QUESTION PAPER CODE 65/1/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{dy}{dx} = \cos x$ $\frac{1}{2}$

Slope of tangent at (0, 0) is 1

Equation of tangent is y = x $\frac{1}{2}$

2. π 1

3. Order of AB is 3 × 4 1

4. Putting (1 + log x) or log x = t $\frac{1}{2}$

$\log|1 + \log x| + C$ $\frac{1}{2}$

SECTION B

5. Differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a$$ 1

Eliminating 4a, we get

$$y^2 = 2y\frac{dy}{dx}\cdot x$$

or $$2xy\frac{dy}{dx} - y^2 = 0$$ 1

6. Let number of large vans = x

and number of small vans = y

Minimize cost z = 400x + 200y $\frac{1}{2}$

Subject to constraints

$$\left.\begin{array}{l} 200x + 80y \geq 1200 \text{ or } 5x + 2y \geq 30 \\ x \leq y \\ 400x + 200y \leq 3000 \text{ or } 2x + y \leq 15 \end{array}\right\}$$ $1\frac{1}{2}$

$x \geq 0,\ y \geq 0$

7. $R_2 \to R_2 + R_1$ implies

$$\begin{pmatrix} 2 & 3 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & -3 \\ 17 & -7 \end{pmatrix}$$ 1+1

1 mark for pre matrix on LHS and 1 mar for matrix on RHS

8. Integrating factor is $e^{\int 2dx} = e^{2x}$ $\frac{1}{2}$

$\therefore$ Required solution is

$$y.e^{2x} = \int e^{3x} \cdot e^{2x} dx$$ $\frac{1}{2}$

$$y.e^{2x} = \frac{e^{5x}}{5} + C$$ 1

or $$y = \frac{e^{3x}}{5} + Ce^{-2x}$$

9. $\frac{dr}{dt} = -3$ cm/min, $\frac{dh}{dt} = 2$ cm/min $\frac{1}{2}$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left[r^2 \frac{dh}{dt} + 2hr\frac{dr}{dt}\right]$$ 1

$$\left(\frac{dV}{dt}\right)_{at\ r=9,\ h=6} = -54\pi \text{ cm}^3/\text{min}$$ $\frac{1}{2}$

$\Rightarrow$ Volume is decreasing at the rate 54π cm^3/min.

10. $$11\hat{i} - 3\hat{j} = \frac{3(12\hat{i} + \mu\hat{j}) + 1(\lambda\hat{i} + 3\hat{j})}{4}$$ 1

$$44 = 36 + \lambda, \; -12 = 3\mu + 3$$

$$\lambda = 8, \; \mu = -5$$ $\frac{1}{2} + \frac{1}{2}$

11. Given integral becomes

$$\int \sqrt{1-(x-1)^2}\, dx$$ 1

$$= \frac{(x-1)}{2}\sqrt{2x-x^2} + \frac{1}{2}\sin^{-1}(x-1) + C$$ 1

12. $\lim_{x \to 0} f(x) = f(0)$ $\frac{1}{2}$

$$\lim_{x \to 0} \frac{4 \times 2\sin^2 2x}{4x^2} = p$$ $1\frac{1}{2}$

$p = 8$ $\frac{1}{2}$

SECTION C

13. Let $\frac{x^2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$ 1

$$A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$$ $1\frac{1}{2}$

Thus integral becomes

$$\frac{1}{2}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{xdx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{2}\log|x-1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$ $1\frac{1}{2}$

14. $\overrightarrow{AB} = \hat{i} + (x-3)\hat{j} + 4\hat{k}$

$\overrightarrow{AC} = \hat{i} - 3\hat{k}$

$\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ $1\frac{1}{2}$

As A, B, C & D are coplanar

$\therefore \quad \overrightarrow{AB}\cdot(\overrightarrow{AC} \times \overrightarrow{AD}) = 0$

i.e. $\begin{vmatrix} 1 & x-3 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$ $1\frac{1}{2}$

which gives

$x = 6$ 1

15. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y\cos\frac{y}{x} + x}{x\cos\frac{y}{x}} \qquad \text{....(i)}$$ $\frac{1}{2}$

Clearly it is homogenous

Let $\frac{y}{x} = v,\ \frac{dy}{dx} = v + \frac{dv}{dx}$ 1

(1) becomes

$$v + x\frac{dv}{dx} = v + \sec v$$

$$\Rightarrow \quad \cos v\, dv = \frac{dx}{x}$$ 1

integrating both sides we get

$$\sin v = \log|x| + C$$ 1

$$\sin\frac{y}{x} = \log|x| + C$$ $\frac{1}{2}$

16. Putting $x = \cos\theta$ 1

LHS becomes

$$\tan^{-1}\left(\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta/2 - \sqrt{2}\sin\theta/2}{\sqrt{2}\cos\theta/2 + \sqrt{2}\sin\theta/2}\right)$$ 1

$$= \tan^{-1}\left(\frac{1-\tan\theta/2}{1+\tan\theta/2}\right)$$ 1

$$= \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$ $\frac{1}{2}$

$$= \frac{\pi}{4} - \frac{\theta}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x = \text{RHS}$$ $\frac{1}{2}$

17. Taking x, y, z common from C_1, C_2, C_3 respectively, we get

$$xyz\begin{vmatrix} a/x & b/y-1 & c/z-1 \\ a/x-1 & b/y & c/z-1 \\ a/x-1 & b/y-1 & c/z \end{vmatrix} = 0$$ 1

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a/x+b/y+c/z-2 & b/y-1 & c/z-1 \\ a/x+b/y+c/z-2 & b/y & c/z-1 \\ a/x+b/y+c/z-2 & b/y-1 & c/z \end{vmatrix} = 0$$ 1

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 1 & b/y & c/z-1 \\ 1 & b/y-1 & c/z \end{vmatrix} = 0$$ $\frac{1}{2}$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$ 1

$$\therefore \quad \left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\cdot 1 = 0 \Rightarrow \frac{a}{x}+\frac{b}{y}+\frac{c}{z} = 2$$ $\frac{1}{2}$

OR

We know that

$IA = A$ 1

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ 0 & -5 \end{pmatrix}$$ 1

$R_2 \to \frac{R_2}{-5}$

$$\begin{pmatrix} 1 & 0 \\ 2/5 & -1/5 \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$ $\frac{1}{2}$

$R_1 \to R_1 - 2R_2$

$$\begin{pmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ 1

$$\therefore \quad A^{-1} = \begin{pmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{pmatrix}$$ $\frac{1}{2}$

Full marks for finding correct A^{-1} using column transformations with $AI = A$

18. Differentiating $y = \cos(x + y)$ wrt x we get

$$\frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$$ 1

Slope of given line is $\frac{-1}{2}$ $\frac{1}{2}$

As tangent is parallel to line $x + 2y = 0$

$$\therefore \quad \frac{-\sin(x+y)}{1+\sin(x+y)} = \frac{-1}{2}$$

$\Rightarrow \quad \sin(x + y) = 1$

$\Rightarrow \quad x + y = n\pi + (-1)^n \frac{\pi}{2}, n \in Z$ ...(1) 1

Putting (1) in $y = \cos(x + y)$

we get $y = 0$

$\Rightarrow \quad x = n\pi + (-1)^n \pi/2, n \in Z$

$x = \frac{-3\pi}{2} \in [-2\pi, 0]$ $\frac{1}{2}$

$\therefore$ Required equation of tangent is

$$y = \frac{-1}{2}\left(x + \frac{3\pi}{2}\right)$$

or $\quad 2y + x + \frac{3\pi}{2} = 0$ 1

19. Given equation of lines can be written as

$$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{1} \quad ...(1) \qquad 1$$

$$\frac{x-1}{-3p/7} = \frac{y-5}{-1} = \frac{z-11}{-7} \quad ...(2) \qquad 1$$

(1) & (2) are perpendicular

So $-3\left(\frac{-3p}{7}\right) + \frac{2p}{7}(-1) + 1(-7) = 0$ 1

which gives $p = 7$ 1

OR

Required equation of plane is $x + y + z - 1 + \lambda(2x + 3y + 4z - 5) = 0$ for some λ. 1

i.e. $(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z = 1 + 5\lambda$

according to question

$$2\left(\frac{1+5\lambda}{1+3\lambda}\right) = 3\left(\frac{1+5\lambda}{1+4\lambda}\right) \qquad 1$$

Solving we get $\lambda = -1$ 1

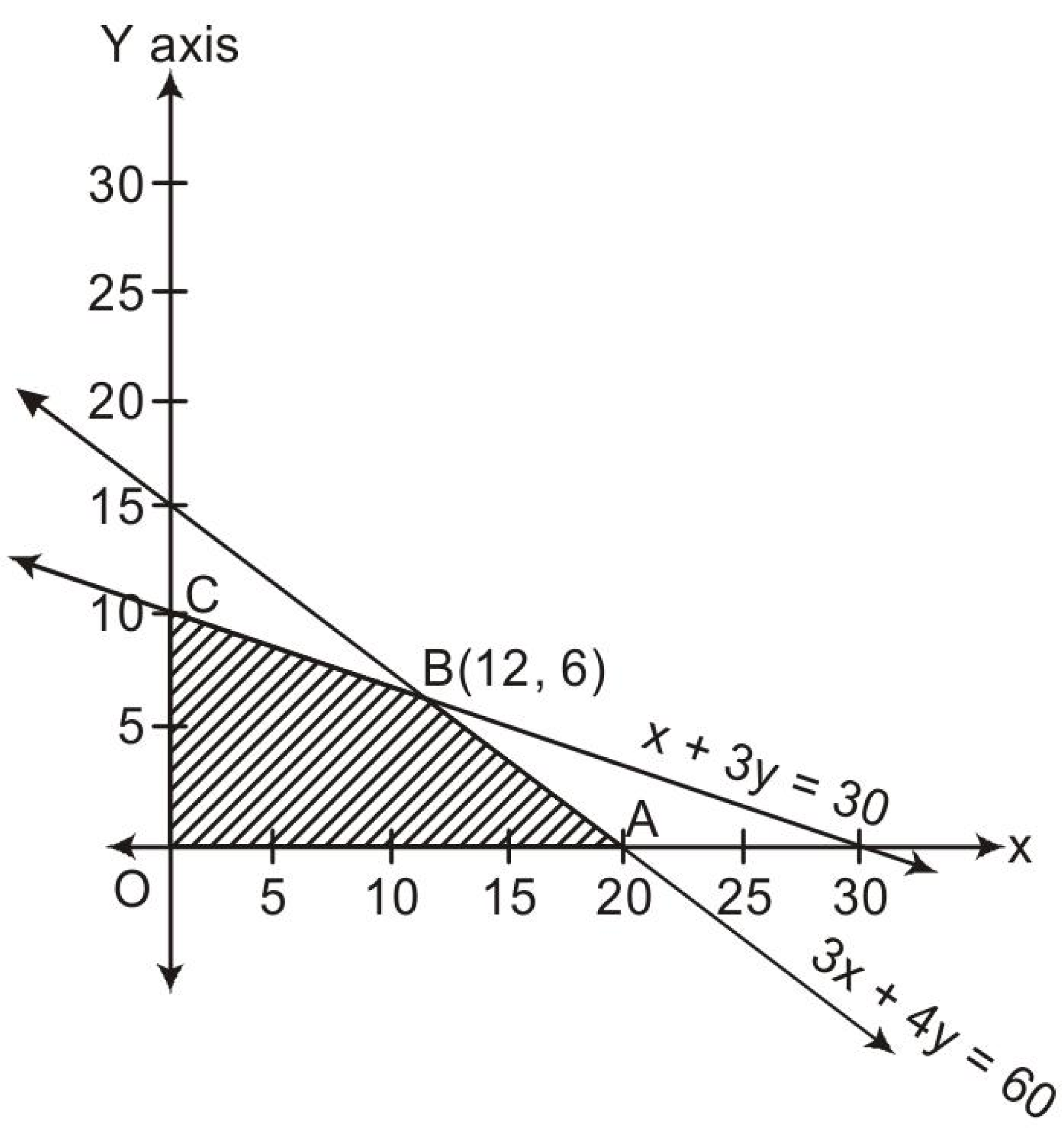
Thus the equation of required plane is

$-x - 2y - 3z = -4$

or $x + 2y + 3z = 4$ 1

20. **Correct lines** 2

Correct shading 1



Corner points	Value of Z	
O(0, 0)	0	
A(20, 0)	160000	
B(12, 6)	168000 → max	$\frac{1}{2}$
C(0, 10)	120000	

Maximum Z = 168000

at x = 12, y = 6 $\frac{1}{2}$

21. Given integral $= \int \frac{x+7}{(3x+4)(x+7)}\,dx$ 2

$= \int \frac{1}{3x+4}\,dx$

$= \frac{1}{3}\log|3x+4| + C$ 2

22. $\frac{dx}{dt} = -3\sin t + 6\cos^2 t \sin t$

$= +3\sin t \cos 2t$ $1\frac{1}{2}$

$\frac{dy}{dt} = 3\cos t - 6\sin^2 t - \cos t$

$= 3\cos t \cos 2t$ 1

$\frac{dy}{dx} = \cot t$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2 t \frac{dt}{dx} = \frac{-1}{3}\frac{\operatorname{cosec}^3 t}{\cos 2t}$ 1

23. E_1: Student selected from category A

E_2: Student selected from category B

E_3: Student selected from category C

S: Student could not get good marks

$P(E_1) = \frac{1}{6}$ $P(E_2) = \frac{3}{6}$ $P(E_3) = \frac{2}{6}$ 1

$P(S/E_1) = 0.002$ $P(S/E_2) = 0.02$, $P(S/E_3) = 0.2$

$$P(E_3/S) = \frac{P(E_3)\,P(S/E_3)}{P(E_1)\,P(S/E_1) + P(E_2)\,P(S/E_2) + P(E_3)\,P(S/E_3)}$$

$$= \frac{\frac{2}{6}\times 0.2}{\frac{1}{6}\times .002 + \frac{3}{6}\times .02 + \frac{2}{6}\times 0.2}$$ 1

$$= \frac{200}{231}$$ 1

Value: Hardwork and Regularity 1

SECTION D

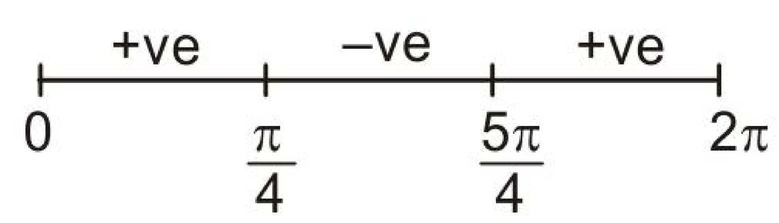
24. $f(x) = \sin x + \cos x \qquad 0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$ 1

$f'(x) = 0 \Rightarrow \cos x = \sin x$ 1

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ 1

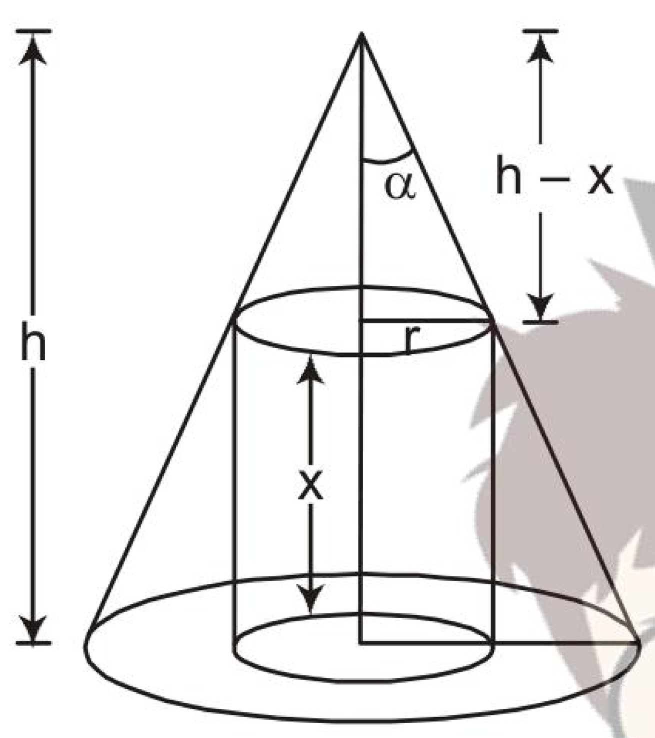
Sign of $f^1(x)$



2

So f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ and strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ 1

OR



For Figure 1

$\frac{r}{h-x} = \tan\alpha$ 1

$r = (h - x)\tan\alpha$

Volume of cylinder

$V = \pi r^2 x$

$V = \pi(h-x)^2 x \tan^2\alpha$ $\frac{1}{2}$

$\frac{dV}{dx} = \pi\tan^2(h-x)(h-3x)$

$\frac{dV}{dx} = 0 \Rightarrow h = x \text{ or } h = 3x$

i.e. $x = \frac{h}{3}$ $1\frac{1}{2}$

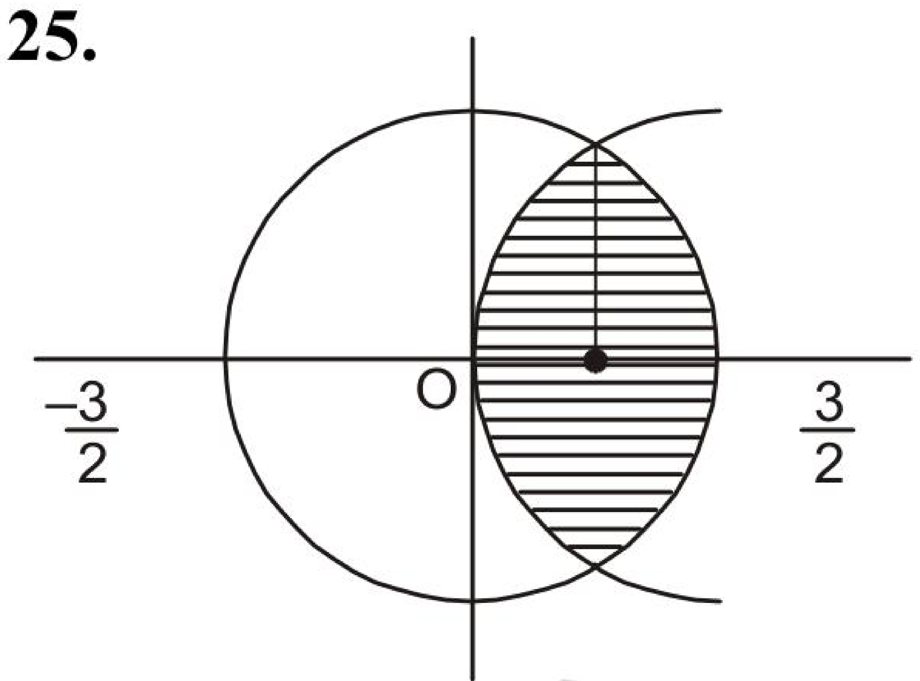
$$\frac{d^2V}{dx^2} = \pi \tan^2\alpha\,(6x - 4h)$$

$$\therefore \quad \frac{d^2V}{dx^2} < 0 \quad \text{at} \quad x = \frac{h}{3}$$

$$\therefore \quad \text{V is maximum at } x = \frac{h}{3}$$ 1

and maximum volume is $V = \frac{4}{27}\pi h^3 \tan^2 \alpha$ 1

25.



x coordinate of point of intersection is, $x = \frac{1}{2}$ 1

For Figure 1

Required area

$$= 2\left(\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \int_{\frac{1}{2}}^{3/2} \sqrt{\frac{9}{4} - x^2}\,dx\right)$$ 2

$$= 2\left[\frac{4}{3}x^{3/2}\Big|_0^{\frac{1}{2}} + \frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\Big|_{\frac{1}{2}}^{3/2}\right]$$ $\frac{1}{2}+1$

$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right) \quad \text{or} \quad \frac{\sqrt{2}}{6} + \frac{9}{4}\cos^{-1}\frac{1}{3}$$ $\frac{1}{2}$

26. $n = 8,\ P = \frac{1}{2},\quad q = \frac{1}{2}$ 1

(i) $P(X = 5) = 8C_5\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 = \frac{7}{32}$ $1\frac{1}{2}$

(ii) $P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8)$

$$= {}^8C_6\left(\frac{1}{2}\right)^8 + {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8$$ 2

$$= \frac{37}{256}$$ $\frac{1}{2}$

(iii) $P(X \le 6) = 1 - [P(X=7) + P(X=8)]$

$$= 1 - \frac{9}{256} = \frac{247}{256}$$ 1

OR

Let X denote number of red cards drawn

X(xi)	P(X)	p_i	$p_i x_i$	$p_i x_i^2$
0	${}^3C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	0	0
1	${}^3C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	${}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1$	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	${}^3C_3 \left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$

Correct table 4

$$\text{mean} = \Sigma p_i x_i = \frac{12}{8} = \frac{3}{2}$$ 1

$$\text{Variance} = \Sigma p_i x_i^2 - (\text{mean})^2$$

$$= 3 - \frac{9}{4} = \frac{3}{4}$$ 1

27. For one-one

Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ such that

$$f(x_1) = f(x_2)$$

$$\Rightarrow \quad \frac{4x_1}{3x_1 + 4} = \frac{4x_2}{3x_2 + 4}$$

$$\Rightarrow \quad 12 \cancel{x_1} x_2 + 16x_1 = 12 \cancel{x_1} x_2 + 16x_2$$

$$\Rightarrow \quad x_1 = x_2$$

$\therefore$ f is one-one 3

Clearly f: $R - \left\{-\frac{4}{3}\right\} \rightarrow$ Range f is onto 1

Let f(x) = y

i.e. $\frac{4x}{3x+4} = y$

$\Rightarrow \quad x = \frac{4y}{4-3y}$ 1

So f^{-1}: Range f $\rightarrow R - \left\{-\frac{4}{3}\right\}$ is

$f^{-1}(y) = \frac{4y}{4-3y}$ 1

OR

(a, b) * (c, d) = (a + c, b + d)

(c, d) * (a, b) = (c + a, d + b)

(a, b) * (c, d) = (c, d) * (a, b)

∴ * is commutative 2

((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)

(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)

As ((a, b) * (c, d)) * (e, f) = (a, b) * ((c, d) * (e, f))

∴ * is associative 2

Let (e_1, e_2) be identity

$(a, b) * (e_1, e_2) = (a, b)$

$(a + e_1, b + e_2) = (a, b)$

$e_1 = 0, e_2 = 0$

$(0, 0) \in R \times R$ is the identity element. 2

28. Clearly order of X is 3 × 2 1

Let $X = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ 1

So $\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \\ 11 & 10 & 9 \end{pmatrix}$

$$\left.\begin{matrix} a+4b=-7 & c+4d=2 & e+4f=11 \\ 2a+5b=-8 & 2c+5d=4 & 2e+5f=10 \end{matrix}\right\}$$ 2

Solving we get

$a = 1, \quad b = -2, \quad c = 2, \quad d = 0, \quad e = -5 \quad f = 4$ 1

Thus $X = \begin{pmatrix} 1 & -2 \\ 2 & 0 \\ -5 & 4 \end{pmatrix}$

29. Consider $\begin{vmatrix} -1+3 & 2-1 & 5-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$ 2

$= \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0$ 1

$\therefore$ Lines are coplanar.

Equation of plane is given by

$$\begin{vmatrix} x+3 & y-1 & z-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0$$ 2

Which gives

$-x + 2y - 3 = 0$ or $x - 2y + z = 0$ 1