

JEE Main 2022 B.E./B.Tech June 29- Shift 1- Mathematics

Question ID:101761

Topic Name:Mathematics-Section A

Question:

The probability that a randomly chosen 2×2 matrix with all the entries from the set of first 10 primes, is singular, is equal to :

A $\frac{133}{10^4}$

B $\frac{18}{10^3}$

C $\frac{19}{10^3}$

D $\frac{271}{10^4}$

Answer Given By Candidate:Not Attempted

Question ID:101762

Topic Name:Mathematics-Section A

Let the solution curve of the differential equation

Question: $x \frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$. Then $y(2)$ is equal to :

A 15

B 11

C 13

D 17

Answer Given By Candidate:Not Attempted

Question ID:101763

Topic Name:Mathematics-Section A

Question:

If the mirror image of the point $(2, 4, 7)$ in the plane $3x - y + 4z = 2$ is (a, b, c) , then $2a + b + 2c$ is equal to :

A 54

B 50

C -6

D -42

Answer Given By Candidate:C

Question ID:101764

Topic Name:Mathematics-Section A

Question:

Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be a function defined by :

$$f(x) = \begin{cases} \max_{t \leq x} \{t^3 - 3t\} & ; x \leq 2 \\ x^2 + 2x - 6 & ; 2 < x < 3 \\ [x - 3] + 9 & ; 3 \leq x \leq 5 \\ 2x + 1 & ; x > 5 \end{cases}$$

where $[t]$ is the greatest integer less than or equal to t . Let m be the number of points where
 f is not differentiable and $I = \int_{-2}^2 f(x) dx$. Then the ordered pair (m, I) is equal to :

A $\left(3, \frac{27}{4}\right)$

B $\left(3, \frac{23}{4}\right)$

C $\left(4, \frac{27}{4}\right)$

D $\left(4, \frac{23}{4}\right)$

Answer Given By Candidate:Not Attempted

Question ID:101765

Topic Name:Mathematics-Section A

Question:

Let $\vec{a} = \alpha \hat{i} + 3 \hat{j} - \hat{k}$, $\vec{b} = 3 \hat{i} - \beta \hat{j} + 4 \hat{k}$ and $\vec{c} = \hat{i} + 2 \hat{j} - 2 \hat{k}$ where $\alpha, \beta \in \mathbf{R}$, be threevectors. If the projection of \vec{a} on \vec{c} is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6 \hat{i} + 10 \hat{j} + 7 \hat{k}$, then the value of $\alpha + \beta$ is equal to :

A 3

B 4

C 5

D 6

Answer Given By Candidate:A

Question ID:101766

Topic Name:Mathematics-Section A

Question:

The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}x$ that lies outside the triangle formed by $y = \sqrt{2}x$, $x = 1$, $y = 2\sqrt{2}$, is equal to :

- A $\frac{16\sqrt{2}}{6}$
- B $\frac{11\sqrt{2}}{6}$
- C $\frac{13\sqrt{2}}{6}$
- D $\frac{5\sqrt{2}}{6}$

Answer Given By Candidate: **Not Attempted**

Question ID:101767

Topic Name:Mathematics-Section A

If the system of linear equations

$$2x + y - z = 7$$

$$x - 3y + 2z = 1$$

$$x + 4y + \delta z = k, \text{ where } \delta, k \in \mathbf{R}$$

has infinitely many solutions, then $\delta + k$ is equal to :

- Question:
- A -3
- B 3
- C 6
- D 9

Answer Given By Candidate: **B**

Question ID:101768

Topic Name:Mathematics-Section A

Question:

Let α and β be the roots of the equation $x^2 + (2i - 1)x = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to :

- A 50
- B 250
- C 1250
- D 1500

Answer Given By Candidate: **A**

Question ID:101769

Topic Name:Mathematics-Section A

Question:

Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q) \Delta ((p \vee q) \Rightarrow q)$ is a tautology. Then Δ is equal to :

- A \wedge
- B \vee
- C \Rightarrow
- D \Leftrightarrow

Answer Given By Candidate:Not Attempted

Question ID:101770

Topic Name:Mathematics-Section A

Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all $i, j = 1, 2, 3$. Then, the matrix $A^2 + A^3 + \dots + A^{10}$ is equal to :

Question:

- A $\left(\frac{3^{10} - 3}{2}\right)A$
- B $\left(\frac{3^{10} - 1}{2}\right)A$
- C $\left(\frac{3^{10} + 1}{2}\right)A$
- D $\left(\frac{3^{10} + 3}{2}\right)A$

Answer Given By Candidate:Not Attempted

Question ID:101771

Topic Name:Mathematics-Section A

Question:

Let a set $A = A_1 \cup A_2 \cup \dots \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j, 1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i \text{ if and only if } x \in A_i, 1 \leq i \leq k\}$. Then, R is :

- A reflexive, symmetric but not transitive
- B reflexive, transitive but not symmetric
- C reflexive but not symmetric and transitive
- D an equivalence relation

Answer Given By Candidate:Not Attempted

Question ID:101772

Topic Name:Mathematics-Section A

Question:

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$.

Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to :

A $\frac{6}{343}$

B $\frac{7}{216}$

C $\frac{8}{343}$

D $\frac{49}{216}$

Answer Given By Candidate: **Not Attempted**

Question ID: **101773**

Topic Name: Mathematics-Section A

Question:

The distance between the two points A and A' which lie on $y=2$ such that both the line segments AB and A' B (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to :

A 10

B $\frac{48}{5}$

C $\frac{52}{5}$

D 3

Answer Given By Candidate: **Not Attempted**

Question ID: **101774**

Topic Name: Mathematics-Section A

Question:

A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is :

A $\frac{22}{9 + 4\sqrt{3}}$

B

$$\frac{66}{9 + 4\sqrt{3}}$$

C
$$\frac{22}{4 + 9\sqrt{3}}$$

D
$$\frac{66}{4 + 9\sqrt{3}}$$

Answer Given By Candidate: **Not Attempted**

Question ID:101775

Topic Name:Mathematics-Section A

The domain of the function $\cos^{-1} \left(\frac{2 \sin^{-1} \left(\frac{1}{4x^2-1} \right)}{\pi} \right)$ is :

Question:

A
$$\mathbf{R} - \left\{ -\frac{1}{2}, \frac{1}{2} \right\}$$

B
$$(-\infty, -1] \cup [1, \infty) \cup \{0\}$$

C
$$\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$$

D
$$\left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$$

Answer Given By Candidate:C

Question ID:101776

Topic Name:Mathematics-Section A

Question:

If the constant term in the expansion of $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k \cdot l$, where l is an odd integer, then the value of k is equal to :

A 6

B 7

C 8

D 9

Answer Given By Candidate: **Not Attempted**

Question ID:101777

Topic Name:Mathematics-Section A

$$\int_0^5 \cos \left(\pi \left(x - \left[\frac{x}{2} \right] \right) \right) dx,$$

Question: where $[t]$ denotes greatest integer less than or equal to t , is equal to :

- A -3
- B -2
- C 2
- D 0

Answer Given By Candidate: **Not Attempted**

Question ID:101778

Topic Name:Mathematics-Section A

Question:

Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3, 0)$. Let the line segment PQ be also a focal chord of the ellipse

E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is

equal to :

- A $1 + \sqrt{2}$
- B $3 + 2\sqrt{2}$
- C $1 + 2\sqrt{3}$
- D $4 + 5\sqrt{3}$

Answer Given By Candidate: **Not Attempted**

Question ID:101779

Topic Name:Mathematics-Section A

Question:

Let the tangent to the circle $C_1 : x^2 + y^2 = 2$ at the point $M(-1, 1)$ intersect the circle $C_2 : (x-3)^2 + (y-2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to :

- A $\frac{1}{2}$
- B $\frac{2}{3}$
- C $\frac{1}{6}$
- D

$$\frac{5}{3}$$

Answer Given By Candidate: **Not Attempted**

Question ID:101780

Topic Name:Mathematics-Section A

Question:

Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively.

If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a + x_5)$ is equal to :

- A 13
- B 15
- C 17
- D 18

Answer Given By Candidate: **Not Attempted**

Question ID:101781

Topic Name:Mathematics-Section B

Question:

Let $S = \{z \in \mathbb{C} : |z - 2| \leq 1, z(1 + i) + \bar{z}(1 - i) \leq 2\}$. Let $|z - 4i|$ attains minimum and maximum values, respectively, at $z_1 \in S$ and $z_2 \in S$. If $5(|z_1|^2 + |z_2|^2) = \alpha + \beta\sqrt{5}$, where α and β are integers, then the value of $\alpha + \beta$ is equal to _____.

Answer Given By Candidate: **Not Attempted**

Question ID:101782

Topic Name:Mathematics-Section B

Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^4 x - \cos 2x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}, 0 < x < \frac{\pi}{2} \text{ with } y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}.$$

If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18} e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to _____.

Question:

Answer Given By Candidate: **Not Attempted**

Question ID:101783

Topic Name:Mathematics-Section B

Question:

Let d be the distance between the foot of perpendiculars of the points $P(1, 2, -1)$ and $Q(2, -1, 3)$ on the plane $-x + y + z = 1$. Then d^2 is equal to _____.

Answer Given By Candidate: **26**

Question ID:101784**Topic Name:**Mathematics-Section B**Question:**

The number of elements in the set $S = \{\theta \in [-4\pi, 4\pi] : 3 \cos^2 2\theta + 6 \cos 2\theta - 10 \cos^2 \theta + 5 = 0\}$ is _____.

Answer Given By Candidate:34

Question ID:101785**Topic Name:**Mathematics-Section B**Question:**

The number of solutions of the equation $2\theta - \cos^2 \theta + \sqrt{2} = 0$ in \mathbf{R} is equal to _____.

Answer Given By Candidate:Not Attempted

Question ID:101786**Topic Name:**Mathematics-Section B**Question:**

$50 \tan \left(3 \tan^{-1} \left(\frac{1}{2} \right) + 2 \cos^{-1} \left(\frac{1}{\sqrt{5}} \right) \right) + 4\sqrt{2} \tan \left(\frac{1}{2} \tan^{-1} (2\sqrt{2}) \right)$ is equal to _____.

Answer Given By Candidate:Not Attempted

Question ID:101787**Topic Name:**Mathematics-Section B**Question:**

Let $c, k \in \mathbf{R}$. If $f(x) = (c+1)x^2 + (1-c^2)x + 2k$ and $f(x+y) = f(x) + f(y) - xy$, for all $x, y \in \mathbf{R}$, then the value of $|2(f(1) + f(2) + f(3) + \dots + f(20))|$ is equal to _____.

Answer Given By Candidate:Not Attempted

Question ID:101788**Topic Name:**Mathematics-Section B**Question:**

Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, a > 0, b > 0$, be a hyperbola such that the sum of lengths of the

transverse and the conjugate axes is $4(2\sqrt{2} + \sqrt{14})$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then the value of $a^2 + b^2$ is equal to _____.

Answer Given By Candidate:Not Attempted

Question ID:101789**Topic Name:**Mathematics-Section B**Question:**

Let $P_1: \vec{r} \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 4$ be a plane. Let P_2 be another plane which passes through the points $(2, -3, 2), (2, -2, -3)$ and $(1, -4, 2)$. If the direction ratios of the line of intersection of P_1 and P_2 be $16, \alpha, \beta$, then the value of $\alpha + \beta$ is equal to _____.

Answer Given By Candidate:Not Attempted

Question ID:101790**Topic Name:**Mathematics-Section B

Question:

Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, \dots, 100\}$ for $1 \leq i \leq 4$ and $b_i \neq b_j$ for $i \neq j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1b_2b_3b_4$ is equal to _____.

Answer Given By Candidate: **Not Attempted**