

JEE-Main-27-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: Let $a_1, a_2, a_3, \dots, a_n$ be in A.P. The ratio of sum of first five term to the sum of first nine terms is $5:17$. Also $110 < a_{15} < 120$. Find the sum of first 10 terms of the A.P. (where all a_i ($i = 1, 2, 3, \dots, n$) are integers)

Options:

- (a) 330
- (b) 460
- (c) 290
- (d) 380

Answer: (d)

Solution:

Let first term be ' a ' and common difference be ' d ' for the A.P.

$$\frac{S_5}{S_9} = \frac{5}{17}$$

$$\Rightarrow \frac{\frac{5}{2}(2a+4d)}{\frac{9}{2}(2a+8d)} = \frac{5}{17}$$

$$\therefore 4a = d$$

$$a_{15} = a + 14d = 57a$$

It is given that $110 < a_{15} < 120$

$$\Rightarrow 110 < 57a < 120$$

For integral terms of the A.P, $a = 2$

Sum of 10 terms of A.P:

$$S_{10} = \frac{10}{2}(4 + 9 \times 8) = 380$$

Question: Let S be sample space for 5 digit numbers. If p is probability of a number being randomly selected which is multiple of 7 but not divisible by 5, then $9p$ is equal to:

Options:

- (a) 1.0146
- (b) 1.2085
- (c) 1.0285
- (d) 1.1521

Answer: (c)

Solution:

Five digit number line from 10000 to 99999

$$\therefore S = 90000$$

$$\text{Number divisible by 7} = \frac{90000}{7}$$

$$\text{Number divisible by 7 and multiple by 5} = \frac{90000}{35}$$

$$\therefore \text{Required Probability} = \frac{\frac{90000}{7} - \frac{90000}{35}}{90000}$$

$$\Rightarrow p = \frac{4}{35}$$

$$\therefore 9p = \frac{36}{35} = 1.02857$$

Question: Let $A = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}$, α & β belongs to real numbers such that $\alpha A^2 + \beta A = 2I$,

where I is an identity matrix of order 2×2 . Then the value of $\alpha + \beta$ is equal to:

Options:

- (a) -10
- (b) -6
- (c) 6
- (d) 10

Answer: (d)

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix}$$

It is given that $\alpha A^2 + \beta A = 2I$

$$\alpha \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix} + \beta \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$-3\alpha + \beta = 2 \quad \& \quad -8\alpha + 2\beta = 0$$

$$\alpha = 2, \beta = 8$$

$$\Rightarrow \alpha + \beta = 10$$

Question: $(p \wedge r) \Leftrightarrow (p \wedge \sim q)$ which is equivalent to $\sim p$. Then r will be:

Options:

- (a) p
- (b) $\sim p$
- (c) q
- (d) $\sim q$

Answer: (c)

Solution:

The truth table is

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$p \wedge q \Leftrightarrow p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

Clearly $(p \wedge r) \Leftrightarrow (p \wedge \sim q) \equiv \sim p$

$\therefore r = q$

Question: The remainder of $(2021)^{2022} + (2022)^{2021}$ when divided by 7 is:

Answer: 0.00

Solution:

$$\text{Let } S = (2021)^{2022} + (2022)^{2021}$$

$$\Rightarrow (2023 - 2)^{2022} + (2023 - 1)^{2021}$$

$$= 7k_1 + 2^{2022} + 7k_2 - 1$$

$$= 7(k_1 + k_2) + 8^{674} - 1$$

$$= 7(k_1 + k_2) + (7 + 1)^{674} - 1$$

$$= 7(k_1 + k_2) + 7k_3 + 1 - 1$$

$$= 7(k_1 + k_2 + k_3)$$

Therefore, S is divisible by 7

Question: The mean and variance of 10 observation was 15 and 15. The mistake was 25 instead of 15. The new standard deviation is:

Answer: 0

Solution:

$$\frac{x_1 + \dots + x_9 + 25}{10} = 15$$

$$\frac{x_1 + \dots + x_9 + 15}{10} = \text{correct mean} = m$$

$$\frac{25}{10} - \frac{15}{10} = 15 - m$$

$$m = 4$$

Correct mean is 4.

$$\frac{x_1^2 + \dots + x_9^2 + 25^2}{10} - 15^2 = 15$$

$$\frac{x_1^2 + \dots + x_9^2 + 15^2}{10} - 14^2 = \text{correct variance} = \text{new SD}$$

$$\frac{25^2 - 15^2}{10} - (15^2 - 14^2) = 15 - v$$

Variance is 4, SD is 2.

Question: Let $f(x) = 2x^2 - x - 1$ and $S = \{n : |f(n)| \leq 800\}$ where $n \in \mathbb{Z}$, then $\sum_{n \in S} f(n) =$

Answer: 10620.00

Solution:

$$-800 \leq f(n) \leq 800$$

$$-800 \leq 2n^2 - n - 1 \leq 800$$

$$2n^2 - n + 799 \geq 0$$

$$a > 0$$

$$D = 1 - 4(2)(799) < 0$$

Always true

$$n \in \mathbb{R}$$

$$2n^2 - n - 801 \leq 0$$

$$n = \frac{1 \pm \sqrt{1 + 4(2)(801)}}{4}$$

$$= \frac{1 \pm \sqrt{6408}}{4}$$

$$n \approx \frac{1 \pm 80}{4}$$

$$n = \frac{-79}{4}, \frac{81}{4}$$

$$n \in [-19.75, 20.25]$$

$$n \in \{-19, -18, -17, \dots, -1, 0, 1, \dots, 20\}$$

$$f(n) = 2n^2 - n - 1$$

$$f(-19) = 2(-19)^2 - (-19) - 1$$

\vdots

$$f(19) = 2(19)^2 - (19) - 1$$

$$f(20) = 2(20)^2 - (20) - 1$$

$$= 2[(-19)^2 + (-18)^2 + \dots + (-1)^2 + 0^2 + (1)^2 + \dots + 19^2 + 20^2]$$

$$= -[(-19) + (-18) + \dots + (-1) + 0 + (1) + \dots + (19) + (20)] - 40$$

$$= 2 \left[400 + 2 \left(\frac{19 \times 20 \times 39}{6} \right) \right] - 20 - 40$$
$$= 10620$$