65/6/3

QUESTION PAPER CODE 65/6/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

Question numbers 1 to 6 carry 2 marks each.

1. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}, \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

Ans. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c})(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ 1

$\Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ $\frac{1}{2}$

$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-3}{2}$ $\frac{1}{2}$

2. (a) Find the general solution of the differential equation x cos y dy = (x log x + 1) e^x dx.

OR

(b) Find the value of (2a – 3b), if a and b represent respectively the order the degree of the differential equation $x\left[y\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}\frac{dy}{dx}\right] = 0.$

Ans. x cos y dy = (x log x + 1)e^x dx

$\Rightarrow \int \cos y \, dy = \int \left(\log x + \frac{1}{x}\right) \cdot e^x \, dx$ $\frac{1}{2}$

$\Rightarrow \sin y = \log x . e^x + C$ $\left(\because \int [f(x) + f'(x)] \cdot e^x dx = f(x) \cdot e^x + C\right)$ $1\frac{1}{2}$

OR

order = 2, degree = 3 $1 + \frac{1}{2}$

$\therefore$ 2a – 3b = 4 – 9 = –5 $\frac{1}{2}$

3. Evaluate:

$$\int_1^2 \log\left(\frac{3}{x}-1\right)dx$$

Ans. $I = \int_1^2 \log\left(\frac{3}{x}-1\right)dx$

$= \int_1^2 [\log(3-x)-\log x]\,dx$ $\quad \frac{1}{2}$

$= \int_1^2 \log(3-x)\,dx - \int_1^2 \log x\,dx$

$= \int_1^2 \log(3-x)\,dx - \int_1^2 \log(3-x)\,dx$ $\quad (\because$ Using $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx)$ $\quad 1$

$= 0$ $\quad \frac{1}{2}$

4. If $\vec{a} = \hat{i}+\hat{j}-2\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{c} = -\hat{i}+2\hat{j}-\hat{k}$ are three vectors, then find a vector perpendicular to both the vectors $(\vec{a}+\vec{b})$ and $(\vec{b}-\vec{c})$.

Ans. $\vec{a}+\vec{b} = 3\hat{j}$, $\vec{b}-\vec{c} = 3\hat{k}$ $\quad \frac{1}{2}+\frac{1}{2}$

Vector perpendicular to $\vec{a}+\vec{b}$ and $\vec{b}-\vec{c}$

$= (3\hat{j})\times(3\hat{k}) = 9\hat{i}$ $\quad 1$

5. One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.

Ans. **Case I:** White ball is transferred from bag I to bag II

P(white ball from bag II) = $\frac{4}{9}\times\frac{7}{14}$ $\quad \frac{1}{2}$

Case II: Black ball is transferred from bag I to bag II

P(white ball from bag II) = $\frac{5}{9}\times\frac{6}{14}$ $\quad \frac{1}{2}$

Total Probability = $\frac{4}{9} \times \frac{7}{14} + \frac{5}{9} \times \frac{6}{14}$ $\frac{1}{2}$

$= \frac{29}{63}$ $\frac{1}{2}$

6. A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, without replacement. Find the probability that the number on each card is a multiple of 7.

Ans. Multiples of 7 from 1 to 25 are 7, 14, 21 $\frac{1}{2}$

P (number on each card is a multiple of 7)

$= \frac{3}{25} \times \frac{2}{24} = \frac{1}{100}$ $1\frac{1}{2}$

SECTION B

Question numbers 7 to 10 carry 3 marks each.

7. Find the distance between the point (3, 4, 5) and the point where the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ meets the plane x + y + z = 17.

Ans. Any point on the given line is $(\lambda + 3, 2\lambda + 4, 2\lambda + 5)$. 1

If this point is the point of intersection of the line and plane,

then $(\lambda + 3) + (2\lambda + 4) + (2\lambda + 5) = 17 \Rightarrow \lambda = 1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (4, 6, 7) $\frac{1}{2}$

Required distance = $\sqrt{(4-3)^2 + (6-4)^2 + (7-5)^2} = 3$ 1

8. (a) Find the distance between the following parallel lines:

$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$

$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$

OR

(b) Find the coordinates of the point where the line through the points (–1, 1, –8) and (5, –2, 10) crosses the ZX-plane.

Ans. (a) $\vec{a}_2 - \vec{a}_1 = -\hat{i} - 3\hat{j} + 2\hat{k},\ \vec{b} = \hat{i} + \hat{j} - \hat{k}$ $\frac{1}{2}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 2 \\ 1 & 1 & -1 \end{vmatrix} = \hat{i} + \hat{j} + 2\hat{k}$$ 1

Required distance = $\frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$

$$= \frac{\sqrt{1+1+4}}{\sqrt{1+1+1}} = \sqrt{\frac{6}{3}} = \sqrt{2}$$ $1\frac{1}{2}$

OR

(b) Equation of line through (–1, 1, –8) and (5, –2, 10)

is $\frac{x+1}{5-(-1)} = \frac{y-1}{-2-1} = \frac{z+8}{10-(-8)}$

i.e. $\frac{x+1}{6} = \frac{y-1}{-3} = \frac{z+8}{18} = \lambda$ 1

Any point on this line is $(6\lambda - 1, -3\lambda + 1, 18\lambda - 8)$ $\frac{1}{2}$

Line crosses ZX-plane i.e. $y = 0$

$\Rightarrow -3\lambda + 1 = 0 \Rightarrow \lambda = \frac{1}{3}$ 1

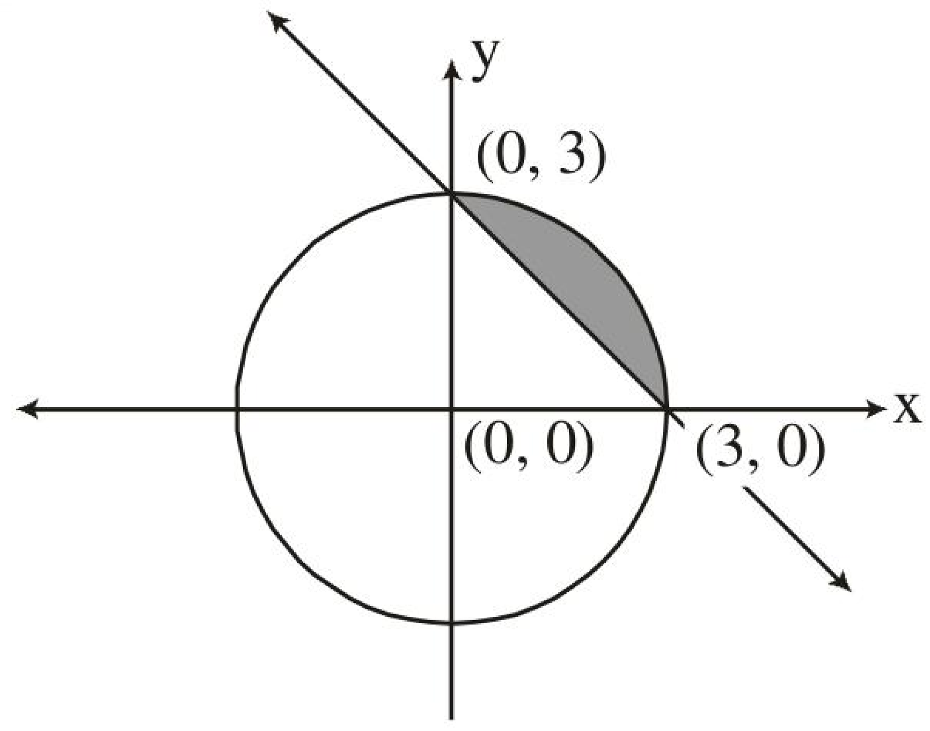
Required point is (1, 0, –2) $\frac{1}{2}$

9. (a) Find the area of the region $\{(x, y) : x^2 + y^2 \le 9,\ x + y \ge 3\}$, using integration.

OR

(b) Using integration, find the area of the region bounded by the parabola $y^2 = 4x$, the lines $x = 0$ and $x = 3$ and the x-axis.

Ans. Point of intersection (3, 0) and (0, 3)



Correct figure $\frac{1}{2}$

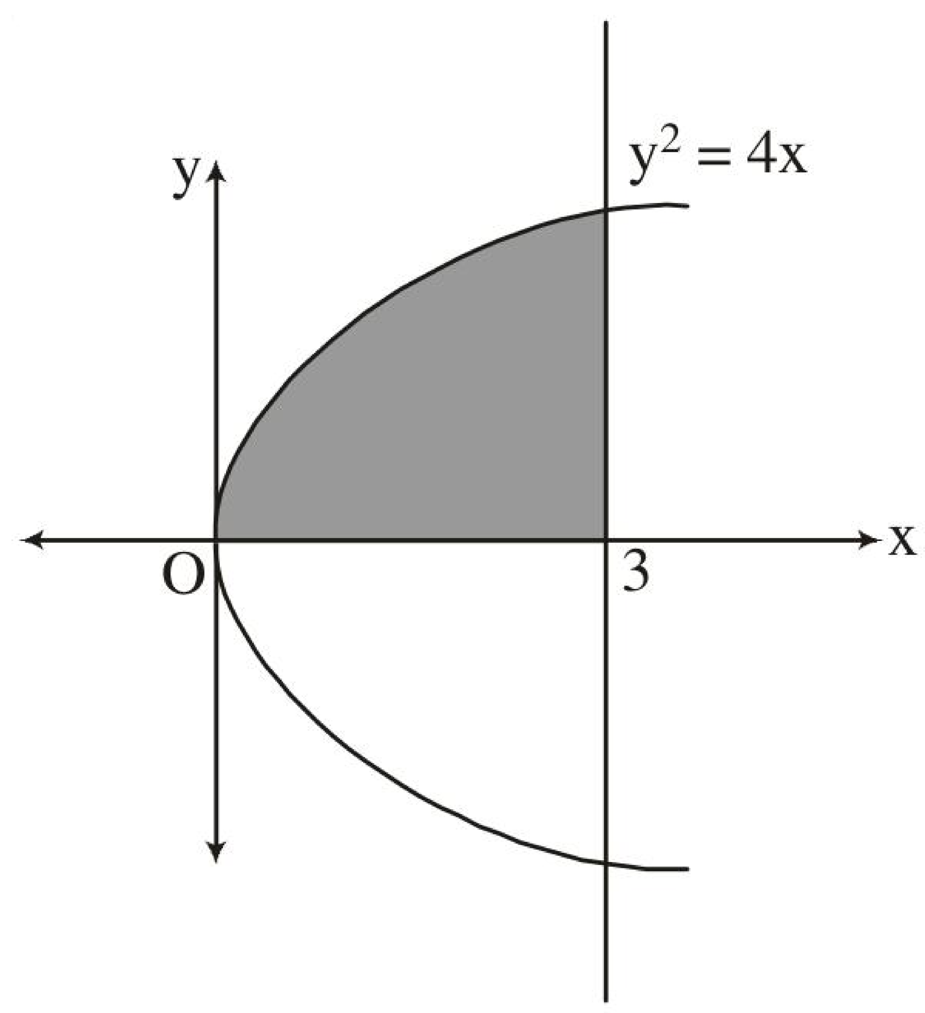
Required Area

$$= \int_0^3 \sqrt{9-x^2}\,dx - \int_0^3 (3-x)dx$$ 1

$$= \left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3 - \left[\frac{(3-x)^2}{-2}\right]_0^3$$ 1

$$= \frac{9}{2}\sin^{-1}1 - \frac{9}{2} = \frac{9}{2}\left(\frac{\pi}{2} - 1\right)$$ $\frac{1}{2}$

OR



Correct Figure 1

Required Area $= \int_0^3 2\sqrt{x}\, dx$ 1

$= 2 \times \frac{2}{3}[x^{3/2}]_0^3$ $\frac{1}{2}$

$= \frac{4}{3} \times 3^{3/2} = 4\sqrt{3}$ $\frac{1}{2}$

10. Find:

$\int \sin 2x \sin 3x\, dx$

Ans. $I = \int \sin 2x \sin 3x\, dx$

$= \frac{1}{2}\int 2 \sin 2x \sin 3x\, dx$ 1

$= \frac{1}{2}\int (\cos x - \cos 5x)\, dx$ 1

$= \frac{1}{2}\left[\sin x - \frac{\sin 5x}{5}\right] + C$ 1

SECTION C

Question numbers 11 to 14 carry 4 marks each.

11. (a) Find:

$\int \cos x \bullet \tan^{-1}(\sin x)\, dx$

OR

(b) Find:

$\int \frac{e^x}{(e^x + 1)(e^x + 3)} dx$

Ans. $I = \int \cos x \cdot \tan^{-1}(\sin x)\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$ 1

$\therefore\ I = \int \tan^{-1} t \cdot 1\, dt$

$$= \tan^{-1} t \cdot t - \frac{1}{2}\int \frac{2t}{1+t^2}\,dt$$ $1\frac{1}{2}$

$$= t\cdot\tan^{-1} t - \frac{1}{2}\log|1+t^2| + C$$ 1

$$= \sin x\cdot\tan^{-1}(\sin x) - \frac{1}{2}\log|1+\sin^2 x| + C$$ $\frac{1}{2}$

OR

$$I = \int \frac{e^x}{(e^x+1)(e^x+3)}\,dx$$

Put $e^x = t \Rightarrow e^x\,dx = dt$ 1

$$I = \int \frac{dt}{(t+1)(t+3)}$$

$$= \frac{1}{2}\int\left(\frac{1}{t+1} - \frac{1}{t+3}\right)dt$$ $1\frac{1}{2}$

$$= \frac{1}{2}[\log|t+1| - \log|t+3|] + C$$ 1

$$= \frac{1}{2}[\log|e^x+1| - \log|e^x+3|] + C$$ $\frac{1}{2}$

or $\frac{1}{2}\log\left|\frac{e^x+1}{e^x+3}\right| + C$

12. Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \tan x$, given $y(0) = 1$.

Ans. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\tan x}{1+x^2}$$ $\frac{1}{2}$

$$\text{I.F.} = e^{\int\frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1+x^2$$ 1

Solution is given by

$$y\cdot(1+x^2) = \int \frac{\tan x}{1+x^2}\cdot(1+x^2)\,dx$$ 1

$= \int \tan x \, dx$

$= \log |\sec x| + C$ $\frac{1}{2}$

When $x = 0$, $y = 1$ gives $C = 1$ $\frac{1}{2}$

Required particular solution is $y \cdot (1 + x^2) = \log |\sec x| + 1$ $\frac{1}{2}$

or $y = \dfrac{\log |\sec x|}{1+x^2} + \dfrac{1}{1+x^2}$

13. Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i}+2\hat{j}-3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i}+5\hat{j}-3\hat{k}) = 9$ and through the point (2, 1, 3).

Ans. Equation of required plane is

$\vec{r} \cdot [(2\hat{i}+2\hat{j}-3\hat{k}) + \lambda(2\hat{i}+5\hat{j}+3\hat{k})] = 7 + 9\lambda$ 1

or $\vec{r} \cdot [(2+2\lambda)\hat{i} + (2+5\lambda)\hat{j} + (-3+3\lambda)\hat{k})] = 7 + 9\lambda$

As the plane passes through (2, 1, 3), we have

$(2\hat{i}+\hat{j}+3\hat{k}) \cdot [(2+2\lambda)\hat{i} + (2+5\lambda)\hat{j} + (-3+3\lambda)\hat{k}] = 7 + 9\lambda$ 1

$\Rightarrow 2(2 + 2\lambda) + 1(2 + 5\lambda) + 3(-3 + 3\lambda) = 7 + 9\lambda$

$\Rightarrow 9\lambda = 10 \Rightarrow \lambda = \dfrac{10}{9}$ 1

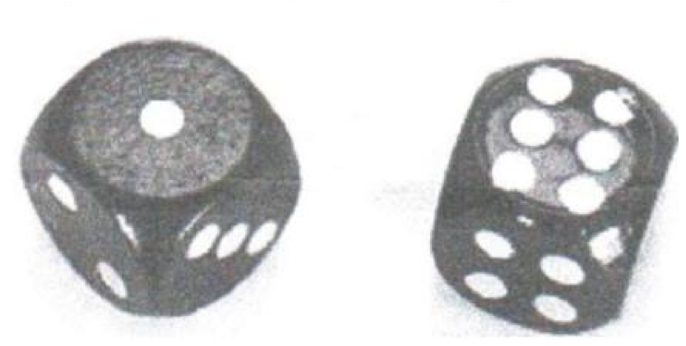
Required plane is $\vec{r} \cdot \left(\dfrac{38}{9}\hat{i} + \dfrac{68}{9}\hat{j} + \dfrac{3}{9}\hat{k}\right) = 17$ 1

or $\vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$

Case-Study Based Question

14. A biased die is tossed and respective probabilities for various faces to turn up are the following:

Face	1	2	3	4	5	6
Probability	0.1	0.24	0.19	0.18	0.15	K



Based on the above information, answer the following questions:

(a) What is the value of K?

(b) If a face showing an even number has turned up, then what is the probability that it is the face with 2 or 4?

Ans. (a) $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$ 1

$\Rightarrow$ $0.1 + 0.24 + 0.19 + 0.18 + 0.15 + K = 1$

$\Rightarrow$ $K = 0.14$ 1

(b) A: face shows 2 or 4

B: even face have turned up

Required probability = $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

here $P(A \cap B) = P(2) + P(4)$

$= 0.24 + 0.18 = 0.42$ 1

$P(B) = P(2) + P(4) + P(6) = 0.56$ $\frac{1}{2}$

$\therefore$ $P(A/B) = \dfrac{0.42}{0.56} = \dfrac{3}{4}$ $\frac{1}{2}$