CBSE Class–12 Mathematics

NCERT solution

Chapter -12

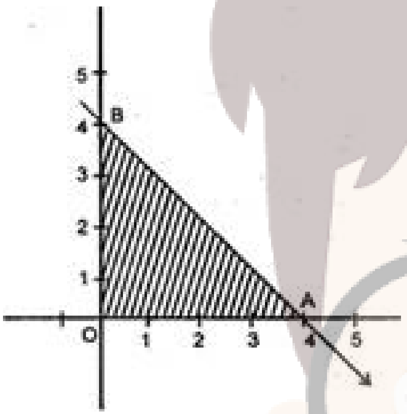
Linear Programming - Exercise 12.1

Solve the following Linear Programming Problems graphically:

1. Maximize Z = $3x+4y$ **subject to the constraints:** $x+y\leq 4, x\geq 0, y\geq 0$.

Ans. As $x\geq 0, y\geq 0,$ therefore we shall shade the other inequalities in the first quadrant only.

Now $x+y\leq 4$



Let $x+y=4$

$$\Rightarrow \frac{x}{4}+\frac{y}{4}=1$$

Thus the line has 4 and 4 as intercepts along the axes. Now, (0, 0) satisfies the inequation, i.e., $0+0\leq 4.$ Therefore, shaded region OAB is the feasible solution.

Its corners are O (0, 0), A (4, 0), B (0, 4)

At O (0, 0) Z = 0

At A (4, 0) Z = 3 x 4 = 12

At B (0, 4) Z = 4 x 4 = 16

Hence, max Z = 16 at $x=0, y=4$.

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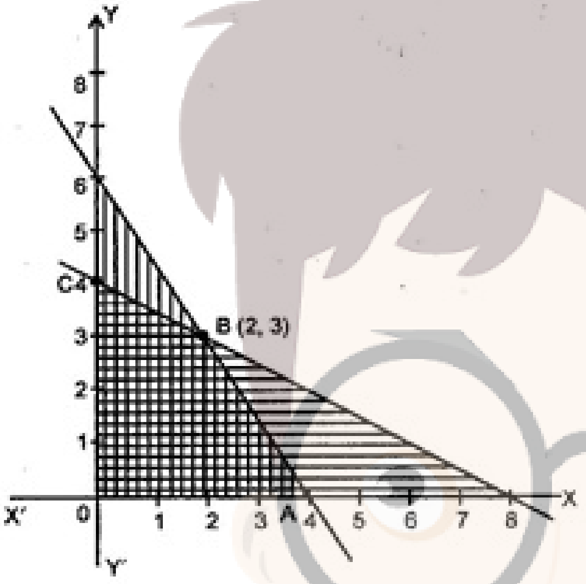
2. Minimize Z = $-3x+4y$ **subject to** $x+2y\le 8, 3x+2y\le 12, x\ge 0, y\ge 0$.

Ans. Consider $x+2y\le 8$

Let $x+2y=8$

$$\Rightarrow \frac{x}{8}+\frac{y}{4}=1$$

$\therefore a=8, b=4$



Since, (0, 0) satisfies the inequaitons $x+2y\le 8$

Therefore, its solution contains (0, 0)

Again $3x+2y\le 12$

Let $3x+2y=12$

$$\Rightarrow \frac{x}{4}+\frac{y}{6}=1$$

Again, (0, 0) satisfies $3x+2y\le 12$

Therefore its solution contains (0, 0).

The feasible region is the solution set which is double shaded and is OABCO.

At O (0, 0) Z = 0

At A (4, 0) Z = –3 x 4 = –12

At B (2, 3) Z = –3 x 2 + 4 x 3 = 6

At C (0, 4) Z = 4 x 4 = 16

Hence, minimum Z = –12 at $x = 4, y = 0.$

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3. Maximize Z = $5x + 3y$ **subject to** $3x + 5y \le 15, 5x + 2y \le 10, x \ge 0, y \ge 0.$

Ans. We first draw the graph of equation $3x + 5y = 15$

$$\Rightarrow x = \frac{15 - 5y}{3}$$

For $y = 3, \quad x = 0$

And for $y = 0, \quad x = 5$

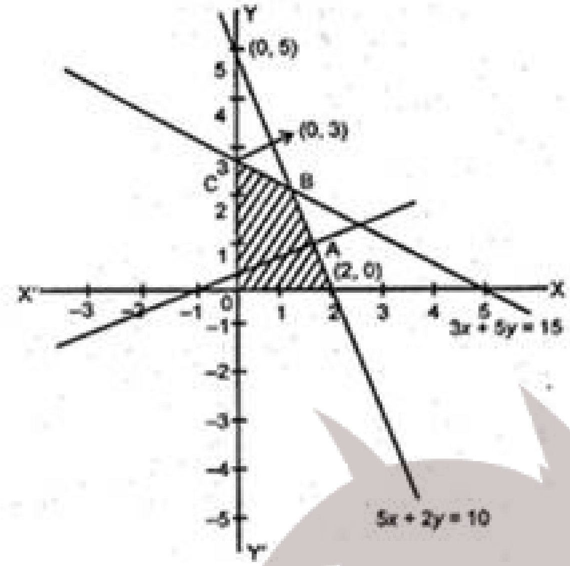
x	0	5
y	3	0

Similarly, for equation $5x + 2y = 10$, the points are (2, 0) and (0, 5).

x	2	0
y	0	5

As (0, 0) satisfies both the inequations and also $x \ge 0, y \ge 0,$ then the feasible require contains the half-plane containing (0, 0).

Therefore, the feasible portion is OABC which is shown as shaded in the graph.



Co-ordinates of point B can be obtained by solving $3x+5y=15$ and $5x+2y=10$ and it is B$\left(\frac{20}{19}, \frac{45}{19}\right)$.

Thus, co-ordinates of O, A, B and C are (0, 0), (2, 0), $\left(\frac{20}{19}, \frac{45}{19}\right)$ and (0, 3).

Z = $5x+3y=0$ (if $x=0, y=0$)

Z = 5 x 2 + 3 x 0 = 10 (if $x=2, y=0$)

Z = 5 x $\frac{20}{19}$ + 3 x $\frac{45}{19}$ = $\frac{235}{19}$ (if $x=\frac{20}{19}, y=\frac{45}{19}$)

Z = 5 x 0 + 3 x 3 = 9 (if $x=0, y=3$)

Hence, Z = $\frac{235}{19}$ is maximum when $x=\frac{20}{19}, y=\frac{45}{19}$.

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4. Minimize P = $3x+5y$ **such that** $x+3y\geq 3, x+y\geq 2, x, y\geq 0.$

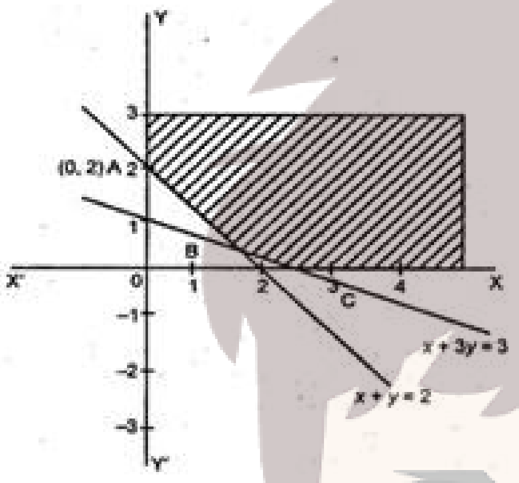
Ans. For plotting the graphs of $x+3y=3$ and $x+y=2$, we have the following tables:

x	0	3
y	1	0

x	1	0
y	1	2

The feasible portion represented by the inequalities

$x+3y \geq 3, x+y \geq 2$ and $x, y \geq 0$ is ABC which is shaded



in the figure. The coordinates of point B are $\left(\frac{3}{2}, \frac{1}{2}\right)$

Which can be obtained by solving $x+3y=3$ and $x+y=2$.

At A (0, 2)

Z = 3 x 0 + 5 x 2 = 10

At B $\left(\frac{3}{2}, \frac{1}{2}\right)$

Z = $3\times\frac{3}{2}+5\times\frac{1}{2}=\frac{9}{2}+\frac{5}{2}=\frac{14}{2}=7$

At C (3, 0)

Z = 3 x 3 + 5 x 0 = 9

Hence, Z is minimum is 7 when $x = \frac{3}{2}$ and $y = \frac{1}{2}$.

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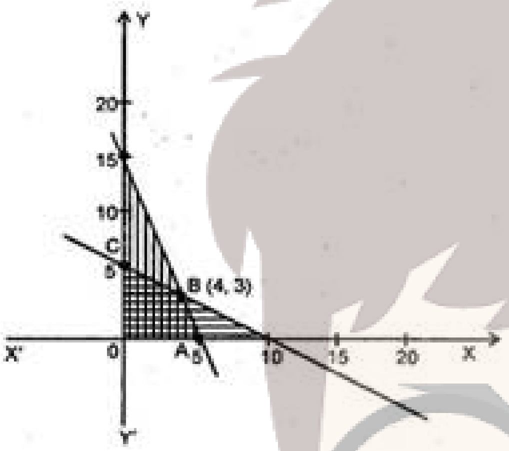
5. Maximize Z = $3x + 2y$ **subject to** $x + 2y \le 10, 3x + y \le 15, x, y \ge 0.$

Ans. Consider $x + 2y \le 10$

Let $x + 2y = 10$

$$\Rightarrow \frac{x}{10} + \frac{y}{5} = 1$$



Since, (0, 0) satisfies the inequation, therefore the half plane containing (0, 0) is the required plane.

Again $3x + 2y \le 15$

Let $3x + y = 15$

$$\Rightarrow \frac{x}{5} + \frac{y}{15} = 1$$

It also satisfies by (0, 0) and its required half plane contains (0, 0).

Now double shaded region in the first quadrant contains the solution.

Now OABC represents the feasible region.

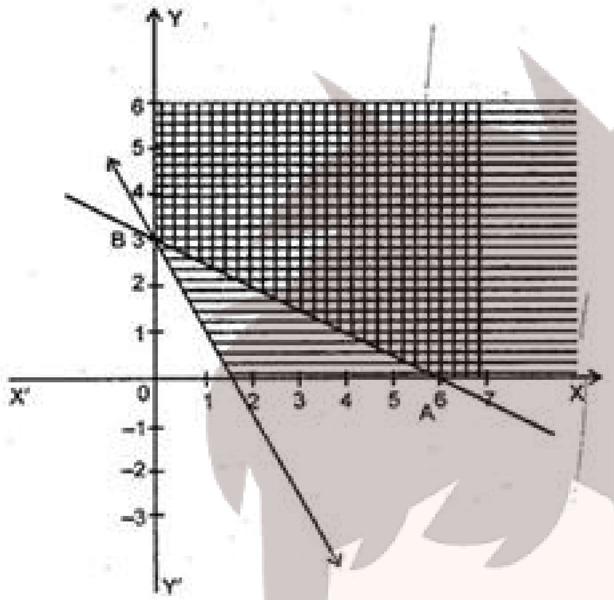
Z = $3x + 2y$

At O (0, 0) Z = 3 x 0 + 2 x 0 = 0

At A (5, 0) Z = 3 x 5 + 2 x 0 = 15

At B (4, 3) Z = 3 x 4 + 2 x 3 = 18

At C (0, 5) Z = 3 x 0 + 2 x 5 = 10



Hence, Z is maximum i.e., 18 at $x = 4, y = 3$.

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6. Minimize Z = $x + 2y$ **subject to** $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0$. **Show that the minimum of Z occurs at more than two points.**

Ans. Consider $2x + y \geq 3$

Let $2x + y = 3 \Rightarrow y = 3 - 2x$

x	0	1	–1
y	3	1	5

(0, 0) is not contained in the required half plane as (0, 0) does not satisfy the inequation $2x + y \geq 3$.

Again $x + 2y \geq 6$

Let $x + 2y = 6$

$$\Rightarrow \frac{x}{6}+\frac{y}{3}=1$$

Here also (0, 0) does not contain the required half plane. The double shaded region XABY is the solution set. Its corners are A (6, 0) and B (0, 3).

At A (6, 0) Z = 6 + 0 = 6

At B (0, 3) Z = 0 + 2 x 3 = 6

Therefore, at both points the value of Z = 6 which is minimum. In fact at every point on the line AB makes Z = 6 which is also minimum.

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7. Minimize and Maximize Z = $5x+10y$ **subject to** $x+2y\le 120, x+y\ge 60, x-2y\ge 0, x, y\ge 0.$

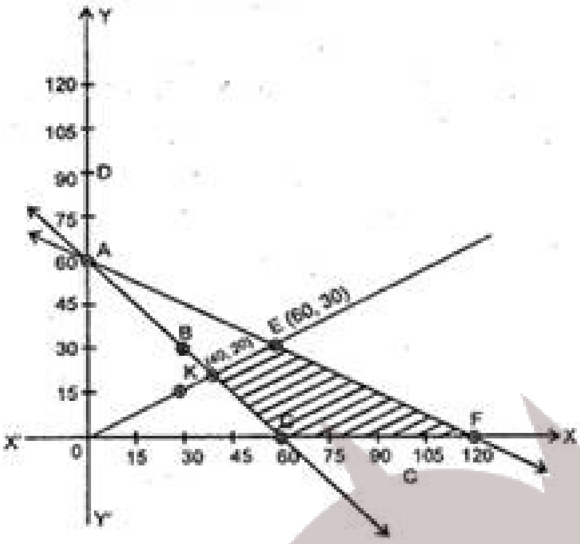
Ans. Consider $x+2y\le 120$

Let $x+2y=120$

$$\Rightarrow \frac{x}{120}+\frac{y}{60}=1$$

The half plane containing(0, 0) is the required half plane as (0, 0) makes $x+2y\le 120$, true.

x	0	30	60
y	60	45	30



Again $x + y \geq 60$

Let $x + y = 60$

Also the half plane containing (0, 0) does not make $x + y \geq 6$ true.

Therefore, the required half plane does not contain (0, 0).

Again $x - 2y \geq 0$

Let $x - 2y = 0 \Rightarrow x = 2y$

Let test point be (30, 0).

x	0	30	60
y	0	15	30

$\Rightarrow x - 2y \geq 0 \Rightarrow 30 - 2 \times 0 \geq 0$ It is true.

Therefore, the half plane contains (30, 0).

The region CFEKC represents the feasible region.

At C (60, 0) Z = 5 x 60 = 300

At F (120, 0) Z = 5 x 120 = 600

At E (60, 30) Z = 5 x 60 + 10 x 30 = 600

At K (40, 20) Z = 5 x 40 + 10 x 20 = 400

Hence, minimum Z = 300 at $x = 60, y = 0$ and maximum Z = 600 at $x = 120, y = 0$ or $x = 60, y = 30.$

8. Minimize and Maximize Z = $x + 2y$ subject to $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y \geq 0.$

Ans. Consider $x + 2y \geq 100$

Let $x + 2y = 100 \Rightarrow \frac{x}{100} + \frac{y}{50} = 1$

$x + 2y \geq 100$ represents which does not include (0, 0) as it does not made it true.

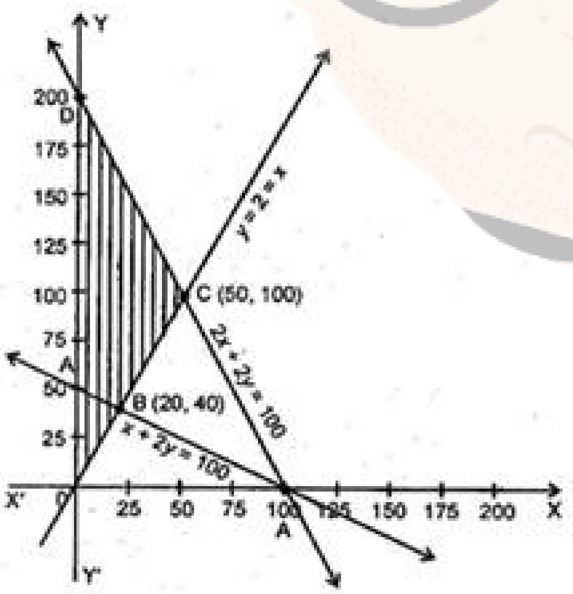
x	0	25	50	100
y	0	50	100	200

Again consider $2x - y \leq 0$

Let $2x - y = 0 \Rightarrow y = 2x$

Let the test point be (10, 0).

$\therefore$ 2 x 10 – 0 $\leq$ 0 which is false.



Therefore, the required half does not contain (10, 0).

Again consider $2x + y \leq 200$

Let $2x + y = 200$

$$\Rightarrow \frac{x}{100} + \frac{y}{200} = 1$$

Now (0, 0) satisfies $2x + y \leq 200$

Therefore, the required half place contains (0, 0).

Now triple shaded region is ABCDA which is the required feasible region.

At A (0, 50)

Z = $x + 2y$ = 0 + 2 x 50 = 100

At B (20, 40) Z = 20 + 2 x 40 = 100

At C (50, 100) Z = 50 + 2 x 100 = 250

At D (0, 200) Z = 0 + 2 x 200 = 400

Hence maximum Z = 400 at $x = 0, y = 200$ and minimum Z = 100 at $x = 0, y = 50$ or $x = 20, y = 40$.

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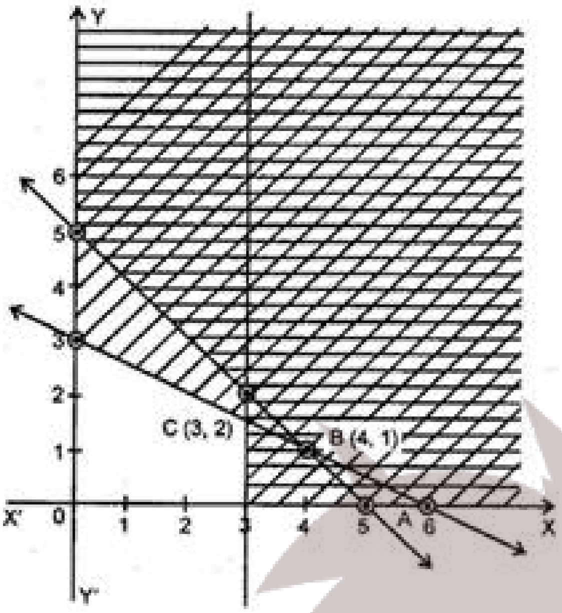
9. Maximize Z = $-x + 2y$ **subject to the constraints:** $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$.

Ans. Consider $x \geq 3$

Let $x = 3$ which is a line parallel to $y-$axis at a positive distance of 3 from it.



Since $x \geq 3$, therefore the required half-plane does not contain (0, 0).

Now consider $x+y \geq 5$

Let $x+y=5$

$$\Rightarrow \frac{x}{5}+\frac{y}{5}=1$$

Now (0, 0) does not satisfy $x+y \geq 5$, therefore the required half plane does not contain (0, 0).

Again consider $x+2y \geq 6$

Let $x+2y=6$

$$\Rightarrow \frac{x}{6}+\frac{y}{3}=1$$

Here also (0, 0) does not satisfy $x+2y \geq 6$, therefore the required half plane does not contain (0, 0).

The corners of the feasible region are A (6, 0), B (4, 1) and C (3, 2).

At A (6, 0) Z = –6 + 2 x 0 = –6

At B (4, 1) Z = –4 + 2 x 1 = –2

At C (3, 2) Z = –3 + 2 x 2 = 1

Hence, maximum Z = 1 at $x = 3, y = 2$.

10. Maximize Z = $x + y$ **subject to** $x - y \leq -1, -x + y \leq 0, x, y \geq 0$.

Ans. Consider $x - y \leq -1$

Let $x - y = -1$

$\Rightarrow x = y - 1$

	A	B	C	D
x	-1	0	2	3
y	0	1	2	4

If (0, 0) is the test point then $x - y \leq -1 \Rightarrow 0 \leq -1$ which is false and thus the required plane does not include (0, 0).

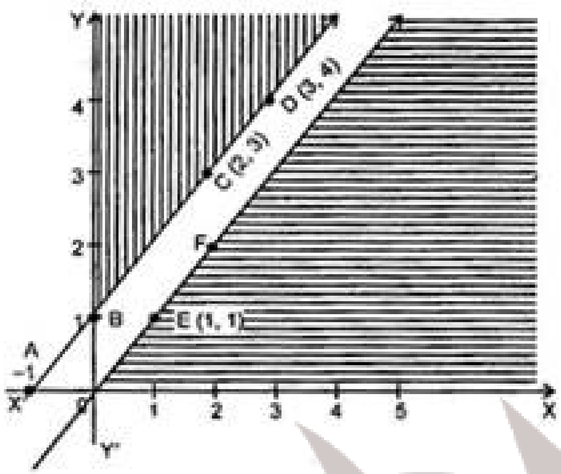
Again $-x + y \leq 0$

Let $-x + y = 0$

$\Rightarrow y = x$

	O	E	F
x	0	1	2
y	0	1	2

For (1, 0) – 1 $\leq$ 0 which is true, therefore the required half-plane include (1, 0).



It is clear that the two required half planes do not intersect at all, i.e., they do not have a common region.

Hence there is no maximum Z.