QUESTION PAPER CODE 65/1/F

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\vec{a} + \vec{b} = 6\hat{i} + \hat{k}$ ½ m

$\therefore$ Reqd. unit vector $= \frac{6}{\sqrt{37}}\hat{i} + \frac{1}{\sqrt{37}}\hat{k}$ ½ m

2. Reqd. area $= \left|\vec{a} \times \vec{b}\right|$ ½ m

$\therefore \left|12\hat{i} - 4\hat{j} + 8\hat{k}\right| = \sqrt{144+16+64} = \sqrt{224}$ or $4\sqrt{14}$ sq. units ½ m

3. Getting $x-\text{intercept} = \frac{5}{2}$, $y-\text{intercept} = 5$, $z-\text{intercept} = -5$ ½ m

$\therefore$ Their sum $= \frac{5}{2}$ ½ m

4. $\text{co}-\text{factor of } a_{21} = 3$ 1 m

5. Degree = Order = 2 any one correct ½ m

$\therefore$ Degree + order = 4 ½ m

6. $2^y\, dy = dx \Rightarrow \frac{2^y}{\log 2} = x + c$ ½+½ m

SECTION - B

7. Getting $A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$ 1 m

$$4A - 3I = \begin{bmatrix} 8-3 & -4 \\ -4 & 8-3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} = A^2$$ 1 m

$A^2 = 4A - 3I$ ....(i)

Multiply both sides by A^{-1} ½ m

$$A = 4I - 3A^{-1} \text{ or } A^{-1} = \frac{1}{3}(4I - A) = \frac{1}{3}\begin{pmatrix} 4-2 & 0+1 \\ 0+1 & 4-2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$ 1½ m

OR

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},\ B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a^2+b & a-1 \\ b(a-1) & b+1 \end{bmatrix}$$ 1½ m

$$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a) - 2(2+b) & 4 \end{pmatrix} \text{ .... (i)}$$ 1½ m

$$A^2 + B^2 = \begin{pmatrix} a^2+b-1 & a-1 \\ b(a-1) & b \end{pmatrix} \text{ ....(ii)}$$

Equating (i) and (ii), we get $b = 4, a = 1$ 1 m

8. Using $C_1 \to C_1 + C_2 + C_3$ and taking $a^2 + a + 1$ common from C_1

$$\Delta = (a^2 + a + 1)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}, \text{ using } R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$$ 1½ m

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & a(a-1) & (1-a)(1+a) \end{vmatrix} = (a^2+a+1)(1-a)^2\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & 1+a \end{vmatrix}$$ 1½ m

$$\left.\begin{aligned} &= (a^2+a+1)(1-a)^2(1+a+a^2) \\ &= [(1-a)(1+a+a^2)]^2 = (1-a^3)^2 \end{aligned}\right\}$$ 1 m

9. Let $x+a=t \Rightarrow dx = dt$ and $x = t-a \Rightarrow x-a = t-2a$ 1 m

$$\therefore I = \int \frac{\sin(t-2a)\,dt}{\sin t} = \int \frac{(\sin t \cos 2a - \cos t \cdot \sin 2a)\,dt}{\sin t}$$ 1 m

$$= \cos 2a \int dt - \sin 2a \int \cot t\,dt = \cos 2a\ t - \sin 2a . \log|\sin t| + c$$ 1 m

$$= \cos 2a\,(x+a) - \sin 2a \log|\sin(x+a)| + c$$ 1

OR

consider $\dfrac{x^2}{(x^2+4)(x^2+9)}$ · Let $x^2 = t$ ½ m

$$\therefore \frac{t}{(t+4)(t+9)} = -\frac{4}{5}\,\frac{1}{t+4} + \frac{9}{5}\,\frac{1}{t+9}$$ 1 m

$$I = \int \frac{t\quad dt}{(t+4)(t+9)} = -\frac{4}{5}\int \frac{dt}{t+4} + \frac{9}{5}\int \frac{dt}{t+9}$$ ½ m

$$= \frac{-4}{5}\log|t+4| + \frac{9}{5}\log|t+9| + c$$ 1½ m

$$\therefore I = -\frac{4}{5}\log|x^2+4| + \frac{9}{5}\log|x^2+9| + c$$ ½ m

10. Writing given integral as 1 m

$$I = \int_{-\pi/2}^{0} \frac{\cos x}{1+e^x}\,dx + \int_{0}^{\pi/2} \frac{\cos x}{1+e^x}\,dx$$ Let $x = -t$, $dx = -dt$ 1½ m

when $x = -\dfrac{\pi}{2}$, $t = \pi/2$

$x = 0$, $t = 0$

$$\therefore\ I = \int_0^{\frac{\pi}{2}} \frac{e^t \cos t}{1+e^t}\,dt + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx = \int_0^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx$$ 1½ m

$$I = \int_0^{\frac{\pi}{2}} \frac{(1+e^x)\cos x\,dx}{(1+e^x)} = \int_0^{\frac{\pi}{2}} \cos x\,dx = (\sin x)_0^{\frac{\pi}{2}} = 1$$ 1 m

11. Let B_1, B_2, B_3 be the events that the bolts produced by machines E_1, E_2, E_3 and A be the event that the selected bulb is defective ½ m

$$\therefore\ P(B_1) = \frac{1}{2},\ P(B_2) = P(B_3) = \frac{1}{4}$$

$$P\left(A/B_1\right) = \frac{1}{25},\ P\left(A/B_2\right) = \frac{1}{25},\ P\left(A/B_3\right) = \frac{1}{20}$$ 1½ m

$$P(A) = \sum_{c=1}^{3} P(B_c)\,P\left(A/B_c\right) = \frac{1}{2}\times\frac{1}{25} + \frac{1}{4}\times\frac{1}{20} + \frac{1}{4}\times\frac{1}{25}$$ 1+½ m

$$= \frac{17}{400}$$ ½ m

OR

$$\therefore\ P(x=3) = \frac{1}{6}\times\frac{1}{5}\times 2 = \frac{1}{15},\ P(x=4) = \frac{2}{6}\times\frac{1}{5}\times 2 = \frac{2}{15}$$

Simlarly $P(x=5) = \frac{3}{15},\ P(x=6) = \frac{4}{15},\ P(x=7) = \frac{5}{15}$

Prob. distribution is

x :	3	4	5	6	7
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

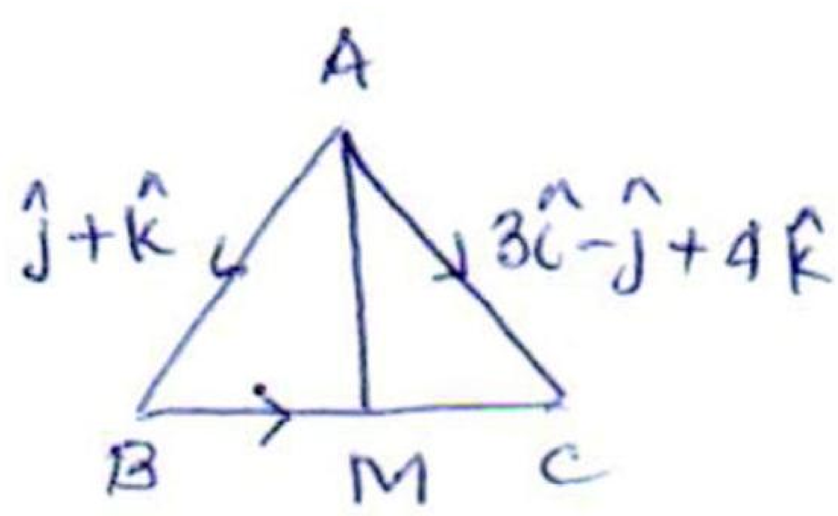
2 m

$x \cdot P(x)$:	$\frac{3}{15}$	$\frac{8}{15}$	$\frac{15}{15}$	$\frac{24}{15}$	$\frac{35}{15}$
$x^2 P(x)$:	$\frac{9}{15}$	$\frac{32}{15}$	$\frac{75}{15}$	$\frac{144}{15}$	$\frac{245}{15}$

$$\text{Mean} = \sum x_i \cdot P(x_i) = \frac{85}{15} = \frac{17}{3}$$ 1 m

$$\text{Variance} = \sum x_i^2 \; P(x_i) - (\text{Mean})^2 = \frac{101}{3} - \frac{289}{9} = \frac{14}{9}$$ 1 m

12.



$$\vec{BC} = \left(3\hat{i} - \hat{j} + 4\hat{k}\right) - \left(\hat{j} + \hat{k}\right) = 3\hat{i} - 2\hat{j} + 3\hat{k}$$ 1½ m

$$\therefore \; \vec{BM} = \frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}$$ 1 m

$$AM = \left| \hat{j} + \hat{k} + \frac{3\hat{i} - 2\hat{j} + 3\hat{k}}{2} \right| = \left| \frac{3\hat{i} + 5\hat{k}}{2} \right| = \frac{\sqrt{34}}{2}$$ 1½ m

13. Any plane through given point is $a(x-3) + b(y-6) + c(z-4) = 0$..........(i) 1 m

with $a + 5b + 4c = 0$ ....(A) ½ m

(i) passes through (3, 2, 0) $\Rightarrow -4b - 4c = 0$ or $b + c = 0$ ....(B) ½ m

From (A) and (B) $a + b + (4b + 4c) = 0 \Rightarrow a = -b$ 1 m

$$\left.\begin{array}{l} \therefore \; a = -b = c \\ \therefore \; \text{Required eqn. of plane is } x - y + z - 1 = 0 \end{array}\right\}$$ 1 m

14. $\text{LHS} = \tan^{-1}\left(\frac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}\frac{2\cos\theta}{\sin^2\theta}$ 2 m

$\therefore \tan^{-1}\frac{2\cos\theta}{\sin^2\theta} = \tan^{-1}\left(\frac{2}{\sin\theta}\right)$ 1 m

$\Rightarrow \cot\theta = 1 \text{ or } \theta = \pi/4$ 1 m

OR

The given equation can be written

$\left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)+\left(\tan^{-1}4-\tan^{-1}3\right)+.....+\tan^{-1}(n+1)-\tan^{-1}n=\tan^{-1}\theta$ 2 m

$\Rightarrow \tan^{-1}(n+1)-\tan^{-1}1 = \tan^{-1}\theta$ 1 m

$\Rightarrow \tan^{-1}\frac{n+1-1}{1+(n+1)} = \tan^{-1}\theta \Rightarrow \theta = \frac{n}{n+2}$ 1 m

15. $9y^2 = x^3 \Rightarrow 18y\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \text{slope of tangent} = \frac{x^2}{6y}$

$\therefore$ Slope of normal $= -\frac{6y}{x^2}$ 1+½ m

As the intercepts by normal on both axes are equal

$\therefore$ Slope of normal $= \pm 1 \Rightarrow \frac{-6y}{x^2} = \pm 1 \Rightarrow y = \pm\frac{x^2}{6}$ 1 m

$\therefore 9\left(\frac{x^4}{36}\right) = x^3 \Rightarrow x = 4 \text{ and } y^2 = \frac{64}{9} \Rightarrow y = \pm 8/3$ 1 m

$\therefore$ The points are $\left(4, \frac{8}{3}\right), \left(4, -8/3\right)$ ½ m

16. $\frac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left[1+\frac{x}{\sqrt{1+x^2}}\right] = \frac{n}{\sqrt{1+x^2}}\left[x+\sqrt{1+x^2}\right]^n = \frac{ny}{\sqrt{1+x^2}}$ 1½ m

$\therefore \sqrt{1+x^2}\,\frac{dy}{dx} = ny$............(i) ½ m

$\therefore \sqrt{1+x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{x}{\sqrt{1+x^2}} = n\frac{dy}{dx}$ 1 m

$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{1+x^2}\,\frac{dy}{dx} = n\cdot ny$ (from (i))

$= n^2 y$ 1 m

17. L H D at $x=1$: $\lim_{x\to 1^-}\left(\frac{x-1}{x-1}\right) = 1$

R H D at $x=1$, $\lim_{x\to 1^+}\frac{2-x-1}{x-1} = -1$

2 m

$\therefore$ f is not differentiable at $x=1$

L H D at $x=2$, $\lim_{x\to 2^-}\frac{2-x-0}{x-2} = -1$

R H D at $x=2$, $\lim_{x\to 2^+}\frac{-2+3x-x^2}{(x-2)} = \lim_{x\to 2^+} -\frac{(x-1)(x-2)}{(x-2)} = -1$

2 m

$\therefore$ f is diff. at $x=2$

18. Communication Matrix $A = \begin{pmatrix}140\\200\\150\end{pmatrix}\begin{matrix}\text{Telephone}\\\text{House calls}\\\text{Letters}\end{matrix}$

$$\begin{array}{cc} & \begin{array}{ccc}\text{Tele} & \text{House calls} & \text{Letters}\end{array} \\ \text{Cost Matrix } B = & \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{array}{l}\text{City x} \\ \text{City y}\end{array}\end{array}$$

$$\therefore \text{ Total cost Matrix } = \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix}\begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} = \begin{pmatrix} 990000 \\ 2120000 \end{pmatrix}$$ 3 m

any relevant value 1 m

19. $I = \int e^{2x} \sin(3x+1)\, dx = \left[\frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \int e^{2x} \cos(3x+1)\, dx\right]$ 1½ m

$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \cdot \frac{e^{2x}}{2} \cdot \cos(3x+1) - \frac{9}{4} \int e^{2x} \sin(3x+1)\, dx \Big]$ 1 m

$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) - \frac{9}{4} I$ 1 m

$$\left.\begin{array}{l} \frac{13}{4} I = \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) \\ I = \frac{4}{13}\left[\frac{e^{2x}}{2}\left(-\frac{3}{2} \cos(3x+1) + \sin(3x+1)\right)\right] + c \end{array}\right\}$$ ½ m

SECTION - C

20. Let $x_1, x_2, \in R$ such that $f(x_1) = f(x_2) \Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ 1½+½ m

$\Rightarrow 4(x_1 - x_2)[x_1 + x_2 + 3] = 0 \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one – one

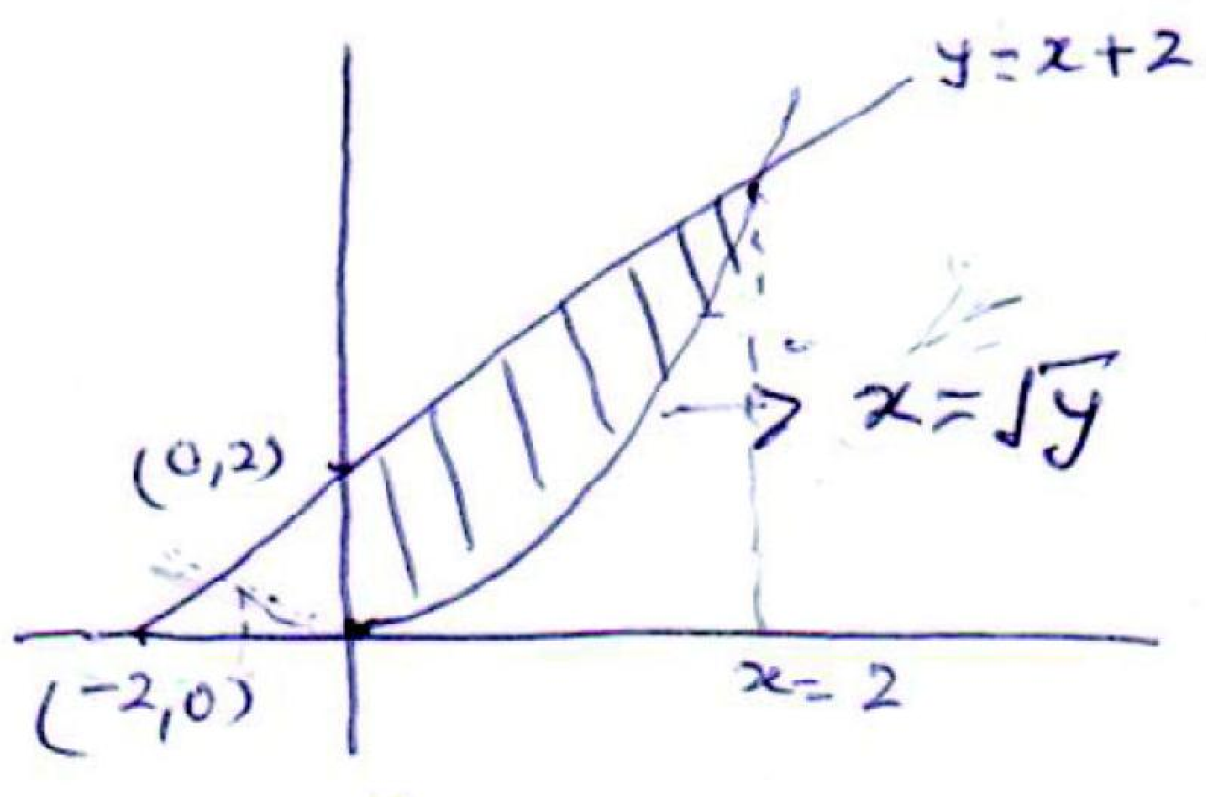
f is clearly onto and hence intertible 1 m

Let y be an arbitrary element of S

$f(x) = y = 4x^2 + 12x + 15 = (2x+3)^2 + 6$ 1 m

$\therefore f^{-1}: R \to S$ is given by $f^{-1}(y) = \left(\frac{\sqrt{y-6} - 3}{2}\right)$ 2 m

21.



Correct Figure | 1m

Points of intersection

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$ (-1 is rejected) | 1½ m

$\therefore$ Reqd. area $= \int_0^2 \{(x+2) - x^2\}dx$ | 1½ m

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_0^2$

$= \left(2 + 4 - \frac{8}{3}\right) = \frac{10}{3}$ sq.units | 2 m

22. Let $z = ax + by$, also $xy = c^2 \Rightarrow y = \frac{c^2}{x}$

$\therefore \quad z = ax + \frac{bc^2}{x}$ | ½ + ½ m

$\therefore \frac{dz}{dx} = a + bc^2\left(\frac{-1}{x^2}\right), \frac{dz}{dx} = 0 \Rightarrow bc^2 = ax^2$

or $x = \sqrt{\frac{b}{a}}\, c$ | 1½ m

showing $\frac{d^2z}{ax^2}$ at $x = \sqrt{\frac{b}{a}}\, c > 0 \Rightarrow$ minima | 1½ m

$y = \frac{c^2}{x} = \frac{c^2}{c}\sqrt{\frac{a}{b}} = c\sqrt{\frac{a}{b}}$ | 1 m

$\therefore$ minimum $z = a\sqrt{\frac{b}{a}}\, c + bc\sqrt{\frac{a}{b}}\, c = 2\sqrt{ab}\, c$ | 1 m

OR

$y = x^2 + 7x + 2$, $3x - y - 3 = 0$ ....(i)

$\therefore\ 3x - \left(x^2 + 7x + 2\right) - 3 = 0$ 1 m

Distance of (x, y) from (i)

$$D = \left| \frac{3x - \left(x^2 + 7x + 2\right) - 3}{\sqrt{10}} \right| \quad \text{or} \quad D = \left| \frac{\left(-x^2 - 4x - 5\right)}{\sqrt{10}} \right| = \left| \frac{(x+2)^2 + 1}{\sqrt{10}} \right|$$ 2 m

$$\frac{dD}{dx} = \frac{2}{\sqrt{10}}\,(x+2), \quad \frac{dD}{dx} = 0 \text{ at } x = -2$$ 1 m

$$\frac{d^2D}{dx^2} > 0 \Rightarrow \text{minima}$$ 1 m

$\therefore$ D is minimum at x = – 2

at x = – 2, y = – 8 1 m

$\therefore$ The required pt. on the parabola is $(-2, -8)$

23.

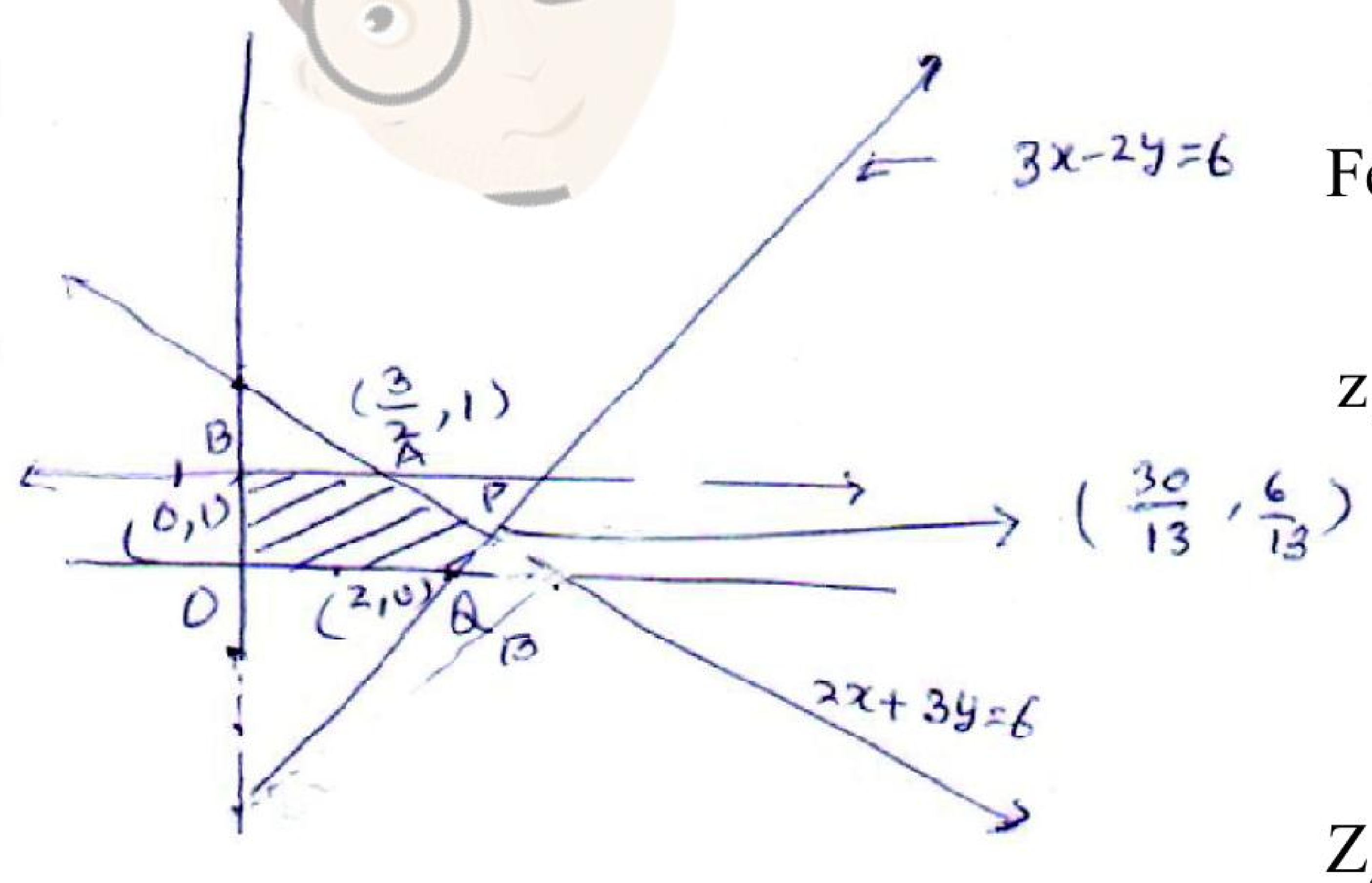


Figure 3 m

Feasible region is B A PQO 1 m

$$z_B = 9,\ z_P = \frac{240}{13} + \frac{54}{13} = \frac{294}{13} = 22\frac{8}{13}$$ 1 m

$Z_Q = 16$

$\therefore$ Z is maximum at $\left(\dfrac{30}{13}, \dfrac{6}{13}\right)$

and maximum value $= 22\dfrac{8}{13}$ 1 m

24. Any line through $(1,-2,3)$ with d. r's as $2, 3 - 6$ is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$ 1 ½ m

$$\therefore\ x = 2\lambda + 1,\ y = 3\lambda - 2,\ z = -6\lambda + 3$$ 1½ m

It lies on the plane $x - y + z = 5$

$$\therefore\ 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$$

$$\Rightarrow\ \lambda = \frac{1}{7}$$ 1 m

Reqd. point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$ 1 m

$$\therefore\ \text{Reqd distance} = \sqrt{\left(\frac{9}{7} - 1\right)^2 + \left(-\frac{11}{7} + 2\right)^2 + \left(\frac{15}{7} - 3\right)^2} = \frac{1}{7}\sqrt{4 + 9 + 36} = \frac{7}{7} = 1$$ 1 m

25. $$\frac{dy}{dx} = \frac{2x \sin\left(\frac{y}{x}\right) - y \cos\left(\frac{y}{x}\right)}{y - x\cos\left(\frac{y}{x}\right)} = \frac{2\sin\left(\frac{y}{x}\right) - \frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{y}{x} - \cos\left(\frac{y}{x}\right)}$$ 1 m

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$$\therefore\ v + x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v} \Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v^2}{v - \cos v}$$ 1 m

$$\Rightarrow \frac{v - \cos v}{-2\sin v + v^2} = \frac{-dx}{x} \text{ or } \frac{1}{2}\left[\frac{2v - 2\cos v}{-2\sin v + v^2}\right]dv = -\frac{dx}{x}$$ 1 m

$$\left.\begin{aligned}&\Rightarrow \frac{1}{2}\log\left|v^2-2\sin v\right| = -\log x+\log c\\ &\text{or } \log\left|\sqrt{v^2-2\sin v}\right| = \log c-\log x\end{aligned}\right\}$$ 1 m

$$\sqrt{v^2-2\sin v} = \frac{c}{x}$$

$$\text{or } x\sqrt{\frac{y^2}{x^2} - 2\sin\frac{y}{x}} = c$$ ½ m

$$y^2-2x^2\sin\left(\frac{y}{x}\right) = c'$$ ½ m

OR

$$\left(\sqrt{(1+x^2)(1+y^2)}\right)dx + xy\,dy = 0$$

$$\Rightarrow \frac{y}{\sqrt{1+y^2}}\,dy + \frac{\sqrt{1+x^2}}{x}\,dx = 0$$ 1 m

$$\frac{1}{2}\int\frac{2y}{\sqrt{1+y^2}}\,dy + \int\frac{\sqrt{1+x^2}}{x}\,dx = 0$$

$$\sqrt{1+y^2} + \int\frac{(1+x^2)}{x\sqrt{1+x^2}}\,dx = c$$ 1½ m

$$I_1 = \int\frac{1}{x\sqrt{1+x^2}}\,dx + \int\frac{x}{\sqrt{1+x^2}}\,dx = I_2+\sqrt{1+x^2}$$ 1 m

For I_2, Let $x=\frac{1}{t}$, $dx = \frac{-1}{t^2}\,dt$ 1 m

$$I_2 = \int\frac{-1}{t^2\cdot\frac{1}{t}\sqrt{1+\frac{1}{t^2}}}\,dt = -\int\frac{dt}{\sqrt{t^2+1}} = -\log\left[t+\sqrt{1+t^2}\right]$$ 1 m

$$= -\log\left[\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right] = -\log\left[\frac{1+\sqrt{1+x^2}}{x}\right]$$

$\therefore$ The solution is $\sqrt{1+x^2}+\sqrt{1+y^2} - \log\left(\frac{1+\sqrt{1+x^2}}{2}\right) = c$ ½ m

26. $P(\text{Doublet}) = \frac{1}{6}$, $P(\text{not a doublet}) = 5/6$ 1 m

The random variate x can take values 0, 1, 2, 3, 4

x	0	1	2	3	4	
P(x)	$\left(\frac{5}{6}\right)^4$	$4\,\frac{1}{6}\left(\frac{5}{6}\right)^3$	$6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$	$4\left(\frac{1}{6}\right)^3\frac{5}{6}$	$\left(\frac{1}{6}\right)^4$	
	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$	2½ m

Mean $= \sum x\,P(x) = \frac{500+300+60+4}{1296} = \frac{864}{1296} = \frac{2}{3}$ 1 m

$\sum x^2\,P(x) = \frac{500+600+180+16}{1296} = \frac{1296}{1296} = 1$ 1 m

$\therefore$ Variance $= 1-\left(\frac{2}{3}\right)^2 = \frac{5}{9}$ ½ m