NUMBER SYSTEM

- **Number:** A number is a mathematical object used to count, measure and label.
- **Number System:** Number system or system of numeration is a writing system of numbers or symbols in a consistent manner. In simple words we can say number system deals with writing numbers.
- Depending on base or numbers used in the system, number system can be classified as:
    - Binary Number System
    - Octal Number System
    - Decimal Number System
    - Hexadecimal Number System
- These number systems are usually used in computer these days. But in "Mathematics" we use Decimal Number System.
- **Decimal Number System:** "Deci" means 10 thus "Decimal number system" use 0 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). We can form any number using these digits. The base of this system is also 10.
- **Notation of a Number:** Notation of a number can be done as:

  $231 = 2\times10^2 + 3\times10^1 + 1\times10^0$

  While doing notation of a number we take number from right to left for which we use two system to represent in India:
    - Indian System/ (Hindu-Arabic System)
    - International System

Indian System: We use it to read number from left to right but count it from right to left Arab, Ten Crores, Crores, Ten Lakh, Lakhs, Ten Thousand, Thousands, Hundred, Tens, Ones
10^9, 10^8, 10^7, 10^6, 10^5, 10^4, 10^3, 10^2, 10^1, 10^0

International System: We use it to read number from left to right but count it from right to left Billion, Hund. M, Ten M, Million, Hund. Thou., Ten Thou., Thousands, Hundred, Tens, Ones
10^9, 10^8, 10^7, 10^6, 10^5, 10^4, 10^3, 10^2, 10^1, 10^0

- While representing numbers we use two terms for them:
    - Face Value
    - Place Value
- **Face Value:** Face value is the actual value of the digit in the number.

  Ex-1: Face Value of 7 in 38786 is 7 only
- **Place Value:** Place value is the value of the digit with the position at which they occur in the number.

  Ex-2: Place Value of 7 in 38786 is 700 i.e. 7×100

Types of Numbers

- **Imaginary Number:** Imaginary Numbers are those number that are expressed as a square root of a negative number. These numbers are usually represented as "ai" where 'a' is a number and 'i' is symbol for imaginary part, where value of 'i' is $(-1)^2$. Imaginary number are usually plot on the vertical number line plane.

  Ex: 2i, 7i

- **Real Number:** Real Numbers are numbers that can be represented on number line. In simple words all numbers other than imaginary numbers are Real number.

  Ex: 1, -4, 2.3, $\frac{2}{5}$ $\sqrt{2}$ $\frac{1}{\sqrt{3}}$, ,

- **Irrational Number:** Irrational numbers are those real numbers that cannot be expressed as $"\frac{p}{q}"$ because either "p" or "q" is a non-terminating term.

  Ex: $\sqrt{2}$ $\frac{1}{\sqrt{3}}$,

  Note: Perfect square root are not irrational numbers for example, $\sqrt{4}$is not consider as irrational number since it has terminating answer, i.e., 2

- **Rational Number:** Rational numbers are those real numbers that can be expressed as $"\frac{p}{q}"$ where q is a non-zero number. Ex: 1, -4, 2.3

Integers

Integers are those numbers in which denominator is '1' or you can say numbers which are not in form of fraction or in decimal. Integers can be further classified as:

- **Positive Integers:** Integers that lie on positive side of number line.

  Ex: 1,2,3 … ∞

- **Negative Integers:** Integers that lie on negative side of number line.

  Ex: - ∞. -3, -2, -1

- **Zero:** Center of a number line which represents no value.

  Ex: 0

Fraction

Fraction means a part of the whole. Fraction usually written as $\frac{p}{q}$ where 'p' is called numerator and 'q' is called denominator. Here "p & q" both are non-zero term.

Fraction can be further classified as:

- **Proper Fraction:** Proper Fraction are those fractions where p < q.

  Ex: $\frac{2}{3}$ $\frac{5}{11}$

- **Improper Fraction:** Improper Fraction are those fractions where p > q.

  Ex: $\frac{4}{3}$ $\frac{15}{11}$,

- **Mixed Fraction:** Mixed Fraction are those fractions which can be expressed improper fraction as proper fraction and integers.

  Ex: $\frac{4}{3}$ can be expressed as 1 $\frac{1}{3}$ where 1 is an integer and $\frac{1}{3}$ proper fraction.

Note: We can form infinite number of fractions between any integers.

Decimal

Decimal is a fraction whose denominator is a power of 10. Ex: 1.5, 2.75, 3.873

Recurring Decimal:

- Recurring Decimal is decimal where a digit or a group of digit recur indefinitely after decimal. Ex: 0.66… …, 0.373737……
- Recurring decimal are usually known as bar.
- A bar or a line is put over a digit to show that the digit will be repeating itself indefinitely. 0.66…. = $\overline{6}$ 0.

Convert a recurring decimal into a fraction:

When we remove a decimal and bar from any digit then from the denominator we subtract 1, where denominator is written in of power of 10 and where power of 10 depends upon the digit or number of digit that are recurring

0. $\overline{6}$ $\frac{6}{10-1} = \frac{6}{9} = \frac{2}{3}$ =

0. $\overline{37}$ $\frac{37}{100-1} = \frac{37}{99}$ =

So to simply this we can say we put as many 9 in denominator as many digit are under bar.

If the decimal has a mixed recurring decimal i.e. 0.5 , t$\overline{6}$en we use zero for non-bar numbers and 9 for bar number in denominator and subtract the non-bar number from whole number in numerator. Ex: 0.5 $\overline{6}$

0.5 $\overline{6}$= $\frac{56-5}{90} = \frac{51}{90} = \frac{17}{30}$

Natural Numbers: Natural numbers can be said the number we used for counting or a set of all positive integers. Ex: 1, 2 , 3 ,….. ∞

Whole Numbers: When zero is also included in natural numbers than the set of numbers are called whole numbers. Ex:0, 1, 2 , 3 ,….. ∞

Even Numbers: Set of natural numbers which are exactly divisible by 2 is known as even number. Ex: 2, 4, 6, 8…. ∞

Odd Numbers: Set of natural number except even numbers are called odd numbers. Ex: 1, 3, 5…. ∞

Prime Numbers: Prime numbers are set of those natural numbers which have exactly two factors, those are '1' and itself. Ex: 2, 3, 5, 7, 11……

Composite Numbers: Composite numbers are set of those natural numbers which have more than two factors. Ex: 4,6,8,9,10,12…….

Unit: As we know 1 has only one factor thus it is neither prime nor composite thus classified as unit. Ex: 1

Twin Prime Numbers: Two prime numbers which differ by 2 is known as twin prime numbers. Ex: (3,5), (5,7) (11,13)

Co-prime Numbers: Co prime numbers are a set of those numbers which have no common factor between them except '1'. In other word we can say a set of numbers with H.C.F 1 are co-prime numbers. Ex: (6,35), (12,25)

Note:

- 2 is the only even prime number.
- There are 15 prime numbers between 1 and 50 and their sum is 328
- There are 25 prime numbers between 1 and 100 and their sum is 1060
- 2 and 3 are the only consecutive prime numbers.
- 3, 5 and 7 are the only triplet of twin prime numbers.

Perfect Number: A number is said to be perfect number, if the sum of their factors except that number is equal to that number. Ex: 6, 28

Factors of 6 are 1, 2, 3, 6.

1 + 2 + 3 = 6

Factors of 28 are 1, 2, 4, 7, 14, 28

1 + 2 + 4 + 7 + 14 = 28

Note:

Sum of reciprocal of the factors of a perfect number is always 2.

Ex:

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=2$$

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}=2$$

Divisibility Test

By 2: If the unit digit of a number is even then the number is divisible by 2 or the last digit is 0.

Ex: 47896 is divisible by 2 as 6 is an even number

47895 is not divisible by 2 as 5 is an odd number.

By 3: If the sum of the digits of a number is a multiple of 3.

Ex: 729834

7 + 2 + 9 + 8 + 3 + 4 = 33

3 + 3 =6

So, it is divisible by 3

425786

4 + 2 + 5 + 7 + 8 + 6 = 32

3 + 2 =5

Thus, it is not divisible by 3

By 4: When the number formed by the last two digit is divisible by 4 or the last two digits are 0.

Ex: 384764

64 is divisible by 4 thus the number is divisible by 4

53877

77 is not divisible by 4 thus the number is not divisible by 4

By 5: If the last digit of a number is 5 or 0 then the number is divisible by 5.

Ex: 4375815

Since last digit is 5 than the number is divisible by 5.

By 6: A number which follows divisibility test of both 2 and 3 then the number is divisible by 6.

Ex: 729834

7 + 2 + 9 + 8 + 3 + 4 = 33

3 + 3 =6

So, it is divisible by 3.

And last digit is even so it is divisible by 2 as well thus the number is divisible by 6.

By 7: Lets understand with an example.

Ex: 3402

Step 1: 340 – 2 × 2 =336

Step 2: 33 – 2 × 6 = 21

Step 3: Check 21 is divisible by 7 or not.

As it is divisible by 7 thus number is divisible by 7.

By 8: When the number formed by the last three digit is divisible by 8 or the last three digits are 0.

Ex: 847648

648 is divisible by 8 thus the number is divisible by 8

By 9: If the sum of the digits of a number is a multiple of 9

Ex: 6329834

6 + 3 + 2 + 9 + 8 + 3 + 4 = 36

3 + 6 = 9

So, it is divisible by 9 124786

1 + 2 + 4 + 7 + 8 + 6 = 28

2 + 8 = 10

Thus, it is not divisible by 9.

By 10: If the last digit of a number is 0 then the number is divisible by 10.

Ex: 375810

Since last digit is 0 than the number is divisible by 10.

By 11: Let's understand it with an example.

Ex: 273691

(2 + 3 + 9) – (7 + 6 + 1) = 0

Thus, the number is divisible by 11

If the difference between the sum of digits at even position in a number and sum of digits at odd position in a number is equal to "0 or multiple of 11", then the number is divisible by 11.

By 12: A number which follows divisibility test of both 3 and 4 then the number is divisible by 12.

Ex: 4298364

4 + 2 + 9 + 8 + 3 + 6 + 4 = 36

3 + 6 = 9

So, it is divisible by 3

And last two digit is divisible by 4 so it is divisible by 4 as well thus the number is divisible by 12.

Note:

- If a six digit number is formed by repeating a digit then the number is divisible by 3, 7, 11, 13, and 37.

  Ex: 111111, 222222, 333333

- If a six digit number is formed by repeating a two digit number then the number is divisible by 3, 7, 13, and 37.

  Ex: 272727, 353535, 565656

- If a six digit number is formed by repeating a three digit number then the number is divisible by 7, 11, and 13.

  Ex: 273273, 135135, 456456

- If the difference of a number of its thousand and remainder of its divisible by 1000 is divisible by 7 then number is divisible by 7.

  Ex: 596638

  638 – 596 = 42

  42 is divisible by 7 thus number is divisible by 7

- If the difference of a number of its thousand and remainder of its divisible by 1000 is divisible by 13 then number is divisible by 13

  Ex: 265213

  265 – 213 = 52

  52 is divisible by 13 thus number is divisible by 13

- $a^n + b^n$ is completely divisible by (a + b) if n is odd

- $a^n - b^n$ is completely divisible by (a + b) and (a – b) if n is even.

Counting Zero:

Usually in exam question are asked on counting the number of zeros at the end of a digit formed by product of numbers.

The number of zeros at the end of any number depends upon the number of power of 10 that can be formed in the number for which we have to remember that 10 is a product of 2 and 5.

$2^n \times 5^n = 10^n$

So, to find the number of zeros in any expression we have to find out the number of multiple of 2 and 5 that comes in the expression and the number of power of 10 will be equal to the power of 5 or 2 which ever power is less.

Ex: Find the number of zeros at the end of 24 × 15 × 14 × 75.

Solution:

$24 \times 15 \times 14 \times 75 = 2^3 \times 3 \times 3 \times 5 \times 2 \times 7 \times 5^2 \times 3$

$2^4 \times 5^3 \times 3^3 \times 7$

So, the minimum power between power of 2 and power of 5 is 3 so the power of 10 will be 3.

Thus, the number of zeros at the end of the number is 3.

Ex: Find the number of zeros at the end of 12 × 25 × 42 × 125.

Solution:

Power of 2 is 3, power of 5 is 5

Thus, power of 10 will be 3.

Factorial

Factorial is a product of all natural numbers from first natural number till the number.

In simple words we can say factorial of number is equal to the product of all natural number equal and less than the number.

$N! = 1 \times 2 \times 3 \ldots (N-1) \times N.$

Number of zeros in N! = $\frac{N}{5^1} + \frac{N}{5^2} + \frac{N}{5^3} + \ldots + \frac{N}{5^a}$

The process will carry on till $5^a > N$ and we will only add quotient, neglecting the remainder.

Ex: Find the number of zeros in 210!

Solution:

$\frac{210}{5} + \frac{210}{25} + \frac{210}{125}$

$42 + 8 + 1 = 51$

OR

Step 1: $\frac{210}{5} = 42$

Step 2: $\frac{42}{5} = 8$

Step 3: $\frac{8}{5} = 1$

Number of zeros = 42 + 8 +1 = 51

Ex: Find the number of zeros in 1000!

Solution:

$\frac{1000}{5} + \frac{1000}{25} + \frac{1000}{125} + \frac{1000}{625}$

200 + 40 + 8 +1 = 249

Unit Digit

Several time in exam we have been asked about the unit digit of an expression. It is very simple to calculate unit digit of an expression if the numbers in expression are not given in a form of power.

Ex: Find the unit digit of 33 × 77 × 62 × 89 × 44 + 12834

Solution:

We will only use the unit digit of the numbers in the expression i.e. 7 × 2 × 9 × 4 + 4 = 4 + 4 = 8
We only consider unit of product also.

But it become difficult if the expression has number with power like 37^{31} as now we cannot determine the unit digit of this number. To calculate this easily we have to understand the cyclicity of unit digit with power of number.

Unit digit of every digit repeat itself and follows a cycle of 1,2 and 4.

Let's understand with an example.

If consider 2 so its cyclicity is 4, it means after every 4th power unit digit will be same.

$2^1 = 2$, $2^5 = 32$, $2^9 = 512$

$2^2 = 4$, $2^6 = 64$, $2^{10} = 1024$

$2^3 = 8$, $2^7 = 128$, $2^{11} = 2048$

$2^4 = 16$, $2^8 = 256$, $2^{12} = 4096$

From above example we can understand the cyclicity easily. As the biggest cycle is of 4 terms thus we try to remember cyclicity of 4 terms for all the digits.

Power **Number**	**1**	**2**	**3**	**4**
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1
0	0	0	0	0

Note:

From the above table we can say that,

- For 0, 1, 5, 6 their unit digits are 0, 1, 5, 6 respectively irrespective to their power.
- If the power of even number except number with unit digit zero is 4n then unit digit will be

6.

- If the power of odd number except number with unit digit 5 is 4n then unit digit will be 1.

Ex: What will be the unit digit of 37^{31}?

Remainder when power is divided by 4 i.e., $\frac{31}{4}$

Remainder will be 3

So, unit digit can be determined by 7^3 which will be 3

Ex: Find the unit digit $22^{21} \times 23^{22} \times 24^{23} \times 26^{25}$

Solution:

$22^{21} \times 23^{22} \times 24^{23} \times 26^{25}$

$2^1 \times 3^2 \times 4^3 \times 6$

$2 \times 9 \times 4 \times 6$

Unit digit will be 2.

Ex: Find the unit digit of the product of all the prime numbers less than 99

Solution:

$2 \times 3 \times 5 \times 7 \ldots\ldots 97$

Unit digit will be 0

Since product of 2 and 5 is 10, which means unit digit will be 0 for their product and anything multiplied by 0 will give zero, thus the unit digit will be 0 only.

Ex: Find the unit digit of 1! + 2! + 3! + …….. 99!.

Solution:

Unit digit From 5! will be zero, so we can say that the unit digit of this expression will be sum of unit digit of 1!, 2!, 3!, 4!

1 + 2 + 6 +4 =13

Unit digit will be 3.

Relation between Dividend, Divisor, Quotient and Remainder

Dividend = Divisor × Quotient + Remainder

Remainder is always as positive value but sometimes we also consider negative remainder to solve question fast.

How to calculate Negative remainder

Negative Remainder = Remainder – Divisor

Ex: What will be negative remainder if divide 1489 by 23.

Solution: Remainder = 17

Negative Remainder = 17 – 23 = – 6

Negative remainder is hypothetical condition used just to do fast calculation.

Ex: The divisor is 25 times the quotient and 5 times the remainder. If the remainder is 15, then find the dividend.

Solution:

Divisor = 15 × 5 = 75

Quotient = $\frac{75}{25} = 3$

Dividend = 75 × 3 + 15 = 240

Ex: A number when divided by 56 it leaves 41 as a remainder. Find the remainder if the same number is divided by 7.

Solution:

Number = 56 × X + 41

Now if the same number is divided by 7, then

$\frac{56x + 41}{7} = \frac{56x}{7} + \frac{41}{7}$ Remainder will be 6

OR

Just check whether the previous divisor is divisible by new divisor if it is divisible then remainder will be equal to the remainder we will get after dividing the former remainder by new divisor else the answer will be cannot be determined.

In the above question,

56 is divisible by 7 thus remainder can easily be find by dividing 41 by 7.

41 divide by 7 gives 6 as remainder and that is our answer.

Ex: A number when divided by 65 it leaves 15 as a remainder. Find the remainder if the same number is divided by 15.

Solution: Since 65 is not divisible by 15.

Answer will be cannot be determined.

Ex: A number when divided by 7 it leaves 4 as a remainder. Find the remainder if the square of the same number is divided by 7.

Solution: Number = 7x + 4

Square = $(7x+4)^2 = 49x^2 + 56x + 16$

$$\frac{49x^2}{7} + \frac{56x}{7} + \frac{16}{7}$$

Remainder will be 2

OR

In such question, you can just square the former remainder and divided it with the divisor to get the required remainder.

$4^2 = 16$

16 divided by 7 leaves remainder 2.

Ex: When a number is divided by 27 it leaves 8 as the remainder. If the cube of the same number is divided by 27 then find the remainder.

Solution: $8^3 = 512$

Remainder left when 512 is divided by 27 is 26.

Thus, the required remainder will be 26.

Ex: Two numbers when divided by 18. Leaves remainder 12 and 7 respectively if the sum of these two numbers is divided by 18 then the remainder will be –

Solution:

First number = 18x + 12

Second number = 18y + 7

Sum = 18x + 12 + 18y + 7 = 18 (x + y) + 19

$$\frac{18x}{18} + \frac{19}{18}$$

Remainder will be 1

OR

Divisor = $\text{Remainder}_1 + \text{Remainder}_2 - \text{Remainder}_3$

Required Remainder = 12 + 7 – 18 = 1

This formula always works because sum of two remainders by same divisor cannot be equal or more than twice of the divisor.

Ex: When two different numbers are divided by the same number they leaves remainder 27 and 21 respectively. If the sum of both the numbers are divided by the same divisor the remainder will be 13 then find the divisor.

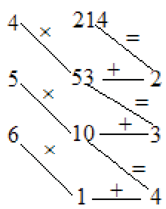
Solution: Divisor = 27 + 21 – 13 = 35

Successive Division

If in a division process quotient is used as a next dividend and the same process is carried on then the division is known as successive division. For example, if we divide 150 by 5 quotient will be 30 and remainder will be 0, now if we divide 30 by 2 quotient will be 15 and remainder will be 0, if we again divide 15 by 3 the quotient will be 5 and remainder will be 0 and now if we divide by 5 by 4 quotient will be 1 and remainder will also be 1

Ex: The least possible number when successively divided by 4, 5 and 6 leaves remainder 2, 3 and 4 respectively.

Solution:



To solve such question we consider the last quotient to be 1 and solve the question in reverse of the order i.e. we will find second last quotient first then third last and in final the smallest dividend or the number from which the successive division is started.

Second last quotient = 6 × 1 + 4 = 10

Third last quotient = 5 × 10 + 3 = 53

Smallest dividend = 4 × 53 + 2 = 214

Or {(6 + 4) × 5 + 3} × 4 + 2 =214

Ex: A least number when successively divided by 2, 3 and 5 it leaves remainder 1, 2 and 4 respectively. Find the remainder if the same number is divided by 6.

Solution: Number = {(2 + 1) × 3 + 2} × 5 + 4 = 44

If we divide 44 by 6 then remainder will be 2

Note:

- If $(1 + a)^n$ is divided by "a" then the remainder will be 1. It can also be expressed as if a^n is divided by (a – 1) the remainder will be 1.
- If a^n is divided by (1 + a) gives remainder 1 when n is even.

- If a^n is divided by $(1 + a)$ gives remainder a when n is odd.
- If $a^n + a$ is completely divisible by $(1 + a)$ when n is even.
- If $a^n + a$ is divided by $(1 + a)$ gives remainder $a – 1$ when n is odd.

Practice Questions:

Q1. If $49^{15} – 1$ is exactly divisible by:

1) 5 2) 6 3) 7 4) 9

Q2. Find the remainder when $29^{47} + 17^{47}$ is divided by 46.

1) 0 2) 1 3) 7 4) 13

Q3. Find the unit digit of $13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234$.

1) 6 2) 7 3) 0 4) 4

Q4. Find the unit digit of $278^{9235!} + 222^{9235!} + 666^{9235!}$

1) 6 2) 2 3) 8 4) 0

Q5. Find the number of zeroes at the end of the product 25! × 32! ×45!.

1) 10 2) 23 3) 22 4) 7

Q6. Find the number of zeroes at the end of 41×42×43…..100

1) 14 2) 15 3) 16 4) 17

Q7. If 5724A is divisible by 11 then find the value of A.

1) 0 2) 1 3) 2 4) 4

Q8. When a natural number N is divided by 3, the remainder will be 1 and when N + 1 is divided by 5, the remainder will be 0. The value of N will be:

1) 65 2) 64 3) 63 4) 62

Q9. A number when divided by 136, leaves 46 as the remainder. If the same number is divided by 34 then the remainder will be:

1) 2 2) 6 3) 12 4) 16

Q10. Two numbers when divided by a certain divisor give remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

1) 425 2) 475 3) 495 4) 535

Solution:

1) $49^{15} - 1 = 7^{2\times15} - 1^{30}$

$a^n - b^n$ is always divisible by a + b and a – b if n is even, so

$7^{30} - 1^{30}$ is always divisible by 7 + 1 = 8, 7 – 1 = 6

Thus, answer will be 6.

2) $29^{47} + 17^{47}$

$a^n + b^n$ is completely divisible by a + b when n is odd

so, 29 + 17 = 46

Thus, remainder will be 0.

3) $13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234$

$3^{4n} \times 8^1 + 4^1 \times 7^{4n} + 4$

$1\times 8 + 4\times1 + 4 = 16$

Thus unit digit will be 6.

4) $278^{9235!} + 222^{9235!} + 666^{9235!}$

9235! Last 2 digit will be 0 it means all the power are in the form of 4n and all the numbers are even so the unit digit in all the case will be 6.

6 + 6 + 6 = 18

Thus, unit digit will be 8.

5) 25! × 32! × 45!

Number of zeros at the end of

25! $\Rightarrow$ 5 + 1 = 6

32! $\Rightarrow$ 6 + 1 = 7

45! $\Rightarrow$ 9 + 1 = 10

So total number of zeros = 6 + 7 + 10 = 23

6) Number of zeros in 40! = 8 +1 = 9

Number of zeros in 100! = 20 + 4 = 24

Number of Zeros in 41×42×43…..100 = 24 – 9 = 15

7) If 5724A is divisible by 11, it means

(5 + 2 + A) – (7 + 4) = 0 or 11

A = 4

8) The condition will be fulfilled with multiple of 4 but not a multiple of 3 and 5. So, by option we ca say 64 is the answer.

9) By dividing 46 by 34 we will get remainder i.e., 12.

10) 473 + 298 – 236 = 535

ANSWER KEY				
1) 2	2) 1	3) 1	4) 3	5) 2
6) 2	7) 4	8) 2	9) 3	10) 4