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JKPSC Exam

Mains Previous Paper

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MATHEMATICS - II

[15]

Time Allowed -3 Hours

Maximum Marks-300

INSTRUCTIONS

- i) Answers must be written in English
- ii) The number of marks carried by each question is indicated at the end of the question.
- iii) The answer to each question or part thereof should begin on a fresh page.
- iv) Your answer should be precise and coherent.
- v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.
- vi) Candidates should attempt any **five** questions
- vii) If you encounter any typographical error, please read it as it appears in the text book.
- viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.
- ix) No Continuation sheets shall be provided to any candidate under any circumstances.
- x) Candidates shall put a cross (X) on blank pages of answer Script.
- xi) No blank page be left in between answer to various questions.
- xii) No programmable Calculator is allowed.
- xiii) No stencil (with different markings) is allowed.
- xiv) In no circumstances help of scribe will be allowed.

Section - A

1. a) Suppose G is a group and $g \neq e$ is an element of G . Under what conditions on g is there a homomorphism $f : \mathbb{Z}_{15} \rightarrow G$ with $f([1]) = g$? Under what conditions on G is there an injective homomorphism $f : \mathbb{Z}_{15} \rightarrow G$? (20)
- b) Show that every group of order 85 is cyclic (20)
- c) Using Sylow Theorems, show that in a group of order pq with two primes p, q and $p < q$, there is only one subgroup of order q . Further show that this subgroup is normal. (20)

2. Let G be an abelian group of order n .

a) If $f : G \rightarrow \mathbb{C}$ is a function, then prove that for all $h \in G$,

$$\sum_{g \in G} f(g) = \sum_{g \in G} f(hg) \quad (20)$$

b) Let \mathbb{C}^* be the multiplicative group of non-zero complex numbers and suppose

$f : G \rightarrow \mathbb{C}^*$ is a homomorphism. Prove that

$$\sum_{g \in G} f(g) = 0 \text{ or } \sum_{g \in G} f(g) = n \quad (30)$$

c) If $f : G \rightarrow \mathbb{C}^*$ is any homomorphism, then prove that $\sum_{g \in G} |f(g)| = n$ (10)

3. Let $R_n = \mathbb{Z}_n[x]$ be the polynomial ring over integers modulo n .

a) In the field $R_2 / (x^2 + x + 1)$ let α be the image of x and compute (in reduced form) $(1 + \alpha - \alpha^2)^{-1}$ (30)

b) In the field $R_3 / (x^3 - x + 1)$, let α be the image of x and compute (in reduced form) $(1 + \alpha)(2 + \alpha - \alpha^2)$ (30)

4. Let (X, d) be a metric space

a) Prove that $d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$ is metric on X . (10)

b) Let A be a nonempty subset of X . For any $x \in X$, we define $d(x, A) = \inf \{d(x, y) : y \in A\}$.

Prove that for any $x, y \in X$, we have $|d(x, A) - d(y, A)| \leq d(x, y)$ (30)

c) Let $a, b \in \mathbb{R}, a < b$ and let

$$C([a, b]) = \{f : [a, b] \rightarrow \mathbb{C} : f \text{ is continuous on } [a, b]\}$$

Show that the map $d : C([a, b]) \times C([a, b]) \rightarrow \mathbb{R}_+$ defined by

$$d(f, g) = \max_{x \in [a, b]} |f(x) - g(x)| \text{ is a metric on}$$

$C([a, b])$. Therefore, $(C([a, b]), d)$ is a metric space (20)

5. a) Prove that if f is holomorphic in the annulus

$$U = \{z \in \mathbb{C} : r < |z - a| < R\},$$

then the integral $I = \int_{|z-a|<\rho} f(z) dz$

has the same value for all ρ with $r < \rho < R$.

(30)

- b) Compute the value of the integral

$$I = \int_{\gamma} \frac{z+1}{z(z-1)} dz$$

Where γ is the circle $|z-2| = \sqrt{2}$, traversed counterclockwise.

(30)

6. a) Find the radius of convergence for the series

$$\sum_{n \geq 1} \frac{n^n z^n}{n!} \quad \text{and} \quad \sum_{n \geq 1} 2^{-n^2} z^n \quad (30)$$

- b) For any positive integer n , prove that

$$\int_0^{2\pi} (1 + 2 \cos t)^n (\cos nt) dt = 2\pi \quad (30)$$

7. a) Let $f_n(x) = x^n$ for $x \in [0, 1]$. Then $\{f_n\}$ converge pointwise but not uniformly.

on $[0, 1]$. Let $g : [0, 1] \rightarrow \mathbb{C}$ be a continuous function such that $g(1) = 0$. Prove that $\{g(x)x^n\}$ converge uniformly on $[0, 1]$ (30)

- b) For $f(t) = 2t^2 + 1$, $\alpha(t) = t + [3t]$ on $[0, 1]$ and P is the partition of $[0, 1]$ consisting of 4 subintervals of equal length, find $U(P, f, \alpha)$ and $L(P, f, \alpha)$. Further.

calculate $\int_0^1 \int d\alpha$ if it exists. (30)

8. a) Use separation of variables to solve the wave equation $u_{xx} - C^2 u_{tt} = 0$ with initial conditions $u(x, 0) = f(x)$, $u_x(x, 0) = g(x)$. (20)

- b) Use the method of characteristics to solve the wave equation and verify that these solutions agree with those obtained in part (a). (20)
- c) Find the general integral of the PDE :

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = 3. \quad (20)$$

SECTION - B

1. a) Derive the Euler-Lagrange equations for the motion of a rigid body under a conservative force. (20)
- b) Consider a simple pendulum of length l vertically suspended from a point. If a horizontal force of $F \cos(\omega t)$ is applied at that point, prove that the pendulum settles into a periodic motion. Find the period of this motion. (20)
- c) Consider a circular turn table inclined at angle of θ to the horizontal, which has a constant angular velocity ω about the fixed centre. A block of wood of mass m lies at rest relative to the turn table. What is the smallest value of θ for which the block will begin to slide at atleast one point in the period of rotation. Assume that the coefficient of friction between the table and the block is μ . (20)
2. a) Describe the test for incompressibility of a flow. How are the stream lines derived from a given flow. (20)
- b) Derive Navier Stokes equation for an incompressible flow. (20)
- c) Give the definitions of the terms source and sink. How do the flow equations change in the presence of these phenomena. (20)
3. a) Find a real root of the polynomial $p(x) = x^5 + 3x^3 - 1$, correct to $\epsilon = 0.05$ using the regula falsi method as well as the Newton-Raphson method. Which has higher convergence and why? (30)
- b) Derive the cubic spline interpolation for the function given by the following table

x	1	3	5	7
$f(x)$	2	6	24	120

Use this to find the values of $f(2)$ and $f(6)$. (30)

(15)-II

(4)

4. a) Solve the I.V.P. $y=(l+y)^2$, $y(0)=1$, in the interval $0 \leq x \leq 1$ by using the Runge Kutta method of order 2 by using $h = 0.2$ (30)
- b) State the n-point Gaussian quadrature formula for weight $w(x)=1$ in the interval $[-1,1]$. Derive the error of approximation for any continuous function $f \in C^{n+1}([-1,1])$. (30)
5. a) A token collector collects tokens number $1,2,\dots,n$ all of which are equally likely. What is the expected number of tokens collected before token number i is collected. What are the expected number of tokens before the collection is complete (viz., at least one token of every number is collected). (30)
- b) Consider a random walk in the set of integers, where the starting point, is 0 and the steps taken at any point are independently chosen from either of 0, $\pm 1, \pm 2$ with equal probability. Let X be the random variable that holds the value of the current position after n steps. Find $E[X]$ and V ar $[X]$. Hence, use the central limit theorem to find the limiting distribution of X as $n \rightarrow \infty$. (30)
6. a) Which continuous distribution has the memory-less property? Give a proof of this statement. (10)
- b) What is the difference between Multiple correlation coefficient and Rank correlation coefficient? Give an example where these values might be different. (20)
- c) Describe the least squares approach to linear regression. Derive the optimal parameters from first principles. (30)

7. a) Let X, Y be random variables having the following joint distribution

$$f(x, y) = \begin{cases} (2\pi)^{-1} & x^2 + y^2 \leq 1 \\ (2\pi)^{-1} & (x-1)^2 + y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal distributions of X and Y. Calculate the value of $P[X+Y \geq 1]$.

Calculate the condition probability density function $f_{Y|X}(y)$. (20)

- b) Out of a batch of 30 LED lamps, 11 were found to be defective. Test the hypothesis that the probability of a lamp being defective is $1/3$ at the both levels of significance $\alpha=0.05$ and $\alpha=0.01$. (Assume that, for $n=1$ degree of freedom, $\xi_{0.95}^2=3.84$ and $\xi_{0.99}^2=6.63$) (20)

- c) i) Define the power of a test. How does it differ from the level of significance of a test? (10)
- ii) Let the population variance be σ_p^2 and let a sample be drawn from this population with sample variance be σ_s^2 . Design a test for the equality of σ_s and σ_p . Would your answer change if we only have the variances of two different samples. (10)

8. a) Derive the Kuhn-Tucker conditions for the solution of a non-linear programming problem. (20)

- b) The arrival of customers at a hair dressing salon follows a Poisson process at the rate of 4 per hour. The salon has two hair dressers, each of whom complete their services in an exponential distribution with the rate $\lambda=0.1$ per minute. Answer the following questions.

- i) What is the average waiting time of a customer? (10)
- ii) How much free time does a hair dresser get on an average? (10)
- iii) Describe the steady-state solution of this queue. If the number of hair dressers increases by one, how would the steady state solution change? (20)



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