1. Consider the functions

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then

(a) f is differentiable at zero but g is not differentiable at zero
(b) g is differentiable at zero but f is not differentiable at zero
(c) f and g are both differentiable at zero
(d) Neither f nor g is differentiable at zero

2. How many ordered pairs of numbers (x, y) are there, where $x, y \in \{1, 2, \ldots, 100\}$, such that $|x - y| \leq 50$?

(a) 2550
(b) 5050
(c) 7550
(d) None of the other options are correct

3. Let ABC be a right angled isosceles triangle with angle $\angle ABC$ being right-angled. Let D be the mid-point of AB, E be the foot of the perpendicular drawn from D to the side AC, and F be the foot of the perpendicular drawn from E to the side BC. What is the value of $\frac{FC}{BC}$?

(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{3}{4}$
(c) $2 - \sqrt{2}$
(d) None of the other options are correct

4. Suppose that there are 30 MCQ type questions where each question has four options: A, B, C, D. For each question, a student gets 4 marks for a correct answer, 0 marks for a wrong answer, and 1 mark for not attempting the question. Suppose in each question, the probability that option A is correct is 0.5, option B is correct is 0.3, option C is correct is 0.2, and option D is correct is 0. Two students Gupi and Bagha have no clue about the right answers. Gupi answers each question randomly, that is, ticks any of the options with probability 0.25. Whereas Bagha attempts each question with probability 0.5, but whenever he attempts a question, he randomly ticks an option. Which of the following is correct?

(a) Both Gupi and Bagha have expected scores more than 30

(b) Gupi's expected score is greater than or equal to 30 and Bagha's expected score is strictly less than 30

(c) Gupi's expected score is less than or equal to 30 and Bagha's expected score is strictly more than 30

(d) None of the other options are correct

5. Evaluate: $\lim_{x\to\infty}\left[e^{3x}-5x\right]^{\frac{1}{x}}$

(a) e^3

(b) 3

(c) 1

(d) None of the other options are correct

6. Suppose $f(x)=\left\{\begin{array}{ll}\frac{|x-3|}{x-3}, & \text{for } x\neq 3\\ 0, & \text{for } x=3\end{array}\right\}$. Then, $\lim_{x\to 3} f(x)$:

(a) is -1

(b) is 0

(c) does not exist

(d) is 1

7. Consider the following system of equations in x, y, z :

$$\begin{aligned} x+2y-3z &= a\\ 2x+6y-11z &= b\\ x-2y+7z &= c \end{aligned}$$

For what values of a, b, c, does the above system have no solution?

(a) $c+2b-5a\neq 0$

(b) $c+2b-5a=0$

(c) $c+2b-4a=0$

(d) None of the other options are correct

8. The sequence x_n is given by the formula: for every positive integer n

$$x_n=n^3-9n^2+631.$$

The largest value of n such that $x_n > x_{n+1}$

(a) is 4

(b) is 5

(c) is 6

(d) None of the other options are correct

9. Suppose an unbiased coin is tossed 10 times. Let D be the random variable that denotes the number of heads minus the number of tails. What is the variance of D?

(a) 10

(b) 1

(c) 0

(d) None of the other options are correct

10. Suppose we are given a 4×4 square matrix A, which satisfies $A_{ij} = 0$ if $i < j$. Suppose the each diagonal entry A_{ii} is drawn uniformly at random from $\{0, 1, \ldots, 9\}$. What is the probability that A has full rank?

(a) $\frac{1}{10^4}$

(b) $\frac{3}{5}$

(c) $1 - \frac{1}{10^4}$

(d) $\left(\frac{9}{10}\right)^4$

11. Let a and b be two real numbers where $b \neq 0$ and $g : \mathbb{R} \to \mathbb{R}$ is a continuous function which satisfies

$$g(g(x)) = ag(x) + bx \qquad \forall\ x \in \mathbb{R}.$$

Which of the following must be true?

(a) g is strictly increasing

(b) g is strictly decreasing

(c) $\lim_{x\to\infty} g(x)$ is finite

(d) Either (a) or (b)

12. For every positive integer n, let $S(n)$ denote the sum of digits in n. For instance, $S(387) = 3 + 8 + 7 = 18$. The value of the sum

$$S(1) + S(2) + \ldots + S(99)$$

is

(a) 450

(b) 495

(c) 900

(d) 990

13. Suppose five cards are randomly drawn without replacement from an ordinary deck of 52 playing cards, with four suits of 13 cards each, which has been well shuffled. Let a flush be the event that all five cards are of the same suit. What is the probability of getting a flush?

(a) $\frac{^4C_1 \ ^{13}C_5}{^{52}C_5}$

(b) $\frac{^4C_2 \ ^{13}C_4}{^{52}C_5}$

(c) $\frac{^4P_1 \ ^{13}C_5}{^{52}P_5}$

(d) $\frac{^4C_1 \ ^{12}C_5}{^{52}C_5}$

14. Evaluate: $\int x^n \ln x dx$, where $n > 1$

(a) $\ln x \frac{x^{n+1}}{n+1} - \frac{1}{(n+1)^2} x^{n+1} + c$

(b) $\ln x \frac{x^{n+1}}{2(n+1)} - \frac{1}{(n+1)^2} x^{n+1} + c$

(c) $\ln x \frac{x^{n+1}}{n+1} - \frac{1}{(n+1)} x^{n+1} + c$

(d) None of the other options are correct

15. The area of the region bounded by the curve $y = \ln(x)$, the $Y - axis$, and the lines $y = 1$ and $y = -1$

(a) is $\frac{e}{2}$

(b) is 2

(c) is $e - \frac{1}{e}$

(d) None of the other options are correct

16. A price discriminating monopolist finds that a person's demand for its product depends on the person's age. The inverse demand function of someone of age y, can be written $p = A(y) - q$ where $A(y)$ is an increasing function of y. The product cannot be resold from one buyer to another and the monopolist knows the ages of its consumers. (This is often the case with online subscriptions.) If the monopolist maximizes its profits, then

(a) older people will pay higher prices and purchase more of this product compared to younger people

(b) everyone pays the same price but older people consume more

(c) older people will pay higher prices compared to the younger people but everyone will consume the same quantity of the product

(d) None of the other options are correct

17. Pam's family consists of herself and her 3 sisters. They own a small farm in the agricultural sector in Agri-land. The value of their total output is \$4000 which is divided equally amongst the four. The urban sector has two kinds of jobs: informal sector (which anyone can get) pays \$500 and formal sector jobs give \$1200. Pam would like to maximize her own total income and calculates her own expected returns to migration. The proportion of formal sector jobs to urban labor force that would deter her from migrating is:

(a) Less than 2/3

(b) More than 5/6

(c) More than $\frac{1}{2}$

(d) Less than 5/7

18. Hu and Li are two dealers of used tractors in a rural area of China. Hu sells high quality second hand tractors while Li sells low quality ones. Hu would be willing to sell his high quality tractor at \$8000 while Lu would sell his low quality one for \$5000. Consumers are willing to pay up to \$10,000 for a high quality tractor and \$7000 for a low quality one. They expect a 50% chance of buying a high quality second-hand tractor. In order to signal the quality of their tractors Hu and Li can offer warranties. The cost of warranty for a high quality tractor is 500Y and 1000Y for a low quality one (Y is the number of years of warranty). What is the optimal number of years of warranty that Hu should offer so that consumers know his tractors are of good quality?

(a) Less than 2 years

(b) 0.5 years

(c) More than 1.5 years

(d) 3 years

19. Suppose the capacity curve for each laborer is described as follows: for all payments up to \$100, capacity is zero and then begins to rise by 2

units for each additional $ paid. This happens until the payment rises to $500. Thereafter, an additional $ payment increases work capacity by only 1.1 units, until total income paid is $1000. At this point additional payments have no effect on work capacity. Assume all income is spent on nutrition. Suppose you are an employer faced by the above capacity curve of your workers. You need 8000 units of work or capacity units. How many workers would you hire and how much would you pay each worker so that you get 8000 units of work at minimum cost?

(a) 5 workers; $1000 per worker

(b) 10 workers; $700 per worker

(c) 10 workers; $500 per worker

(d) 15 workers; $400 per worker

20. Suppose you were to believe that "money illusion" exists, that is as prices and income rise proportionally, then people buy more. Which of the following statements about demand should not be true?

(a) Demand functions are downward sloping

(b) Demand functions are homogenous of degree zero

(c) Demand has a positive vertical intercept

(d) Demand functions are homogenous of degree one

21. Consider a Bertrand price competition model between two profit maximizing widget producers, say A and B. The marginal cost of producing a widget is 4 for each producer. Each widget producer has a capacity constraint to produce only 5 widgets. There are 8 identical individuals who demand 1 widget only, and value each widget at 6. If the firms are maximizing profits, then the following statement is true:

(a) Firm A and B will charge 4

(b) Firm A and B will charge 6

(c) Firm A and B will charge greater than or equal to 5

(d) None of the other options are correct

22. The government estimates the market demand (Q_D) and market supply (Q_S) for turnips to be the following: $Q_D = 30 - 2P$, $Q_S = 4$; where P is the per unit price and Q is the quantity measured in kilograms. The government aims to increase the market price of turnips to $8 per unit

to improve the welfare of domestic producers of turnips. Is is considering three possible choices: (i) a per unit subsidy; (ii) a price floor and purchase of any surplus production, and (iii) a production quota. Which of these policies should the government adopt if it wants to maximize the producers' welfare but minimize the loss of efficiency?

(a) A production quota

(b) A price subsidy

(c) Either a price subsidy or a price floor

(d) Either a production quota or a price floor

23. A monopolist faces a demand curve: $q = \dfrac{5}{p}$. Her cost function is: $C(q) = 3q$. Suppose, in the same market, there are some competitive suppliers ready to sell the good at the price $p = 5$. The monopolist's profit maximizing price and output could be given by

(a) $p = 3$, $q = \frac{5}{3}$.

(b) $p = 3.01$, $q = \frac{5}{3.01}$

(c) $p = 2.99$, $q = \frac{5}{2.99}$

(d) $p = 4.99$, $q = \frac{5}{4.99}$

24. The consumption function is given by $C = AY^{\beta}$ with $\beta = 0.5$ and $A = 0.3$. The marginal propensity to save is

(a) equal to 0.5

(b) increasing in income, Y

(c) equal to 0.3

(d) equal to 0.7

25. The production function is given by $Y = AL$. The wage rigidity constraint is given by $W \geq B$. The labour endowment is given by C. Here A, B, and C are finite and positive constants. Assume that the entire labor endowment is supplied. If $A > B$, then in a labour market equilibrium

(a) $L = C$

(b) $L = 0$

(c) $0 < L < C$

(d) None of the other options are correct

26. Consider the Mundell-Flemming model with perfect capital mobility and a flexible exchange rate in the short run. A monetary expansion leads to a/an _____ in output; a fiscal expansion leads to _____ in output.

(a) decrease; no change

(b) increase; decrease

(c) increase; no change

(d) increase; increase

27. Mr. X has an exogenous income W and his utility from consumption c is $u(c)$. Mr. X knows that an accident can occur with probability p and if it occurs, the monetary equivalent to the damage is T. Mr. X can however affect the accident probability p through the prevention effort e. In particular, e can take two values - zero and a and an assumption is that $p(0) > p(a)$, that is by putting prevention effort, probability of occurring an accident can be reduced. Let us also assume that if Mr. X puts an effort e, the disutility from the effort is Ae^2 where A is the per unit effort cost. What is the critical value of A, A^*, below which the effort will be undertaken, and above which the effort will not be undertaken, by Mr. X ?

(a) $A^* = \frac{[p(a)-p(0)][u(W-T)-u(W)]}{a^2}$

(b) $A^* = \frac{[p(a)-p(0)]a^2}{[u(W-T)-u(W)]}$

(c) $A^* = \frac{\frac{p(a)}{p(0)}}{\frac{u(W-T)}{u(W)}}a^2$

(d) $A^* = \frac{p(a)p(0)a^2}{u(W-T)u(W)}$

28. Labor supply in macro models results from individual decision making. Let c denote an individual's consumption and L denote labor supply. Assume that individuals solve the following optimization problem

$$\underset{\{c,L\}}{Max}\ U(c,L) = \log c - \frac{1}{2}\frac{1}{b}L^2$$

subject to $c + \bar{S} = wL$ where $U(\cdot)$ is the utility function, $b > 0$ is a constant, $\bar{S}$ is a constant exogenous level of savings, and w is the real wage the person can earn in the labor market. Derive the optimal labor supply. It is

(a) increasing in w; increasing in c

(b) decreasing in w; decreasing in c

(c) increasing in w; decreasing in c

(d) decreasing in w; increasing in c

29. Consider a Solow economy that begins in steady state. Then a strong earthquake destroys half the capital stock. The steady state level of capital_____, the level of output____on impact, and the growth rate of the economy_____as the economy approaches its steady state.

(a) decreases; decreases; decreases

(b) remains the same; decreases; decreases

(c) remains the same; decreases; increases

(d) decreases; remains the same, decreases

30. Suppose the economy is characterized by the following equations

$$\begin{aligned} C &= c_0 + c_1 Y_D \\ Y_D &= Y - T \\ I &= b_0 + b_1 Y \end{aligned}$$

where C = Consumption, c_0 = Autonomous Consumption, $c_1 \in [0, 1]$, Y_D = Disposable Income, Y = Aggregate GDP, T = Taxes, I = Investment, b_0 = Autonomous Investment, and $b_1 \in [0, 1]$. For the multiplier to be positive, what condition needs to be satisfied?

(a) $b_1 + c_1 = 0$

(b) $b_1 + c_1 = 1$

(c) $b_1 + c_1 < 1$

(d) $b_1 + c_1 > 1$