}\right) < 0$$ **1**

$\therefore$ Curved surface area of cylinder is Max. iff $r = \frac{R}{2}$ **$\frac{1}{2}$**
