MS(QMS)
2019

1. Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. How many values of k are possible?

   (a) 0
   (b) 1
   (c) 2
   (d) None of the above

2. Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\, dx$ for $a > 0$;

   (a) 1/2
   (b) a/2
   (c) a
   (d) None of the above

3. The nearest among the following options for the value of $(9.02)^{3/2}$ is

   (a) 27.09
   (b) 28.50
   (c) 28.02
   (d) None of the above

4. The number of positive integers which are less than or equal to 1000 and are not divisible by any of the numbers 17, 19 and 23 equals

   (a) 854
   (b) 153
   (c) 160
   (d) None of the above

5. If $\lim_{n\to\infty}(1 + \frac{a}{n} + \frac{b}{n^2})^{2n} = \exp(2)$, then the values of a and b are

   (a) Any real number a, any real number b
   (b) Any real number a, b = 1
   (c) a = 1, any real number b
   (d) None of the above

6. For $-3 < x < 3$, consider the function:-

$f(x) = \{\frac{\log(3+x)-\log(3-x)}{x}\}$, if $x \neq 0$
$\quad\quad = c$, if $x = 0$

Then f is a continuous function if the value of c is

(a) 0
(b) $\frac{1}{3}$
(c) $-\frac{1}{3}$
(d) None of the above

7. Let $x_0 > 0$ and x_n be defined recursively by $x_n = \sqrt{6 + x_{n-1}}$, $n \geq 1$. Then $\lim_{n\to\infty} x_n$

(a) does not exist
(b) $\sqrt{6}$
(c) $\sqrt{6 + x_0}$
(d) None of the above

8. The maximum value of $(\cos\ \alpha_1)(\cos\ \alpha_2)(\cos\ \alpha_3).....(\cos\ \alpha_n)$ under the restrictions $0 \leq \alpha_1, \alpha_2, \alpha_3.....\alpha_n \leq \frac{\pi}{2}$ and $(\cot\ \alpha_1)(\cot\ \alpha_2).....(\cot\ \alpha_n) = 1$ is

(a) $\frac{1}{2^{n/2}}$
(b) $\frac{1}{2^n}$
(c) $\frac{1}{2n}$
(d) None of the above

9. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $\frac{-\pi}{4} \leq x \leq \frac{\pi}{4}$ is equal to

(a) 0
(b) 2
(c) 1
(d) None of the above

10. The value of $2\begin{pmatrix} n \\ 1 \end{pmatrix} + 2^3\begin{pmatrix} n \\ 3 \end{pmatrix} + 2^5\begin{pmatrix} n \\ 5 \end{pmatrix} + ...$, n being an integer, is equal to

    (a) $\frac{3^n+(-1)^n}{2}$

    (b) $\frac{3^n+1}{2}$

    (c) $\frac{3^n-(-1)^n}{2}$

    (d) None of the above

11. Suppose $\alpha, \beta, \gamma \neq 0$, are the roots of $x^3 + px^2 + q = 0$, where $q \neq 0$, then $D = \begin{vmatrix} \frac{1}{\alpha} & \frac{1}{\beta} & \frac{1}{\gamma} \\ \frac{1}{\beta} & \frac{1}{\gamma} & \frac{1}{\alpha} \\ \frac{1}{\gamma} & \frac{1}{\alpha} & \frac{1}{\beta} \end{vmatrix}$ is equal to

    (a) 0

    (b) -$\frac{p}{q}$

    (c) $\frac{1}{q}$

    (d) None of the above

12. Let α and β be the distinct roots of $ax^2+bx+c=0$. Then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

    (a) $-\frac{a^2}{2}(\alpha-\beta)^2$

    (b) $\frac{1}{2}(\alpha-\beta)^2$

    (c) $\frac{a^2}{2}(\alpha-\beta)^2$

    (d) None of the above

13. A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

    (a) $\frac{3}{10}$

    (b) $\frac{2}{5}$

    (c) $\frac{1}{2}$

    (d) None of the above

14. There are 7 horses in a race. Mr. Murthy selects two horses at random and bets on them. What is the probability that Mr. Murthy selected the winning horse?

(a) $\frac{1}{7}$
(b) $\frac{2}{7}$
(c) $\frac{5}{7}$
(d) None of the above

15. For $-1000 \leq x \leq 1000$ consider the function $f(x) = \sum_{i=11}^{31} |x - i|$. The minimum value of this function is

(a) 100
(b) 90
(c) 120
(d) None of the above

16. Let $f(x)$ be differentiable at all x and $f'(x) \geq 2$ for $x \in [0, 7]$. If $f(1) = -2$, then

(a) $f(6) = 4$
(b) $f(6) = 5$
(c) $f(6) = 6$
(d) None of the above

17. The smallest integer satisfying the inequality $\log_{x^2}(2 + x) < 1$, is

(a) 2
(b) 3
(c) 4
(d) None of the above

18. Let $f(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin(\frac{n\pi}{2}) & \cos(\frac{n\pi}{2}) \\ a & a^2 & a^3 \end{vmatrix}$, where $a \neq 0$. The value of $\frac{d^n}{dx^n}[f(x)]$ at $x = 0$ is

(a) -1
(b) 0
(c) 1
(d) None of the above

19. If $|a\sin^2\theta + b\sin\theta\cos\theta + c\cos^2\theta - \frac{1}{2}(a+c)| \leq \frac{k}{2}$, then k^2 is equal to

(a) $b^2 + (a-c)^2$
(b) $a^2 + (b-c)^2$
(c) $c^2 + (a-b)^2$
(d) None of the above

20. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n\theta \, d\theta$, then $I_8 + I_6$ equals to

(a) $\frac{1}{7}$
(b) $\frac{1}{6}$
(c) $\frac{1}{5}$
(d) None of the above

21. The number of positive integral solutions of the equation $x^2 - y^2 = 3906$ is

(a) 2
(b) 1
(c) 0
(d) None of the above

22. For which of the options given below, the following system of linear equations is consistent?

$x + 3y + z = a$
$-x - 2y + z = b$
$3x + 7y - z = c$

(a) $c + b - a = 0$
(b) $c + 2b - a = 0$
(c) $a - b - c = 0$
(d) $a - b + c = 0$

23. When a parabola represented by the equation $y - 2x^2 = 4x + 5$ is translated 2 units to the left and 2 units up, the new parabola has its vertex at the co-ordinate point:

(a) $(1, 5)$

(b) $(-3, 5)$

(c) $(1, 1)$

(d) None of the above

24. The value of $\int_0^\infty \frac{1}{1+e^x} dx$ is equal to

(a) $\log 2 - 1$

(b) $\log 2$

(c) $\log 4 - 1$

(d) None of the above

25. The value of $\int_2^3 \frac{dx}{\sqrt{1+x^3}}$ is

(a) less than 1

(b) greater than 2

(c) lies between 3 and 4

(d) None of the above

26. Two distinct numbers are selected from the set $[1, 2, 3, \ldots, 3n]$. The number of ways in which this can be done, if the sum of the selected numbers is divisible by 3, is

(a) $\frac{3n(3n-1)}{2}$

(b) $\frac{n(3n-1)}{2}$

(c) $\frac{3n(n-1)}{2}$

(d) None of the above

27. Let $b > a$ and $I = \int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$, then I equals to

(a) $\frac{\pi}{2}$

(b) π

(c) $\frac{3\pi}{2}$

(d) None of the above

28. If $f(x) = 3 + x^2 + \tan\frac{\pi x}{2}$, then $(f^{-1})'(3)$ is equal to

(a) π

(b) $\frac{\pi}{2}$

(c) 2π

(d) None of the above

29. A class has 15 students. In a test with 100 questions, each question carrying 1 mark, the average score was 80; no negative or fractional marks were given on any of the questions. If the maximum score obtained by any of the students was 85, what is the minimum possible value for the lowest score obtained?

(a) 10

(b) 11

(c) 12

(d) None of the above

30. Square Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix. Then for $k > 2, (k \in N), A^k$ is equal to

(a) $kA - I$

(b) $kA - (k-1)I$

(c) $2^k A - I$

(d) None of the above