07/11/17

Baye's theorem
1+2+3+5 = 11m
Bernoulli's Trials / PD

PROBABILITY

Conditional Probability :

* The probability of event E is called conditional Prob of E, given that event F has already occurred and is denoted as $P(E|F)$

* $P(E|F) = \frac{P(E\cap F)}{P(F)}$ $P(F) \to$ already occurred / given that / on condition

* Properties :
1] $P(S|F) = 1 = P(F|F)$ (S → sample space)
2] If A and B are any two events of sample space S, and F is an event such that $P(F) \neq 0$, then
$$P((A\cup B)|F) = P(A|F) + P(B|F) - P((A\cap B)|F)$$
** 3] $P(E'|F) = 1 - P(E|F)$ $(\because E\cup E' = S)$

Eg 1] $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{4/13}{9/13} = \frac{4}{9}$

Eg 2] $S = \{(b,b),(b,g),(g,b),(g,g)\}$
$E = \{(b,b)\}$; F = condition $= \{(b,b),(b,g),(g,b)\}$
$E\cap F = \{(b,b)\}$
$P(E\cap F) = \frac{1}{4}$, $P(F) = \frac{3}{4}$

$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{1/4}{3/4} = \frac{1}{3}$

Eg 3] $S = \{1,2,3,4,5,6,7,8,9,10\}$
$A = \{2,4,6,8,10\}$ = even no.
$B = \{4,5,6,7,8,9,10\}$
$A\cap B = \{4,6,8,10\}$
$P(A\cap B) = \frac{4}{10}$; $P(B) = \frac{7}{10}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/10}{7/10} = \frac{4}{7}$

Eg 4] ~~B = 10% of 430 = 43~~

$n(S) = 1000$

~~A(A) =~~ A = student chosen randomly from class XII

B = chosen student is a girl.

$P(A \cap B) = \frac{10\% \text{ of } 430}{1000} = \frac{43}{1000}$

$P(B) = \frac{430}{1000}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{43/1000}{430/1000} = \frac{1}{10}$

Eg 6] $n(S) = 36$

B = sum is 6 = {(1,5), (2,4), (3,3), (4,2), (5,1)}

A = 4 atleast once = {(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (1,4), (2,4), (3,4), (5,4), (6,4)}

~~P(A∩B) = {(2,4)~~ $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$; $P(B) = \frac{5}{36}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{5/36} = \frac{2}{5}$

EXERCISE 13.2

1] (a) $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$

(b) $P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$

2] $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.32}{0.5} = 0.64$

3] (i) $P(A\cap B) = \cancel{P(A|B)\cdot P(B)} = P(B|A)\cdot P(A)$
$= (0.4)(0.8) = 0.32$

(ii) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.32}{0.5} = 0.64$

(iii) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$= 0.8 + 0.5 - 0.32 = 0.98$

4] $2P(A) = \frac{5}{13} \Rightarrow P(A) = \frac{5}{26}$, $P(B) = \frac{5}{13}$

$P(A|B) = \frac{P(A\cap B)}{P(B)}$

$\frac{2}{5}\cdot\frac{5}{13} = \boxed{P(A\cap B) = \frac{2}{13}}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{5}{26} + \frac{3}{13} = \frac{11}{26}$

5] i] $P(A\cap B) = P(A) + P(B) - P(A\cup B)$
$= \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{4}{11}$

ii] $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{4/11}{5/11} = \frac{4}{5}$

iii] $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{4/11}{6/11} = \frac{4}{6} = \frac{2}{3}$

6] $n(S) = 8$

(i) $E = \{HHH, HTH, THH, TTH\}$

$F = \{HHT, HHH\}$

$E \cap F = \{HHH\} \Rightarrow P(E \cap F) = \frac{1}{8}$; $P(F) = \frac{2}{8}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/8}{2/8} = \frac{1}{2}$

(ii) $E = \{HHH, HHT, HTH, THH\} \Rightarrow P(E) = 4/8$

$F = \{HHT, HTH, THH, THT, HTT, TTH, TTT\}$; $P(F) = \frac{7}{8}$

$P(E \cap F) = \frac{3}{8}$; $P(F) = \frac{7}{8}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{3/8}{7/8} = \frac{3}{7}$.

(iii) $E = \{HHH, HHT, HTH, THH, THT, HTT, TTH\}$

$F = \{HHT, HTH, THH, THT, HTT, TTH, TTT\}$

$E \cap F = \{HHT, HTH, THH, THT, HTT, TTH\}$

$P(E \cap F) = \frac{6}{8}$; $P(F) = \frac{7}{8}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{6/8}{7/8} = \frac{6}{7}$

7] i] $n(S) = 2 \times 2 = 4$

$E = \{tt, th, ht\}$, $F = \{hh, ht, th\}$

$E \cap F = \{th, ht\}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2/4}{3/4} = \frac{2}{3}$

ii] $E = \{hh\}$, $F = \{tt\}$

$E \cap F = \{\phi\}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0/4}{1/4} = 0$

8] $n(S) = 216$

$F = \{(6,5,1), (6,5,2), (6,5,3), (6,5,4), (6,5,5), (6,5,6)\}$

$E \cap F = \{(6,5,4)\}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/216}{6/216} = \frac{1}{6}$

** 9] $E = \{SFM, SMF, FMS, MFS\}$, $F = \{SFM, MFS\}$

$n(S) = 3! = 6$

$E \cap F = \{SFM, MFS\}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2/6}{2/6} = 1$

10] (a) $n(S) = 6 \times 6 = 36$

E = Sum greater than 9

$= \{(4,6), (5,5), (6,4), (5,6), (6,5), (6,6)\}$

$F = \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

$E \cap F = \{(5,5), (5,6)\}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2/36}{6/36} = \frac{1}{3}$

(b) $E = \{(2,6), (6,2), (3,5), (5,3), (4,4)\}$

$F = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3) \ldots (6,1), (6,2), (6,3)$

$n(F) = 6 \times 3 = 18$

$(E \cap F) = \{(6,2), (5,3)\}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2/36}{18/36} = \frac{1}{9}$

11] Gn: $E = \{1, 3, 5\}$, $F = \{2, 3\}$, $G = \{2, 3, 4, 5\}$

$n(S) = 6$

i] $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/6}{2/6} = \frac{1}{2}$

$P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{1/6}{3/6} = \frac{1}{3}$

ii] $P(E|G) = \frac{P(E\cap G)}{P(G)} = \frac{2/6}{4/6} = \frac{1}{2}$

$P(G|E) = \frac{P(G\cap E)}{P(E)} = \frac{2/6}{3/6} = \frac{2}{3}$

iii] $E\cup F = \{1, 2, 3, 5\}$

$(E\cup F)\cap G = \{2, 3, 5\}$

$E\cap F = \{3\}$

$(E\cap F)\cap G = \{3\}$

$P((E\cup F)|G) = \frac{P((E\cup F)\cap G)}{P(G)} = \frac{3/6}{4/6} = \frac{3}{4}$

$P((E\cap F)|G) = \frac{P((E\cap F)\cap G)}{P(G)} = \frac{1/6}{4/6} = \frac{1}{4}$

16] $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0}{0}$ = not defined

17] Gn: $P(A|B) = P(B|A)$

$\frac{P(A\cap B)}{P(B)} = \frac{P(B\cap A)}{P(A)}$

$P(A) = P(B)$

12] $n(S) = 4$

$S = \{GG, BB, BG, GB\}$

$E = \{GG\}$

i] $F = \{(G,G), BG\}$

$E\cap F = \{GG\}$

$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{2/4}{2/4} = \frac{1}{2}$

ii] $F = \{(GG), (GB), (B,G)\}$

$E\cap(F) = \{(G,G)\}$ ~~GB, BG~~

$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{3/4}{3/4} = \frac{1}{3}$

**13] $n(S) = 1400\ (300+200+500+400)$

$n(E) = 300+500 = 800$

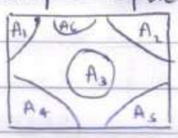
$n(F) = 500+400 = 900$

$E \cap F$ = easy mcq $= 500$

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{500/1400}{900/1400} = \frac{5}{9}$$

Multiplication Theorem on Probability:

- An event occurs and without replacement it does not affect the occurance of 2^{nd} event. For such events we use multiplication theorem.
- If the occurance of one event does not affect the occurance of the other, then they are called independent events.
- ** If A & B are independent events, Then $P(A\cap B) = P(A)\cdot P(B)$
- If the events are independent and do not have anything in common (disjoint set) from the same sample space i.e,



Such sets are called mutually exclusive events.

- ** If A & B are mutually exclusive, then $P(A\cup B) = P(A) + P(B)$

[extension of this principle is Baye's Theorem, when no. of disjoint sets are more and no point is left in the sample space]

- If A & B are two independent events, probability of occurance of atleast one of A and B is equal to $1 - P(A')\cdot P(B')$

EXERCISE 13.2

1] Gn: A & B are independent events

$P(A\cap B) = P(A)\cdot P(B) = \frac{3}{5}\cdot\frac{1}{5} = \frac{3}{25}$

6] $\frac{3}{5}\cdot\frac{3}{10} \neq \frac{1}{5}$ i.e $P(E)\cdot F(F) \neq P(E\cap F)$

$\therefore$ Gn events are not independent.

7] i] $P(A\cup B) = P(A) + P(B)$

$\frac{3}{5} = \frac{1}{2} + p$

$p = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$

ii] $P(A\cap B) = P(A)\cdot P(B)$

$P(A) + P(B) - P(A\cup B) = P(A)\cdot P(B)$

$\frac{1}{2} + p - \frac{3}{5} = \frac{1}{2}\cdot p$

$p - \frac{1}{10} = \frac{p}{2} \Rightarrow \frac{10p-1}{10} = \frac{p}{2} \Rightarrow 10p - 1 = 5p$

$\Rightarrow 5p = 1$

$\Rightarrow p = \frac{1}{5}$

8] i] $P(A\cap B) = P(A)\cdot P(B) = (0.3)(0.4) = 0.12$

ii] $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 0.3 + 0.4 - 0.12 = 0.58$

iii] $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.12}{0.4} = 0.3$

iv] $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{0.12}{0.3} = 0.4$

9] $P(\text{not } A \text{ and not } B) = P(A'\cap B') = P(A\cup B)'$

$= 1 - P(A\cup B)$

$= 1 - [P(A) + P(B) - P(A\cap B)]$

$= 1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{8}$

$= \frac{1}{2} - \frac{1}{4} + \frac{1}{8}$

$= \frac{3}{8}$

10] $P(\text{not } A \text{ or not } B) = P(A' \cup B') = P(A \cap B)' = \frac{1}{4}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{7}{12} = \frac{7}{24} \rightarrow (1)$

$\therefore 1 - P(A \cap B) = \frac{1}{4}$

$P(A \cap B) = \frac{3}{4} \rightarrow (2)$

(1) & (2) $\Rightarrow P(A \cap B) \neq P(A) \cdot P(B)$

$\therefore$ A & B are not independent.

Eg 8] $\cancel{PB}$ $n(S) = 10 + 5 = 15$

$P(B_1) = \frac{10}{15}$, $P(B_2) = \frac{9}{14}$

Total prob $= P(B_1) \cdot P(B_2) = \frac{10}{15} \cdot \frac{9}{14} = \frac{3}{7}$

2] Two cards are drawn at random and without replacement from a pack of 52 cards. Find prob that both are black.

Sol] $n(S) = 52$

$P(B_1) = \frac{26}{52}$; $P(B_2) = \frac{25}{51}$

Total prob $= \frac{26}{52} \cdot \frac{25}{51} = \frac{25}{102}$

3] A box of oranges is inspected by examining 3 selected oranges drawn without replacement. If all 3 oranges are good, the box is approved. Find the prob that box containing 15 oranges out of which 12 are good and 3 are bad will be approved.

Sol] $n(S) = 15$

$P(1^{st} \text{ good orange}) = \frac{12}{15}$, $P(2^{nd} \text{ good orange}) = \frac{11}{14}$

$P(3^{rd} \text{ good orange}) = \frac{10}{13}$

Total prob $= \frac{\cancel{12}^{4}}{\cancel{15}_{5}} \cdot \frac{11}{\cancel{14}_{7}} \cdot \frac{\cancel{10}^{2}}{13} = \frac{44}{91}$

5] A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event the "no. is even", B : the no. is red. Are A and B independent?

Sol] $A = \text{even} = \{2, 4, 6\}$

$B = \{1, 2, 3\}$

$A \cap B = \text{even red} = \{2\}$

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

$P(A \cap B) = \frac{1}{6}$

$P\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \neq \frac{1}{6} \Rightarrow P(A) \cdot P(B) \neq P(A \cap B)$

$\therefore$ A & B are not independent.

11] Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find

i] $P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B) = 0.18$

ii] $P(A \text{ and not } B) = P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.18 = 0.12$

iii] $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.18 = 0.72$

iv] $P(\text{neither } A \text{ and nor } B) = P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - 0.72$

$= 0.28$

** 12] A die is tossed thrice. Find the prob of getting an odd no. atleast once.

Sol] A = getting odd = {1, 3, 5} $\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$; $P(A') = 1 - \frac{1}{2} = \frac{1}{2}$

P(atleast one an odd no.) = 1 − P(no odd)

$= 1 - [P(A') \cdot P(A') \cdot P(A')]$

$= 1 - \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) = 1 - \frac{1}{8} = \frac{7}{8}$

** 14] Prob of solving specific problem independently by A & B are $\frac{1}{2}$ and $\frac{1}{3}$. If both try to solve the problem independently. Find the prob that

i] The problem is solved.

Gn: $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$

P(A not solving) $= P(A') = 1 - \frac{1}{2} = \frac{1}{2}$

P(B not solving) $= P(B') = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ P(A, B not solving) $= P(A') \cdot P(B') = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$

P(problem solved) = 1 − P(problem not solved) $= 1 - \frac{1}{3} = \frac{2}{3}$

ii] exactly one of them solves the problem

P(one solving) $= P(A)\,P(B') + P(A')\,P(B)$

$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

16] In a hostel, 60% read H, 40% read E and 20% read both. A student is selected.

(a) Find the prob that she reads neither H nor E

(b) If she reads H, find prob that she reads E ; P(E|F)

(c) If she reads E, find prob that she reads H ; P(F|E)

Sol] $P(H) = \frac{60}{100} = 0.6$

$P(E) = \frac{40}{100} = 0.4$

$P(H \cap E) = \frac{20}{100} = 0.2$

(a) $P(H' \cap E') = P(H \cup E)' = 1 - P(H \cup E) = 1 - [P(H) + P(E) - P(H \cap E)]$

$= 1 - 0.6 - 0.4 + 0.2$

$= 0.2$

(b) $P(E \mid H) = \frac{P(E \cap H)}{P(H)} = \frac{0.2}{0.6} = \frac{1}{3}$

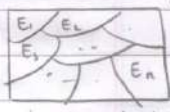
(c) $P(H \mid E) = \frac{P(H \cap E)}{P(E)} = \frac{0.2}{0.4} = \frac{1}{2}$

17] The prob of obtaining an even prime no on each die when a pair of dice is rolled.

Sol] $P((2,2)) = \frac{1}{36}$

Partition of a Sample Space:

S



pair wise disjoint sets (nothing common)

The events $E_1, E_2, E_3 \ldots E_n$.

$P(E_1 \mid A) = \frac{P(E_1 \cap A)}{P(A)}$

$A \rightarrow$ common event to all events E_n.

$E_1, E_2 \ldots E_n$ represent a partition of a sample space S, if they are pairwise disjoint, exhaustive and have non-zero probabilities.

Theorem of total probability:

Considering a partition of sample space S,

$P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2) + \cdots + P(E_n) \cdot P(A \mid E_n)$

i.e $P(A) = \sum_{j=1}^{n} P(E_j) \cdot P(A \mid E_j)$

Here the set $\{E_1, E_2, E_3, \ldots, E_n\}$ be partition of S, each event E has non-zero probability.

Let A be any event associated with S which is common to every event in S.

For any event A, $A = A \cap S$

$= A \cap (E_1 \cup E_2 \cup \cdots E_n)$

$= (A \cap E_1) \cup (A \cap E_2) \cup \cdots (A \cap E_n)$

$P(A) = P(A \cap E_1) + P(A \cap E_2) + \cdots P(A \cap E_n)$

$= P(E_1) \cdot P(A|E_1) + \cdots + P(E_n) \cdot P(A|E_n)$

$\therefore \boxed{P(A) = \sum_{j=1}^{n} P(E_j) \cdot P(A|E_j)}$

Baye's Theorem: (3m)

The prob of particular event E_i in the partition S, given that event A has occurred can be found using total prob P(A).

$$P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{P(A)}$$

EXERCISE 13.3

2] A bag contains 4 red and 4 black balls. Another bag contains 2R and 6B. One of the two bags is selected at random and a ball is drawn, which is found to be red. Find the prob that the ball is drawn from the 1^{st} bag.

Sol] Let E_1 – bag 1 ; E_2 – bag 2

$P(E_1) = P(E_2) = \frac{1}{2}$

$P(A|E_1) = \frac{4}{8} = \frac{1}{2}$

$P(A|E_2) = \frac{2}{8} = \frac{1}{4}$

A = drawing red ball

$P(E_1|A) = ?$

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{4}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{4}} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$$

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7] An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The prob of an accident are 0.01, 0.03 and 0.15 respectively. One of a insured person meets with an accident. What is the prob that he will be a scooter driver?

Sol] $n(S) = 2000 + 4000 + 6000 = 12000$

Scooter driver $= E_1$; A = accident

Car driver $= E_2$

Truck driver $= E_3$

$P(E_1) = \frac{2000}{12000} = \frac{1}{6} = 0.16$

$P(E_2) = \frac{4000}{12000} = \frac{1}{3} = 0.33$

$P(E_3) = \frac{6000}{12000} = \frac{1}{2} = 0.5$

$P(A|E_1) = 0.01$

$P(A|E_2) = 0.03$

$P(A|E_3) = 0.15$

$$P(E_1|A) = ? = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{(0.16)(0.01)}{(0.16)(0.01) + (0.33)(0.03) + (0.5)(0.15)}$$

$$= \frac{0.0016}{0.0016 + 0.0099 + 0.075} = \frac{0.0016}{0.0865}$$

$$= 0.018 = \frac{1}{52}$$

9] Two groups are competing for the position on the board. The prob's that the 1st and 2nd group will win are 0.6 and 0.4 respectively. Further if 1st grp wins, prob of introducing a new product is 0.7 and that of 2nd grp is 0.3. Find the prob that the new product introduced was by 2nd grp.

Sol] E_1 - grp 1 ; E_2 - grp 2

$P(E_1) = 0.6$; $P(E_2) = 0.4$

A = new product

$P(A|E_1) = 0.7$; $P(A|E_2) = 0.3$

$$P(E_2|A) = \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{(0.4)(0.3)}{(0.6)(0.7)+(0.4)(0.3)} = \frac{0.12}{0.42+0.12}$$

$$= \frac{0.12}{0.54} = 0.22$$

Eg 20] A doctor is to visit a patient, it is known that the prob's that he will come by train, bus, scooter, other means are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The prob's that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by other means he will not be late. When he arrives, he is late. What is the prob that he comes by train.

Sol] E_1 - train, E_2 - bus, E_3 - scooter, E_4 - other

$P(E_1) = \frac{3}{10}$ $P(E_2) = \frac{1}{5}$ $P(E_3) = \frac{1}{10}$ $P(E_4) = \frac{2}{5}$

$P(A|E_1) = \frac{1}{4}$ $P(A|E_2) = \frac{1}{3}$ $P(A|E_3) = \frac{1}{12}$ $P(A|E_4) = 0$

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3) + P(E_4) + P(A|E_4)}$$

$$= \frac{\frac{3}{10}\cdot\frac{1}{4}}{\frac{3}{10}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{1}{3} + \frac{1}{10}\cdot\frac{1}{12} + \frac{2}{5}\cdot 0}$$

$$= \frac{\frac{3}{40}}{\frac{3}{40}+\frac{1}{15}+\frac{1}{120}} = \frac{0.075}{0.075+0.066+0.008}$$

$$= \frac{0.075}{0.149} = 0.5 = \frac{1}{2}$$

Eg 17] Given 3 identical boxes 1, 2, 3 each containing 2 coins. Box 1 has 2 gold coins, 2 has 2 silver coins, 3 has 1 gold and 1 silver coins. A person chooses a box and picks a coin. If the coin is gold, what is the prob that the other coin in the box is of gold.

Sol] E_1 – box 1 E_2 – box 2 E_3 – box 3

A = gold coin

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P(A|E_1) = \frac{2}{2} = 1$

$P(A|E_2) = \frac{0}{2} = 0$

$P(A|E_3) = \frac{1}{2}$

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$$

$$= \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} 1 + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot \frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6}} = \frac{1}{3}\left[\frac{1}{\frac{1}{2}}\right]$$

$$= \frac{2}{3}$$

** Eg 21] A man is known to speak true 3 out of 4 times. He throws a die and reports it is a six. Find the prob that it is actually a 6.

Sol] E_1 = 6 occurs E_2 = 6 does not occurs

$P(E_1) = \frac{1}{6}$

$P(E_2) = 1 - \frac{1}{6} = \frac{5}{6}$ A = speaking true

$P(A|E_1) = P$ (6 occurs and man reports 6)

$= P$ (speaking truth) $= \frac{3}{4}$

$P(A|E_2) = P$ (6 does not occur and man reports 6)

$= P$ (speaking lie) $= 1 - \frac{3}{4} = \frac{1}{4}$

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{1}{6}\cdot\frac{3}{4}}{\frac{1}{6}\cdot\frac{3}{4} + \frac{5}{6}\cdot\frac{1}{4}} = \frac{\frac{1}{8}}{\frac{1}{8}+\frac{5}{24}} = \frac{\frac{1}{8}}{\frac{3+5}{24}} = \frac{1}{8}\cdot\frac{24}{8}$$

$$= \frac{3}{8}$$

13] Prob that A speaks truth is $\frac{4}{5}$. A coin is tossed, 'A' reports head appears. What is the prob that head occurs?

Sol] E_1 - occuring head ; $P(E_1) = \frac{1}{2}$

E_2 - not occuring head ; $P(E_2) = 1 - \frac{1}{2} = \frac{1}{2}$

A - speaking truth.

$P(A|E_1) = \frac{4}{5}$; $P(A|E_2) = 1 - \frac{4}{5} = \frac{1}{5}$

$$\therefore P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\, P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{4}{5}}{\frac{1}{2}\cdot\frac{4}{5} + \frac{1}{2}\cdot\frac{1}{5}} = \frac{\frac{2}{5}}{\frac{2}{5}+\frac{1}{10}} \cancel{= \frac{1}{5}\left(\frac{2}{2+\frac{1}{2}}\right) = \frac{4}{5}}$$

$$= \frac{\frac{2}{5}}{\frac{5}{10}} = \frac{4}{5}$$

Eg 18] Suppose that the reliability of a HIV test is specified as follows:

Of people having HIV, 90% of the test detect disease, 10% go undetected.

Of people free of HIV, 99% of the test are judged HIV -ve, 1% are diagnised as HIV +ve.

From a large population of which 0.1% have HIV, one person is selected at random, Given test and reported HIV +ve. What is the prob that person actually has HIV?

Sol] E_1 = person actually has HIV ; $P(E_1) = 0.1\% = \frac{0.1}{100}$

E_2 = person actually not having HIV

$P(E_2) = 1 - \frac{0.1}{100} = \frac{999}{100}$

A = person is ~~det~~ tested and reported +ve.

$P(E_1|A) = ?$

$P(A|E_1)$ = Person actually has HIV and tested +ve.

$= 90\% = 0.9 = \frac{90}{100}$

$P(A|E_2)$ = Person does not HIV but tested +ve.

$= 1\% = \frac{1}{100} = 0.1$

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{\frac{0.1}{100} \cdot \frac{90}{100}}{\frac{0.1}{100} \cdot \frac{90}{100} + \frac{999}{100} \cdot \frac{1}{100}} = \frac{9}{9+999} = \frac{9}{\cancel{1008}\ 108.9} = 0.083$$

Eg 19] In a factor machines A, B, C manufacture 25%, 35% and ~~30~~ 40% of the bolts of their outputs 5%, 4%, 2% are respectively defective. A bolt is drawn and is found to be defective. What is the prob that it is from B?

Sol] E_1 = machine A E_2 = machine B E_3 = machin C

A = defective bolts

$P(E_1) = \frac{25}{100}$ $P(E_2) = \frac{35}{100}$ $P(E_3) = \frac{40}{100}$

$P(A|E_1) = \frac{5}{100}$ $P(A|E_2) = \frac{4}{100}$ $P(A|E_3) = \frac{2}{100}$

$$P(E_2|A) = \frac{P(E_2) \cdot P(A|E_2)}{P(E_1) P(A|E_1) + P(E_2) \cdot P(A|E_2) + \dots} = \frac{\frac{35}{100} \cdot \frac{4}{100}}{\frac{35}{100} \cdot \frac{4}{100} + \frac{25}{100} \cdot \frac{5}{100} + \frac{40}{100} \cdot \frac{2}{100}}$$

$$= \frac{140}{140 + 125 + 80} = \frac{140}{\cancel{265}\ 345} = 0.405$$

11] Manufacturer has 3 operators A, B, C. 1st operator A produces 1% defective items, B - 5%, C - 7%. A is on job 50% of the time, B - 30%, C - 20%. A defective item is produced. What is prob. that it was by A?

Sol] $E_1 = A$ $E_2 = B$ $E_3 = C$

A = defective item

$P(E_1) = \frac{50}{100}$ $P(E_2) = \frac{30}{100}$ $P(E_3) = \frac{20}{100}$

$P(A|E_1) = \frac{1}{100}$ $P(A|E_2) = \frac{5}{100}$ $P(A|E_3) = \frac{7}{100}$

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$$

$$= \frac{\frac{50}{100} \cdot \frac{1}{100}}{\frac{50}{100} \cdot \frac{1}{100} + \frac{30}{100} \cdot \frac{5}{100} + \frac{20}{100} \cdot \frac{7}{100}} = \frac{50}{50 + 150 + 140}$$

$$= \frac{5\cancel{0}}{34\cancel{0}} = \frac{5}{34}$$

** 4] In answering a question on a ~~MCQ~~ MCQ test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the prob that he knows the answer and $\frac{1}{4}$ be the prob that he guesses. Assuming that a student who guesses the answer will be correct with prob $\frac{1}{4}$. What is the prob that student knows the answer given that he answered it correctly.

Sol] E_1 = knows the answer E_2 - guesses the answer

A = answered correctly

$P(E_1) = \frac{3}{4}$ $P(E_2) = \frac{1}{4}$

$P(A|E_1)$ = P(knowing the answer and answered correctly)

$P(A|E_2) = P(\text{guessing the answer and correct})$
$= \frac{1}{4}$

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{3}{4}\cdot 1}{\frac{3}{4}\cdot 1 + \frac{1}{4}\cdot\frac{1}{4}} = \frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}} = \frac{3}{4}\cdot\frac{16}{13} = \frac{12}{13}$$

5] A lab test is 99% effective in detecting a disease, when it is infact present. It yeilds a false +ve result for 0.5% of healthy person tested. If 0.1% of the population actually has disease, what is the prob that a person has a disease and the test result is +ve?

sol] E_1 = person has disease E_2 = person does not have disease
A = result is +ve

$P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001$ $P(E_2) = 1 - \frac{0.1}{100} = \frac{99.9}{100} = 0.999$

$P(A|E_1) = P(\text{has disease and result is +ve}) = 99\% = \frac{99}{100}$
$P(A|E_2) = P(\text{has no disease and result is +ve}) = 0.5\% = \frac{0.5}{100}$

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\,P(A|E_2)}$$

$$= \frac{\frac{0.1}{100}\cdot\frac{99}{100}}{\frac{0.1}{100}\cdot\frac{99}{100} + \frac{99.9}{100}\cdot\frac{0.5}{100}} = \frac{9.9}{9.9 + 49.95} = \frac{9.9}{59.85} = \frac{198}{1197}$$

$= 0.1654$

8] A factor has 2 machines A and B. A produced 60% of items and B 40%, further 2% of items produced by A, 1% by B were defective. One item is chosen and is found to be defective. What is the prob that it was from B?

Sol] E_1 = machine A E_2 = machine B

A = is defective

$P(E_1) = \frac{60}{100}$ $P(E_2) = \frac{40}{100}$

$P(A|E_1) = \frac{2}{100}$ $P(A|E_2) = \frac{1}{100}$

$$P(E_2|A) = \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2)}$$

$$= \frac{\frac{40}{100}\cdot\frac{1}{100}}{\frac{60}{100}\cdot\frac{2}{100} + \frac{40}{100}\cdot\frac{1}{100}} = \frac{40}{120+40} = \frac{40}{160} = \frac{1}{4}$$

Random Variable (RV) and Probability Distribution (PD):

→ A RV is a real valued function (number) whose domain is the sample space of the corresponding random experiment. It is represented by X

→ PD of a RV:

It is a system of no.s as

X	x_1	x_2	x_3	... x_i
P(X)	$P(x_1)$	$P(x_2)$	$P(x_3)$	... $P(x_i)$

$\boxed{\Sigma P(x_i) = 1}$

→ Mean of a RV:

mean, $\mu = E(X) = \Sigma x_i p_i$

$E(X^2) = \Sigma x_i^2 p_i$

→ Variance of X : $Var(X) = E(X^2) - [E(X)]^2$

→ Standard deviation : SD of X $= \sqrt{Var(X)}$

Eg 26] Let X denote the no. of hrs you study during a selected school day. X can take the values as
$$P(X = x) = \begin{cases} 0.1 & ; \text{if } x = 0 \\ kx & ; \text{if } x = 1 \text{ or } 2 \\ k(5-x) & ; x = 3 \text{ or } 4 \\ 0 & ; \text{otherwise} \end{cases}$$

(a) Find k

(b) What is the prob that you study atleast 2 hr's.

(c) " exactly

(d) " atmost "

Sol] PD of X :

X	0	1	2	3	4	otherwise
P(X)	0.1	k	2k	2k	k	0

(a) $\sum P(x_i) = 1$

$\Rightarrow$ ~~$0.1 + kx + kx + k(5-x) + k(5-x) = 1$~~

~~$0.1 + 2kx + 5k -$~~

$0.1 + k + 2k + 2k + k = 0$

$6k = 0.9$

$k = \frac{0.9}{6} = 0.15$

(b) $P(\text{atleast } 2) = P(X \geq 2) = 2k + 2k + k = 5k$

$= 5(0.15) = 0.75$

(c) $P(\text{exact } 2) = P(X=2) = 2k = 2(0.15) = 0.3$

(d) $P(\text{atmost } 2) = P(X \leq 2) = 0.1 + k + 2k = 0.1 + 3(0.15) = 0.55$

EXERCISE 13.4

1] State which are not PD's, give reason

(a)

X	0	1	2
P(X)	0.4	0.4	0.2

$\sum P_i = 0.4 + 0.4 + 0.2 = 1$

$\therefore$ Gn distribution is a PD.

(b)

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	-0.1	0.3

-0.1 is < 0

$\therefore$ Gn distribution is not PD.

(c)

Y	-1	0	1
P(Y)	0.6	0.1	0.2

$\sum P_i = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$

$\therefore$ Gn distribution is not PD.

(d)

Z	3	2	1	0	-1
P(Z)	0.3	0.2	0.4	0.1	0.05

$\sum P_i = 0.3 + 0.2 + 0.4 + 0.05 + 0.1 \neq 1$

$\therefore$ Gn distribution is not PD.

8] A random variable x

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i] $k = ?$

$\Sigma P_i = 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k = 1$

$10k^2 + 9k - 1 = 0$

$k = -1$ not valid, $\therefore k = \frac{1}{10}$

ii] $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

$= 0 + k + 2k = 3\left(\frac{1}{10}\right) = \frac{3}{10}$

iii] $P(x>6) = 7k^2 + k = \frac{7}{100} + \frac{1}{10} = \frac{17}{100}$

iv] $P(0<x<3) = P(x=1) + P(x=2) = 3\left(\frac{1}{10}\right) = \frac{3}{10}$

9] $P(x) = \begin{cases} k; & x = 0 \\ 2k; & x = 1 \\ 3k; & x = 2 \\ 0; & \text{otherwise} \end{cases}$

(a) $k = ?$ (c) $P(X \le 2)$

(b) $P(X<2)$ (d) $P(X \ge 2)$

x	0	1	2	otherwise
P_x	k	2k	3k	0

(a) $k: \Sigma P_i = k + 2k + 3k + 0 = 1$

$6k = 1$

$\boxed{k = \frac{1}{6}}$

(b) $P(x<2) = k + 2k = 3\left(\frac{1}{6}\right) = \frac{1}{2}$

(c) $P(x \le 2) = k + 2k + 3k = 6\left(\frac{1}{6}\right) = 1$

(d) $P(x \ge 2) = 3k + 0 = 3\left(\frac{1}{6}\right) = \frac{1}{2}$

(1m)
** 2] An earn contains 5 R & 2 B balls. 2 balls are drawn. Let X represent black balls. What are possible values of X? Is X a RV?

Sol] X can take the values 0, 1, 2

$\because$ The value of X varies, X is a RV.

and $\because$ there is no order it is a RV

3] Let X represent difference b/w no of heads and no of tails obtained when a coin is tossed 6 times. What are the possible values of X?

Sol] X → diff b/w no of H & no of T

H	6	5	4	3	2	1	0
T	0	1	2	3	4	5	6
Diff	6	4	2	0	2	4	6

$\therefore X = \{0, 2, 4, 6\}$

6] From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the PD of no of defective bulbs.

Sol] $P(\text{defective}) = \frac{6}{30} = \frac{1}{5}$

$P(\text{not defective}) = 1 - \frac{1}{5} = \frac{4}{5}$

$P(x=0) = P(\text{not defective}) = P(\bar{D}\bar{D}\bar{D}\bar{D}) = \frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5} = \frac{256}{625}$

$P(X=1) = P[D\bar{D}\bar{D}\bar{D} + \bar{D}D\bar{D}\bar{D} + \bar{D}\bar{D}D\bar{D} + \bar{D}\bar{D}\bar{D}D]$

$= 4\left(\frac{1}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}\right) = \frac{256}{625}$

$P(X=2) = P[\bar{D}\bar{D}DD + DD\bar{D}\bar{D} + D\bar{D}D\bar{D} + \bar{D}D\bar{D}D + \bar{D}DD\bar{D} + D\bar{D}\bar{D}D]$

$= 6\left(\frac{1}{5}\cdot\frac{1}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}\right) = \frac{96}{625}$

$P(X=3) = P[\bar{D}DDD + D\bar{D}DD + DD\bar{D}D + DDD\bar{D}]$

$= 4\left(\frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{5}\cdot\frac{4}{5}\right) = \frac{16}{625}$

$P(X=4) = P(DDDD) = \frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{5} = \frac{1}{625}$

PD:

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Mean $= E(X) = \Sigma x_i p_i = 0 + \frac{256}{625} + \frac{192}{625} + \frac{48}{625} + \frac{4}{625}$

$= \frac{500}{625} = 0.8$

4] Find the PD of

i] No. of heads in two tosses of a coin.

$P(h) = P(t) = \frac{1}{2}$

X = no. of heads

$P(X = 0h) = P(tt) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(X = 1h) = P(ht + th) = 2\left(\frac{1}{2} \cdot \frac{1}{2}\right) = \frac{1}{2}$

$P(X = 2h) = P(hh) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

PD:

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

ii] No. of tails in simultaneous tosses of 3 coins

$P(h) = P(t) = \frac{1}{2}$

X = no. of tails

$P(X = 0t) = P(hhh) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

$P(X = 1t) = P(thh + hth + hht) = 3\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) = \frac{3}{8}$

$P(X = 2t) = P(tth + tht + htt) = 3\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) = \frac{3}{8}$

$P(X = 3t) = P(ttt) = \frac{1}{8}$

PD:

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

iii] No. of heads in four tosses of a coin.

$P(h) = P(t) = \frac{1}{2}$

$P(X=0h) = P(tttt) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}\cdot\frac{1}{2} = \frac{1}{16}$

$P(X=1h) = P(httt + thtt + ttht + ttth) = 4\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) = \frac{1}{4}$

$P(X=2h) = P(hhtt + htht \cdots) = 6\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) = \frac{3}{8}$

$P(X=3h) = P(hhht + hthh + \cdots) = 4\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) = \frac{1}{4}$

$P(X=4h) = P(hhhh) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{16}$

10] Find the mean no. of heads in 3 tosses of a fair coin.

Sol] $P(h) = P(t) = \frac{1}{2}$

$X =$ no. of heads

$P(X=0h) = P(ttt) = \frac{1}{8}$

$P(X=1h) = P(htt + tht + tth) = \frac{3}{8}$

$P(X=2h) = P(hht + thh + hth) = \frac{3}{8}$

$P(X=3h) = P(hhh) = \frac{1}{8}$

PD :

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$E(X) = \Sigma x_i p_i = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$

$E(X^2) = \Sigma x_i^2 p_i = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = \frac{24}{8} = 3$

$Var(X) = E(X^2) - [E(X)]^2 = 3 - \frac{9}{4} = \frac{3}{4}$

$SD(X) = \sqrt{var(X)} = \frac{\sqrt{3}}{2}$

14] A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19, 20. One student is selected such that every student has the same chance and age X is recorded. What is the PD of X. Find mean variance and standard deviation.

Sol] X - age ; $n(S) = 15$

X	14	15	16	17	18	19	20	21
$P(X)$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

i) $E(x) = \text{mean} = \sum x_i p_i = \frac{28+15+32+51+18+33+60+21}{15}$

$= \frac{263}{15} = 17.53$

$E(x^2) = \frac{392+225+512+\cancel{2889}\,867+324+722+1200+441}{15}$

$= \frac{4683}{15} = 312.2$

$Var(x) = E(x^2) - [E(x)]^2 = 312.2 - (17.53)^2 = 312.2 - 307.3$

$= 4.9$

$SD = \sqrt{Var(x)} = 2.21$

Binomial Distribution :

* Burnoulli's Trials : Trials of a random experiment are called BT if they satisfy the following conditions :

i] The exp should have finite no. of trials.

ii] The prob of success in each trial should remain same.

iii] The trials should be independent.

iv] Each trial has exactly 2 outcomes like T and F or S and F etc.

* It is seen that the outcomes of these Burnoulli's trials are the terms of the binomial expansion like $a^2, 2ab, b^2$ or $a^3, 3a^2b, 3ab^2, b^3$ where 'a' represents happening and 'b' represents not happening.

$a = p$

$b = q = 1-p$

* The above distribution which are terms of binomial expansion is obtained by $P(x=x)$,

$P(x = x) = {}^nC_x p^x q^{n-x}$; n - no. of trials

x - required no.

p - occuring

q - not occuring.

** Eg 31] If a fair coin is tossed 10 times, find the prob of
i] exactly 6 heads iii] atmost 6 heads
ii] atleast 6 heads.

Sol] $n = 10$

$P(h) = \frac{1}{2} = p$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

$P(X=x) = {}^nC_x p^x q^{n-x}$

$P(X=x) = {}^{10}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = {}^{10}C_x \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}\,{}^{10}C_x$

i] $P(X=6) = {}^{10}C_6 \frac{1}{1024} = {}^{10}C_4 \frac{1}{1024}$

$= \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2} \frac{1}{1024} = \frac{210}{1024} = \frac{105}{512}$

ii] $P(\text{atleast } 6) = P(X \geq 6)$

$= P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= \frac{1}{1024}\left[{}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right]$ ${}^nC_n = {}^nC_0 = 1$, ${}^nC_1 = {}^nC_{n-1} = n$

$= \frac{1}{1024}\left[{}^{10}C_4 + {}^{10}C_3 + {}^{10}C_2 + 10 + 1\right]$

$= \frac{1}{1024}\left[210 + \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} + \frac{10 \cdot 9}{2} + 11\right]$

$= \frac{1}{1024}\left[210 + 120 + 45 + 11\right] = \frac{386}{1024} = \frac{193}{512}$

iii] $P(\text{atmost } 6) = P(X \leq 6) = 1 - P(X \geq 6) + P(X=6)$

$= 1 - \frac{193}{512} + \frac{105}{512}$

$= \frac{512 - 193 + 105}{512} = \frac{424}{512}$

$= \frac{212}{256} = \frac{106}{128} = \frac{53}{64}$

EXERCISE 13.5

** 5] The prob that a bulb factory produces a bulb that fuses after 150 days of use is 0.05. Find the prob that out of 5 such bulbs

i] none

ii] not more than 1

iii] more than 1

iv] atleast 1

will fuse after 150 days of use.

Sol] $n = 5$

$p = 0.05$

$q = 1 - 0.05 = 0.95$

$P(X=x) = {}^nC_x\, p^x \cdot q^{n-x}$

$= {}^5C_x (0.05)^x (0.95)^{5-x}$

i] $P(X=0) = {}^5C_0 (0.05)^0 (0.95)^5$

$= 1 \cdot 1 (0.95)^5$

ii] $P(X \le 1) = P(X=0) + P(X=1)$

$= (0.95)^5 + {}^5C_1 (0.05)^1 (0.95)^4$

$= (0.95)^4 \left[0.95 + 0.25\right]$

$= (0.95)^4 (1.2)$

iii] $P(X>1) = 1 - \left[P(X=0) + P(X=1)\right]$

$= 1 - \left[(1.2)(0.95)^4\right]$

iv] $P(X \ge 1) = 1 - P(X=0)$

$= 1 - (0.95)^5$

** 3] There are 5% defective items in a large bulk of items. What is the prob that a sample of 10 items will include not more than 1 defective item.

Sol] $n = 10$

$P(\text{defective}) = 5\% = \frac{5}{100} = 0.05 = p$

$q = 1 - 0.05 = 0.95$

$P(X \le 1) = ?$

$P(X = x) = {}^nC_x p^x q^{n-x}$

$= {}^{10}C_x (0.05)^x (0.95)^{10-x}$

$P(X \le 1) = P(X=0) + P(X=1)$

$= \left[{}^{10}C_0 (0.05)^0 (0.95)^{10}\right] + \left[{}^{10}C_1 (0.05)^1 (0.95)^9\right]$

$= \left[1 \cdot 1 \cdot (0.95)^{10}\right] + \left[10 (0.05)(0.95)^9\right]$

$= (0.95)^9 \left[0.95 + 0.5\right]$

$= (0.95)^9 (1.45)$

** 11] Find the prob of getting 5 exactly twice in 7 throws of a die.

Sol] $n = 7$

$x = 2$

$p = P(\text{getting } 5) = \frac{1}{6}$

$q = 1 - \frac{1}{6} = \frac{5}{6}$

$P(X=2) = {}^7C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5 = \frac{7 \cdot 6}{2 \cdot 1} \left(\frac{1}{36}\right)\left(\frac{3125}{7776}\right)$

$= \frac{21}{36}\left(\frac{5}{6}\right)^5 = \frac{7}{12}\left(\frac{5}{6}\right)^5$

12] Find the prob of throwing atmost 2 sixes in 6th row of a single die.

Sol] $P(X \le 2) = ?$

$n = 6$

$p = P(\text{getting } 6) = \frac{1}{6}$

$q = \frac{5}{6}$

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= \left[{}^6C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^6\right] + \left[{}^6C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^5\right] + \left[{}^6C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4\right]$

$P(X \le 2) = \left(\frac{5}{6}\right)^4 \left[1 \cdot 1 \cdot \frac{25}{36} + 6 \cdot \frac{1}{6} \cdot \frac{5}{6} + \frac{\overset{3}{\cancel{6}} \cdot 5}{\cancel{2}} \left(\frac{1}{6}\right)^2\right]$

$= \left(\frac{5}{6}\right)^4 \left[\frac{25}{36} + \frac{5}{6} + \frac{15}{36}\right] = \left(\frac{5}{6}\right)^4 \left[\frac{25+30+15}{36}\right]$

$= \left(\frac{5}{6}\right)^4 \left(\frac{70}{36}\right) = \left(\frac{5}{6}\right)^4 \left(\frac{35}{18}\right)$

13] It is known that 10% of articles are defective. What is the prob that in a sample of 12 such articles, 9 are defective.

Sol] $n = 12$

$p = 10\% = \frac{10}{100} = 0.1$

$q = 1 - 0.1 = 0.9$

$x = 9$

$P(X = 9) = {}^{12}C_9 (0.1)^9 (0.9)^3$

$= {}^{12}C_3 (0.1)^9 (0.9)^3$

$= \frac{\cancel{12} \cdot 11 \cdot 10}{3 \cdot \cancel{2}} (0.1)^9 (0.9)^3$

$= 220 (0.1)^9 (0.9)^3$

15] The prob that a student is not a swimmer is $\frac{1}{5}$, then the prob that out of 5 students 4 are swimmers is

Sol] $n = 5$

$q = \frac{1}{5}$

$p = 1 - \frac{1}{5} = \frac{4}{5}$

$x = 4$

$P(X = 4) = {}^5C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^1$

$= {}^5C_1 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^1 = \cancel{5} \left(\frac{4}{5}\right)^4 \left(\frac{1}{\cancel{5}}\right)$

$= \frac{256}{625} = 0.4096$

2] A pair of die's is thrown 4 times. If getting a doublets is considered as sucess find the prob of 2 successes.

Sol] $n = 4$

$p = P(\text{getting doublet}) = \frac{6}{36} = \frac{1}{6}$ (1,1),(2,2),(3,3)

$q = 1 - \frac{1}{6} = \frac{5}{6}$

$x = 2$

$P(x=2) = {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$

$= \frac{4 \cdot 3}{2 \cdot 1} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = \frac{1}{6}\left(\frac{5}{6}\right)^2 = \frac{25}{216}$

4] Five cards are drawn successfully with replacement. What is the prob that

i] all 5 are spades iii] none is a spade

ii] only 3 are spades

Sol] $n = 5$

$p = P(\text{spades}) = \frac{13}{52} = \frac{1}{4}$

$q = 1 - \frac{1}{4} = \frac{3}{4}$

$P(x = x) = {}^5C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{5-x}$

i] $x = 5$

$P(x=5) = {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{5-5} = 1 \cdot \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$

ii] $P(x=3) = {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$

$= \frac{5 \cdot 4 \cdot 3}{3 \cdot 2} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 = \frac{10 \cdot 9}{4^5} = \frac{90}{1024} = \frac{45}{512}$

iii] $P(x=0) = {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5 = \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$

9] A bag consists of 10 balls, each marked with one of the digits 0 - 9. If 4 balls are drawn, what is the prob that none is marked with 0?

Sol] $n = 4$

X = no. of balls marked with 0

$p = \frac{1}{10}$

$q = \frac{9}{10}$

$x = 0$

$$P(x=0) = {}^{4}C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^4 = \left(\frac{9}{10}\right)^4$$

** 10] A person by's a lottery ticket in 50 lotteries, in each of which his chance of winning is $\frac{1}{100}$ what is the prob that he ~~with~~ will win

i] atleast once iii] atleast twice.

ii] exactly once

Sol] $n = 50$

$p = \frac{1}{100}$

$q = \frac{99}{100}$

$$P(X=x) = {}^{50}C_x\left(\frac{1}{100}\right)^x\left(\frac{99}{100}\right)^{50-x}$$

i] $P(X \geq 1) = 1 - P(X=0)$

$$= 1 - \left[{}^{50}C_0\left(\frac{1}{100}\right)^0\left(\frac{99}{100}\right)^{50}\right] = 1 - \left(\frac{99}{100}\right)^{50}$$

ii] $$P(X=1) = {}^{50}C_1\left(\frac{1}{100}\right)^1\left(\frac{99}{100}\right)^{49} = 50\cdot\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49} = \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

iii] $P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$

$$= 1 - \left[\left(\frac{99}{100}\right)^{50} + \frac{1}{2}\left(\frac{99}{100}\right)^{49}\right] = 1 - \left(\frac{99}{100}\right)^{49}\left[\frac{99}{100} + \frac{1}{2}\right]$$

$$= 1 - \left(\frac{99}{100}\right)^{49}\left(\frac{99+50}{100}\right) = 1 - \left(\frac{99}{100}\right)^{49}\left(\frac{149}{100}\right)$$

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14] In a box containing 100 bulbs, 10 are defective, the prob that out of a sample of 5 bulbs, none is defective is

Sol] $n = 5$

$p = \frac{1}{10}$, $q = \frac{9}{10}$

$x = 0$

$P(X=0) = {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 = \left(\frac{9}{10}\right)^5$

1] A die is thrown 6 times, if getting an odd no. is success, what is the prob of

(a) 5 successes (c) atmost 5 successes.

(b) atleast 5 successes

Sol] $n = 6$

$p = P(\text{getting odd}) = \frac{3}{6} = \frac{1}{2}$

$q = \frac{1}{2}$

$P(X=x) = {}^6C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{6-x} = {}^6C_x \left(\frac{1}{2}\right)^6 = \frac{1}{64}\,{}^6C_x$

(a) $P(X=5) = \frac{1}{64}\,{}^6C_5 = \frac{6}{64} = \frac{3}{32}$

(b) $P(X \geq 5) = P(X=5) + P(X=6)$

$= \frac{6}{64} + \left[\frac{1}{64}\,{}^6C_6\right] = \cancel{\frac{3}{32} + \frac{1}{64}} = \cancel{\frac{4}{64}} = \cancel{\frac{2}{32}}$

$= \cancel{\frac{1}{16}}\ \frac{7}{64}$

(c) $P(X \leq 5) = 1 - P(X=6)$

$= 1 - \left[{}^6C_6\left(\frac{1}{64}\right)\right] = 1 - \frac{1}{64} = \frac{63}{64}$