Booklet No. [illegible]

JELET—2018

for B.Sc. Candidates

Time : 2 Hours

Full Marks : 100

SEAL

Instructions

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full mark *1*. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C or D.
3. Use only **Black/Blue ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Do not make any stray mark on the OMR.
5. Write question booklet number and your roll number carefully in the specified locations of the **OMR**. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your full signature in appropriate boxes in the OMR.
7. The OMRs will be processed by electronic means. Hence it is liable to become invalid if there is any mistake in the question booklet number or roll number entered or if there is any mistake in filling corresponding bubbles. Also it may become invalid if there is any discrepancy in the name of the candidate, name of the examination centre or signature of the candidate vis-a-vis what is given in the candidate's admit card. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, docu-pen, log table, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will be **reported against** and his/her candidature will be summarily cancelled.
9. Rough Work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.
10. Hand over the OMR to the invigilator before leaving the Examination Hall.

SEAL

SPACE FOR ROUGH WORK

1. The complex number z for which $e^z = -1$ is

(A) $i(4n-1)\pi$ (B) $i(4n+1)\pi$ (C) $i(2n+1)\pi$ (D) $i(2n+1)\frac{\pi}{2}$

2. If $1, \alpha_1, \alpha_2, \ldots, \alpha_{14}$ are roots of the equation $x^{15}-1=0$, then the value of $\alpha_1^{31}+\alpha_2^{31}+\cdots+\alpha_{14}^{31}$ is

(A) -1 (B) 0 (C) 14 (D) 1

3. If $x^3+3px+q$ has a factor of the form $(x-\alpha)^2$, then

(A) $q^3+4p^2=0$ (B) $q^3-4p^2=0$

(C) $q^2+4p^3=0$ (D) $q^3+4p^3=0$

4. If α, β, γ are the roots of $x^3+px+q=0$, then $\alpha^3+\beta^3+\gamma^3$ is

(A) $-3q$ (B) p^3-3q (C) $-3(p+q)$ (D) 0

5. Let $p(x)$ be a real polynomial of degree 4 such that all the roots of $p(x)=0$ are real. Then $p'(x)=0$ has

(A) no real root (B) exactly one real root

(C) three real roots (D) three imaginary roots

6. Let α, β, γ be the roots of the equation $x^3-6x^2+5x+9=0$. Then the equation, whose roots are $\alpha-2, \beta-2, \gamma-2$ is

(A) $x^3-7x+3=0$ (B) $x^3-8x^2+3x+7=0$

(C) $x^3-12x^2+41x-33=0$ (D) $x^3+7x-3=0$

7. If A and B are two square matrices of same order such that $A+B=I$ (where I is a square matrix of same order) and $AB=\underline{0}$, then $BA=$

(A) I (B) B (C) $\underline{0}$ (D) A

8. Let A be a skew-symmetric matrix of order 3. Then det A is

(A) 1 (B) −1 (C) 0 (D) non-zero

9. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a matrix. Let A_{ij} be the cofactor of $a_{ij}\,(1 \le i \le 3,\ 1 \le j \le 3)$ in det A.

Then for any $i = 1, 2, 3$; $\sum_{k=1}^{3} a_{ik} A_{ik}$ is equal to

(A) $\det(\text{Adj } A)$ (B) $\det(A^{-1})$ (C) $\det A$ (D) 0

10. If A and B are two real $n \times n$ matrices and c is a scalar, then which one of the following is true?

(A) $\det(cAB) = c \det A \det B$

(B) $\det(cAB) = c^n \det A \det B$

(C) $\det(cAB) = c^{n-1} \det A \det B$

(D) $\det(cAB) = c^{n-2} \det A \det B$

11. Let A, B and C be square matrices of same order 3. Then

(A) $AB = BA$ always hold

(B) $AC = BC \Rightarrow A = B$

(C) $AB = AC \Rightarrow B = C$ when A is orthogonal

(D) $A + B = A + C \Rightarrow B = C$ only if A be non-singular

12. If $x^2 + y^2 + z^2 = 3\,(x, y, z \in \mathbb{R})$, then the least possible value of $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}$ is

(A) 3 (B) 9 (C) 1 (D) 6

13. For the equation $|x|^2 + |x| - 6 = 0$

(A) there is only one root

(B) the sum of the roots is −1

(C) the sum of the roots is 0

(D) the product of the roots is −6

14. If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then $(\beta + \gamma - \alpha)(\gamma + \alpha - \beta)(\alpha + \beta - \gamma)$ is

(A) p (B) $-q$ (C) 0 (D) $8q$

15. If $(1+x)^n = a_0 + a_1 x + a_2 x^2 + \cdots$, then the value of $a_0 - a_2 + a_4 - \cdots$ (n is positive integer) is equal to

(A) $2^{\frac{n}{2}} \cos\left(\frac{n\pi}{4}\right)$

(B) $2^{\frac{n}{2}} \sin\left(\frac{n\pi}{4}\right)$

(C) $-2^{\frac{n}{2}} \cos\left(\frac{n\pi}{4}\right)$

(D) $-2^{\frac{n}{2}} \sin\left(\frac{n\pi}{4}\right)$

16. Let $T = \left\{ \begin{pmatrix} x & y \\ x & y \end{pmatrix} x,\ y \in \mathbb{R},\ x + y \neq 0 \right\}$ and $*$ be the usual matrix multiplication. Then

(A) $(T, *)$ has a left identity which is not a right identity (in T)

(B) $(T, *)$ has both-sided identity (in T)

(C) $(T, *)$ has neither left identity nor right identity

(D) $(T, *)$ is a group

17. Let G be a non-void set $(\subset \mathbb{R})$ in which operation $*$ be defined as $a * b = a + b - ab$. Then

(A) every element of G has inverse in G

(B) elements of G have inverses in G only if 1 be dropped from G

(C) $(G, *)$ has no identity element in G

(D) $(G, *)$ is not a semigroup

18. On non-void set $G \subset \mathbb{R}$, $*$ be defined as $a * b = a + b + 1$ for all $a,\ b \in G$. Then

(A) $(G, *)$ is not a groupoid

(B) $(G, *)$ is a semigroup

(C) $(G, *)$ is not a semigroup

(D) $(G, *)$ is only a groupoid

19. Let G be a non-void set in which a binary operation $*$ be defined as $a * b = a$ for all $a,\ b \in G$. Then

(A) $(G, *)$ is a semigroup but not a group

(B) $(G, *)$ is a group

(C) $(G, *)$ is not a semigroup

(D) $(G, *)$ is a quasi group

20. If $\begin{pmatrix} 5 & 4 \\ 1 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$, then X equals

(A) $\begin{pmatrix} -3 & -10 \\ 4 & 13 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 3 & -14 \\ 4 & -17 \end{pmatrix}$

21. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a real matrix with $\det A = 1$. If the roots of $\det(A - \lambda I_2) = 0$ (λ is real scalar and I_2 is the identity matrix of order 2) be imaginary, then

(A) $(a+d)^2 < 4$ (B) $(a+d)^2 = 4$ (C) $(a+d)^2 > 4$ (D) $(a+d)^2 = 16$

22. Let $M_2(\mathbb{R})$ be the set of all 2×2 matrices over R. Then $(M_2(\mathbb{R}), +, \cdot)$ is

(A) a commutative ring with unity

(B) a non-commutative ring with unity

(C) not a ring with unity

(D) a ring with unity

23. Let $S = \{-1, 1\}$. Consider the set $S \times S$, where the operation '$\cdot$' be defined as $(a, b) \cdot (c, d) = (ac, bd)$. Then

(A) $(S \times S, \cdot)$ is not a group

(B) $(S \times S, \cdot)$ is an abelian group

(C) $(S \times S, \cdot)$ is a semigroup but not a group

(D) elements of $S \times S$ are not invertible with respect to operation '$\cdot$'

24. If $A = \{1, 2, 3, 4, 5\}$, then the number of subsets of A containing 3 but not containing 5 is

(A) 2 (B) 4 (C) 6 (D) 8

25. Let $a, b \in \mathbb{N}$ be any two elements ($\mathbb{N}$ is the set of all natural numbers) and $*$ is defined by $a * b = a - b$. Then

(A) $(\mathbb{N}, *)$ is a commutative group

(B) $(\mathbb{N}, *)$ is a commutative group but not cyclic

(C) $(\mathbb{N}, *)$ is not a semigroup

(D) $(\mathbb{N}, *)$ is a semigroup but not a group

26. Let P be a 3×3 matrix such that $P^3 - P^2 + P + I_3 = \underline{0}$. Then which one of the following is not true?

(A) $P^4 = I_3$ (B) $P^4 = P^3 + P^2 - P$

(C) $P^2 - P + I_3 + P^{-1} = \underline{0}$ (D) $P^{-1} = P^7$

27. If $\begin{vmatrix} 1 & \cos x - \sin x & \cos x + \sin x \\ 1 & \cos y - \sin y & \cos y + \sin y \\ 1 & \cos z - \sin z & \cos z + \sin z \end{vmatrix} = k \begin{vmatrix} 1 & 1 & 1 \\ \sin x & \sin y & \sin z \\ \cos x & \cos y & \cos z \end{vmatrix}$, then the value of k is

(A) 1 (B) −1 (C) 2 (D) −2

28. The vectors (1, 2, 3), (5, 6, 7), (−1, 3, 2) and (−2, 8, −9) in $\mathbb{R}^3$ over $\mathbb{R}$ are

(A) linearly dependent

(B) linearly independent

(C) generators of $\mathbb{R}^3$

(D) generators as well as linearly independent

29. The vectors $(1, 0, 1)$, $(1, 1, 0)$ and (p, q, r) in $\mathbb{R}^3$ are linearly independent if and only if

(A) $p+q-r \neq 0$

(B) $p = q+r$

(C) $p-q+r \neq 0$

(D) $p \neq q+r$

30. Let A be a 3×3 singular matrix and B be a 3×3 invertible matrix. Then det (AB) is

(A) $\det(B)$

(B) $\det(B^{-1})$

(C) 0

(D) $3\det(B)$

31. The origin is shifted to the point $(4, -5)$ without changing the direction of the axes. If the coordinates of a point is $(1, 3)$ in the old system, then its coordinates in new system is

(A) $(-3, 8)$

(B) $(5, -2)$

(C) $(7, -4)$

(D) $(-2, 5)$

32. The equation $y^2 - 2by - 4ax + 4a^2 + b^2 = 0$ represents

(A) a parabola

(B) a hyperbola

(C) an ellipse

(D) a pair of straight lines

33. The polar equation of a circle passing through the pole is

(A) $r = 2a\cos(\theta-\alpha)$

(B) $r = 2a\sec(\theta-\alpha)$

(C) $r = 2a\tan(\theta-\alpha)$

(D) $r = 2a\operatorname{cosec}(\theta-\alpha)$

34. The value of a for which the lines $2x-3y+5z-a=0$, $x+5y-7z+14=0$ intersect on the z-axis is

(A) 4

(B) 6

(C) 8

(D) 10

35. The equation $\frac{8}{r} = 9 - 5\cos\theta$ represents

(A) a hyperbola

(B) an ellipse

(C) a parabola

(D) a circle

36. The value of c for which the straight lines $\frac{x-1}{2}=\frac{y-4}{1}=\frac{z-5}{2}$ and $\frac{x-2}{-1}=\frac{y-8}{c}=\frac{z-11}{4}$ may intersect is

(A) $\frac{2}{3}$ (B) −2 (C) 5 (D) 3

37. If a plane has intercepts l, m, n on the axes and be at a distance p from the origin, then

(A) $l^2+m^2+n^2=p^2$

(B) $l^{-2}+m^{-2}+n^{-2}=p^{-2}$

(C) $l^3+m^3+n^3=p^3$

(D) $l^{-2}+m^{-2}+n^{-2}=p^2$

38. The equation of the angle bisector between the lines $8x^2-10xy+2y^2=0$ is

(A) $5x^2+6xy+5y^2=0$

(B) $5x^2+6xy-5y^2=0$

(C) $5x^2-5y^2=0$

(D) $5x^2-6xy-5y^2=0$

39. The condition that two straight lines $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ and $\frac{x-a'}{l'}=\frac{y-b'}{m'}=\frac{z-c'}{n'}$ are perpendicular is

(A) $ll'+mm'+nn'=0$

(B) $l(m'-n')+m(n'-l')+n(l'-m')=0$

(C) $l'(m-n)+m'(n-l)+n'(l-m)=0$

(D) $(l-m)(l'-m')+(m-n)(m'-n')+(n-l)(n'-l')=0$

40. Let $\vec{p}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{q}=\hat{i}+2\hat{j}+3\hat{k}$ be vectors in $\mathbb{R}^3$. Suppose $\vec{v}=a\hat{i}+b\hat{j}+c\hat{k}$ is a unit vector such that $\vec{v}\cdot\vec{p}=0=\vec{v}\cdot\vec{q}$. The value of $|a+b+c|$ is

(A) 6 (B) 3 (C) 1 (D) 0

41. If a, b, c, d are rational numbers and x is an irrational number, then $\frac{ax+b}{cx+d}$

(A) is always rational

(B) is always irrational

(C) may be rational or irrational

(D) is purely imaginary

42. The series $1+\frac{1}{2^p}+\frac{1}{3^p}+\cdots$ converges for

(A) $p \geq 1$

(B) $p \leq 1$

(C) $p < 1$

(D) $p > 1$

43. The function

$$f(x)=\begin{cases} 1, & \text{if } x \in \mathbb{Q} \\ 0, & \text{if } x \in \mathbb{R}-\mathbb{Q} \end{cases}$$

is

(A) continuous everywhere

(B) continuous nowhere

(C) continuous at $x = 0$

(D) continuous at every rational points

44. The function $f(x) = |x-2| + |x+3|$ is

(A) differentiable everywhere

(B) differentiable nowhere

(C) differentiable except at $x = -3, 2$

(D) differentiable at $x = -3, 2$

45. $\frac{d^n}{dx^n}\{\sin ax\}$ is equal to

(A) $a^n \sin\left(n\frac{\pi}{2}+ax\right)$

(B) $a^n \cos\left(n\frac{\pi}{2}+ax\right)$

(C) $a^n \sin(n\pi + ax)$

(D) $a^n \cos(n\pi + ax)$

(a is non-zero real number)

46. The value of ξ in the mean value theorem $f(b) - f(a) = (b-a) f'(\xi)$, where $f(x) = \sqrt{x}$, $a = 4$, $b = 9$ is

(A) 6 (B) 7 (C) 6·5 (D) 6·25

47. If $f(x) = e^x$ and $g(x) = \log x \, (x > 0)$, then $(g \circ f)'(x)$ where $(g \circ f)(x) = g[f(x)]$ is equal to

(A) 0 (B) 1 (C) e (D) $\frac{e^x}{x}$

48. For the function $f(x) = \sqrt{x} - x$, $0 \le x \le 1$, which of the following is true?

(A) Rolle's theorem is not applicable to f since f is not differentiable at $x = 0$

(B) Rolle's theorem is applicable to f and hence there is an element $C \in (0, 1)$ such that $f'(C) = 0$

(C) Rolle's theorem is not applicable to f though there exists an element $C \in (0, 1)$ such that $f'(C) = 0$

(D) Rolle's theorem is not applicable to f since $f(0) = 0 = f(1)$

49. The series $x + \frac{x}{2!} + \frac{x}{3!} + \frac{x}{4!} + \cdots (x > 0)$ is

(A) divergent if $x = 1$ (B) divergent if $x > 1$

(C) convergent (D) divergent if $x < 1$

50. The function $f(x) = [x]$, where $[x]$ is the greatest integer not exceeding x, is

(A) continuous at $x = 1$

(B) differentiable at $x = 1$

(C) continuous but not differentiable at $x = 1$

(D) not differentiable at $x = 1$

51. If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(\theta h)$, $0 < \theta < 1$; then the value of θ, when $h = 1$ and $f(x) = (1-x)^{5/2}$ is

(A) 16/25 (B) 3/5 (C) 9/25 (D) 4/5

52. At the point (0, 0), the function $f(x, y) = x^2 + y^2 - 4$ has

(A) a minimum and the minimum value is -4

(B) a maximum and the maximum value is -4

(C) neither minimum nor maximum

(D) a minimum and the minimum value is 0

53. If $y = \tan^{-1} x$, $(-\infty < x < \infty)$, then

(A) $(1+x^2)y_{n+1} + 2nxy_n - n(n-1)y_{n-1} = 0$

(B) $(1+x^2)y_{n+1} - 2nxy_n + n(n-1)y_{n-1} = 0$

(C) $(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$

(D) $(1+x^2)y_{n+1} - 2nxy_n - n(n-1)y_{n-1} = 0$

(Symbols have their usual meanings)

54. $f(x) = e^{-x}$ and $g(x) = e^x$ in $[a, b]$, $0 < a < b$. Then in the Cauchy's mean value theorem, the value of c is

(A) $c = \frac{a+b}{ab}$ (B) $c = \frac{a+b}{2}$ (C) $c = \frac{a-b}{2}$ (D) $c^2 = \frac{a+b}{ab}$

55. $f(x) = -2x^3 - 9x^2 - 12x + 1$ is an increasing function in the interval

(A) $-2 < x < 1$ (B) $-2 < x < -1$ (C) $-1 < x < 1$ (D) $0 < x < 1$

56. The value of $\lim_{x \to 0} \frac{1-\cos^3 x}{\tan^2 x}$ is

(A) 1/2 (B) 1 (C) 3/2 (D) 2/3

57. The series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots + \frac{(-1)^{n-1}}{n^2} + \cdots$ is

(A) conditionally convergent (B) divergent

(C) absolutely convergent (D) oscillatory

58. Let $f(x, y) = x^5 y^2 \tan^{-1}\left(\frac{y}{x}\right)$. Then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is equal to

(A) $2f(x, y)$ (B) $3f(x, y)$ (C) $5f(x, y)$ (D) $7f(x, y)$

59. If $y = e^{-x}\cos x$, then the value of $y_4 + 4y$ is

(A) 0 (B) 1 (C) $\sin x$ (D) $\cos x$

60. If $f(x+1) - 2f(x) + f(x-1) = 2$ for all x, then

(A) $f(x) = \sqrt{x}$ (B) $f(x) = x$ (C) $f(x) = x^2$ (D) $f(x) = x^3$

61. $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$ is equal to

(A) $x\tan^{-1} x - \log(\sqrt{1+x^2}) + c$ (B) $2\tan^{-1} x + 2\log\left(\frac{1}{\sqrt{1+x^2}}\right) + c$

(C) $2x\tan^{-1} x + 2\log\left(\frac{1}{\sqrt{1+x^2}}\right) + c$ (D) $2x\tan^{-1} x + 2\log(\sqrt{1+x^2}) + c$

Where c is constant of integration

62. The value of $\int \frac{\cos x}{\sin x + \cos x} dx$ is equal to

(A) $x + \log|\sin x + \cos x| + c$

(B) $\frac{x}{2} + \log|\sin x + \cos x| + c$

(C) $\frac{x}{2} + \frac{1}{2}\log|\sin x + \cos x| + c$

(D) $x + \frac{1}{2}\log|\sin x + \cos x| + c$

63. $\lim_{n \to \infty} \frac{1}{\sqrt{n}} \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \right)$ is equal to

(A) 1/2

(B) 1

(C) 2

(D) 0

64. Let a be a real number. Then $\int_a^\infty \frac{dx}{1+x^2}$ is

(A) always convergent

(B) always divergent

(C) convergent if $a \neq 0$

(D) divergent if $a = 0$

65. The value of $\int_0^1 \frac{dx}{\sqrt[3]{x(1-x)^2}}$ is

(A) $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$

(B) $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{1}{2}\right)$

(C) $\Gamma\left(\frac{1}{2} + \frac{1}{3}\right)$

(D) $\frac{\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{3}\right)}$

(Symbol Γ has its usual meaning)

66. The value of $\frac{1}{2}\int_0^\infty x^7 e^{-\sqrt{x}} dx$ is

(A) $\frac{15!}{2}$

(B) 15!

(C) $2 \times 15!$

(D) $\frac{16!}{2}$

67. $\int \frac{dx}{x^2+x+1}$ is

(A) $\frac{2}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}+c$

(B) $\frac{1}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}+c$

(C) $\frac{2}{\sqrt{3}}\tan\frac{2x+1}{\sqrt{3}}+c$

(D) $\frac{1}{\sqrt{3}}\tan\frac{2x+1}{\sqrt{3}}+c$

(c is constant of integration)

68. $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$ is equal to

(A) $2x\tan^{-1}x-\log(1+x^2)+C$

(B) $2\tan^{-1}x-\log(1+x^2)+C$

(C) $2x\tan^{-1}x-\log(1-x^2)+C$

(D) $2\tan^{-1}x-\log(1-x^2)+C$

(C is constant of integration)

69. Value of $\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos x\log\left(\frac{1-x}{1+x}\right)dx$ is

(A) -2

(B) -1

(C) 0

(D) 1

70. $\int \frac{\sqrt{x}\,dx}{x(x+1)}$ is equal to

(A) $\tan^{-1}x+C$

(B) $2\tan^{-1}\sqrt{x}+C$

(C) $2\tan^{-1}x^{3/2}+C$

(D) $\tan^{-1}x^{3/2}+C$

(C is constant of integration)

71. For the curve $r=ae^{\theta\cot\alpha}$, the length of the polar tangent is

(A) $r\cot\alpha$

(B) $r\sec\alpha$

(C) $r\tan\alpha$

(D) $r\operatorname{cosec}\alpha$

72. The point at which the normal of the parabola $y^2 = 4ax$ passes through the focus is

(A) $(0, 0)$ (B) $(a, 2a)$ (C) $(a, -2a)$ (D) $(a^3, 2a^2)$

73. The angle between the tangent of $r = ae^{\theta \cot \alpha}$ and radius vector is

(A) α (B) 2α (C) $\frac{\pi}{2} - \alpha$ (D) $\frac{\alpha}{2}$

74. The subtangent at any point of the curve $x^2 y = a^3$ varies as

(A) ordinate of the point

(B) abscissa of the point

(C) distance of the point from origin

(D) square of the distance of the point from the origin

75. The area of the rectangle contained by the subtangent and subnormal at any point (x, y) of a curve $f(x, y) = 0$ is

(A) y^2 (B) x^2 (C) $x^2 + y^2$ (D) $(x^2 + y^2)^2$

76. Consider the curve Γ given by $y^2 (a - x) = x^2 (x + a)$, $a \in \mathbb{R} \setminus \{0\}$. Then

(A) for the curve, $-\infty < x < \infty$

(B) there is no tangent at $(-a, 0)$

(C) the curve is symmetrical about x-axis only

(D) the curve is symmetrical about y-axis only

77. Asymptotes of the curve $x^2 - y^2 = a^2$ $(a \neq 0)$ are

(A) $y = \pm x$ (B) $y = 2x$ (C) $y + 3x = 0$ (D) $x = 0, y = 0$

78. The number of asymptotes of the curve $x^2 + y^2 = a^2$ is

(A) two (B) one (C) zero (D) three

79. The function $f(x) = 2x^3 - 21x^2 + 36x - 20$ is

(A) maximum at $x = 1$

(B) minimum at $x = 1$

(C) maximum at $x = 6$

(D) minimum at $x = \frac{7}{2}$

80. The function $x^3 + y^3$

(A) has maxima at (0, 0)

(B) has minima at (0, 0)

(C) has neither a maxima nor a minima at (0, 0)

(D) has maxima at (0, 0) and minima at (– 1, – 1)

81. The maximum value of $\left(\frac{1}{x}\right)^x$, $x > 0$ is

(A) e (B) $e\sqrt{e}$ (C) $\frac{1}{e}$ (D) $e^{\frac{1}{e}}$

82. Let the curve Γ be $y = be^{-x/a}$ and P be the point, where the curve crosses the axis of y, $(b, a \in \mathbb{R} \sim \{0\})$. Then tangent at P is

(A) $x - y = 2a$

(B) $x + y = 2a$

(C) $\frac{x}{a} - \frac{y}{b} = 1$

(D) $\frac{x}{a} + \frac{y}{b} = 1$

83. The curves $y = x^2$ and $y^2 = x$ pass through the point (1, 1). Then the angle of intersection at this point is

(A) $\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{1}{4}\right)$ (C) $\tan^{-1}\left(\frac{1}{3}\right)$ (D) $\tan^{-1}\left(\frac{2}{3}\right)$

84. For the curve $2y^2 = (1+x)^3$, the subnormal at any point varies as

(A) subtangent

(B) square of subtangent

(C) cube of subtangent

(D) square root of subtangent

85. The area $\iint_R (x^2 + y^2)\,dxdy$, where R is the region bounded by $y = x^2$, $x = 2$, $y = 1$ is

(A) $\frac{106}{105}$ sq. units

(B) $\frac{1006}{105}$ sq. units

(C) $\frac{106}{1005}$ sq. units

(D) $\frac{1006}{1005}$ sq. units

86. Which of the following sets is a convex set?

(A) $\{(x, y) \in \mathbb{R}^2 : |x+y| \geq 2\}$

(B) $\{(x, y) \in \mathbb{R}^2 : |x-y| = 2\}$

(C) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4\}$

(D) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 4\}$

87. The minimum value of $z = 5x_1 - 2x_2$, subject to $2x_1 + 3x_2 \geq 6$, $x_1 \geq 0$, $x_2 \geq 0$ is

(A) -6

(B) -4

(C) 0

(D) 15

88. The maximum value of $z = 2x + 5y$, subject to $0 \leq x \leq 4$, $0 \leq y \leq 3$, $x + y \leq 6$ is

(A) 21

(B) 18

(C) 25

(D) 26

89. Optimal solutions of a linear programming problem are

(A) any point of the convex set of feasible solutions

(B) interior points of the convex set of feasible solutions

(C) extreme points of the convex set of feasible solutions

(D) exterior points of the convex set of feasible solutions

90. Any solution to a general linear programming problem, which satisfies the non-negative restriction of the problem is called

(A) a basic solution (B) an optimal solution

(C) a feasible solution (D) a non-degenerate solution

91. The orthogonal trajectories of the rectangular hyperbola $xy = a^2$ is

(A) $x = c^2 y^2$ (B) $x^2 = c^2 y^2$

(C) $x^2 + y^2 = c^2$ (D) $x^2 - y^2 = c^2$

92. Solution of the differential equation $x\,dy - y\,dx = \cos\frac{1}{x}\,dx$ is

(A) $y + x\sin\frac{1}{x} = c$ (B) $\frac{y}{x} - \sin\frac{1}{x} = c$

(C) $\frac{y}{x} + \sin\frac{1}{x} = c$ (D) $xy + x\sin\frac{1}{x} = c$

93. Degree and order of the differential equation

$$\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{2/3} = \frac{d^2y}{dx^2}$$

is

(A) 2, 3 (B) 3, 2 (C) 2, 2 (D) 4, 2

94. Which of the following is not an integrating factor of the equation $x\,dy - y\,dx = 0$?

(A) $\frac{1}{x^2}$ (B) $\frac{1}{xy}$ (C) $\frac{1}{x}$ (D) $\frac{1}{x^2 + y^2}$

95. The particular integral of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x + e^{-x}$ is

(A) $xe^x + \frac{e^{-x}}{2}$ (B) $\frac{1}{2}(xe^x + e^{-x})$

(C) $\frac{1}{2}(e^x + e^{-x})$ (D) $\frac{1}{2}(xe^x + xe^{-x})$

96. In an orbit described under a force to a centre, the velocity at any point is inversely proportional to the distance of the point from the centre of the force. Then the path is

(A) an equiangular spiral

(B) an equilateral hyperbola

(C) a parabola

(D) an ellipse

97. A particle moves in a plane in such a manner that its tangential and normal accelerations are always equal and its velocity varies as $e^{\tan^{-1}\left(\frac{s}{c}\right)}$. Then the equation of its path is

(A) $s = c(\psi + k)$

(B) $s = c\tan(\psi + k)$

(C) $s = e^{c(\psi + k)}$

(D) $s = \tan(ke^{c\psi})$

98. A particle moves along a straight line according to the law $v^2 = 4(x\sin x + \cos x)$, where v is the velocity at a distance x from a fixed point on its path. Then the acceleration is

(A) $2x\cos x$

(B) $2x\sin x$

(C) $4x\sin x$

(D) $4x\cos x$

99. If a particle moves with constant angular velocity about a point O in its plane of motion, the cross-radial acceleration is proportional to its

(A) angular velocity

(B) radial velocity

(C) constant velocity

(D) All of these

100. In trapezoidal rule, the curve $y = f(x)$ is assumed to be

(A) a circle

(B) a straight line

(C) a parabola

(D) a hyperbola

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