QUESTION PAPER CODE 65/1/C

**EXPECTED ANSWERS/VALUE POINTS**

**SECTION - A**

| | | Marks |
|---|---|---|
| 1. | $\vec{a} \times \vec{b} = -17\hat{i} + 13\hat{j} + 7\hat{k}, \left\| \vec{a} \times \vec{b} \right\| = \sqrt{507}$ | ½+½ m |
| 2. | $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\,\|\vec{b}\|}, \ \theta = \dfrac{2\pi}{3}$ | ½ + ½ m |
| 3. | $d = \left\| \dfrac{\vec{a} \cdot \vec{n} - p}{\|\vec{n}\|} \right\|, \ \text{distance} = \dfrac{13}{7}$ | ½ + ½ m |
| 4. | $e^{2x} \sin 2x$ | 1 m |
| 5. | $y = mx, \dfrac{dy}{dx} = \dfrac{y}{x}$ | ½ + ½ m |
| 6. | $\dfrac{dy}{dx} + \dfrac{1}{x \log x} y = \dfrac{2}{x}$, Integrating factor $= \log x$ | ½ + ½ m |



**SECTION - B**

7. $$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$ 1½ m

$$A^2 - 4A - 51 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix} = O$$ 1 m

$$A^2 - 4A - 51 = O \Rightarrow A^{-1} = \frac{1}{5}(A - 4I)$$ 1 m

$$A^{-1} = \frac{1}{5}\left\{\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix} - \begin{bmatrix}4 & 0 & 0\\0 & 4 & 0\\0 & 0 & 4\end{bmatrix}\right\} = \frac{1}{5}\begin{bmatrix}-3 & 2 & 2\\2 & -3 & 2\\2 & 2 & -3\end{bmatrix}$$ ½ m

OR

$$\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} A$$ 1 m

Using elementary row operations to reach at

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix} A$$ 2 m

$$A^{-1} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}$$ 1 m

8. $$\begin{vmatrix}x+2 & x+6 & x-1\\x+6 & x-1 & x+2\\x-1 & x+2 & x+6\end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix}3x+7 & x+6 & x-1\\3x+7 & x-1 & x+2\\3x+7 & x+2 & x+6\end{vmatrix} = 0$$ 1 m

$R_2 \to R_2 - R_1$ , $R_3 \to R_3 - R_1$

$$\begin{vmatrix}3x+7 & x+6 & x-1\\1 & -7 & 3\\1 & -4 & 7\end{vmatrix} = 0$$ 2 m

$(3x+7)(-37) = 0 \Rightarrow x = \frac{-7}{3}$ 1 m

9. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx \Rightarrow 2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx$ 1 m

$$2I = \int_0^{\pi/2} \frac{\sec^2 x/2}{2\tan\frac{x}{2} + 1 - \tan^2 x/2} dx$$

$I = -\int_0^1 \frac{1}{(t-1)^2 - (\sqrt{2})^2} dt$, where $\tan\frac{x}{2} = t$ 1½ m

$I = \left[-\frac{1}{2\sqrt{2}} \log\left|\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right|\right]_0^1$ 1 m

$I = \frac{1}{2\sqrt{2}} \log\left|\frac{1+\sqrt{2}}{1-\sqrt{2}}\right|$ ½ m

OR

$\int_{-1}^{2} (e^{3x} + 7x - 5) dx$ here $h = \frac{3}{n}$ ½ m

$= \lim_{h \to 0} h\,[f(-1) + f(-1+h) + \ldots\ldots\ldots\ldots]$

$= \lim_{h \to 0} h\left[(e^{-3} - 12) + (e^{-3+3h} + 7h - 12) + \ldots\ldots + (e^{-3+\overline{n-1}h} + 7(n-1)h - 12)\right]$ 1 m

$= \lim_{h \to 0} h\left[e^{-3}(1 + e^{3h} + e^{6h} + \ldots\ldots + e^{3(n-1)h}) + 7h(1+2+3+\ldots.\overline{n-1}) - 12\,nh\right]$ 1 m

$= \lim_{h \to 0} h\left[\frac{e^{-3}(e^{3nh} - 1)h}{e^{3h} - 1} + \frac{7(nh)(nh-h)}{2} - 12nh\right]$ 1 m

$= \frac{e^{-3}(e^9 - 1)}{3} + \frac{63}{2} - 36 = \frac{e^9 - 1}{3e^3} - \frac{9}{2}$ ½ m

10. $\int \frac{x^2}{x^4 + x^2 - 2}\, dx$

$\int \frac{x^2}{x^4 + x^2 - 2} = \frac{t}{t^2 + t - 2} = \frac{t}{(t+2)(t-1)}$ where $x^2 = t$ 1½ m

$= \frac{2}{3(t+2)} + \frac{1}{3(t-1)}$ 1½ m

$$\int \frac{x^2}{x^4 + x^2 - 2}\, dx = \int \frac{2}{3(x^2+2)}\, dx + \int \frac{1}{3(x^2-1)}\, dx$$

$$= \frac{2}{3\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{1}{6} \log \left| \frac{x-1}{x+1} \right| + c$$ 1 m

11. Let $E_1$ : two headed coin is chosen

$E_2$ : unbiased coin is chosen

A : All 5 tosses are heads ½ m

$P(E_1) = \frac{1}{5},\ P(E_2) = \frac{4}{5},\ P\left(A/E_1\right) = 1,\ P\left(A/E_2\right) = \frac{1}{32}$ 2 m

$$P\left(E_1/A\right) = \frac{P(E_1)\,P\left(A/E_1\right)}{P(E_1)\,P\left(A/E_1\right) + P(E_2)\,P\left(A/E_2\right)}$$ ½ m

$$P\left(E_1/A\right) = \frac{\frac{1}{5} \times 1}{\frac{1}{5} \times 1 + \frac{4}{5} \cdot \frac{1}{32}} = \frac{8}{9}$$ 1 m

OR

Let the coin is tossed n times

$1 - P(0) > \frac{80}{100}$ 1½ m

$P(0) < \frac{1}{5}$ ½ m

$^{n}C_0 \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^0 < \frac{1}{5}$ 1 m

$\left(\frac{1}{2}\right)^n < \frac{1}{5} \Rightarrow n \geq 3$ 1 m

12. $\overrightarrow{BA} = \hat{i} + (x-1)\hat{j} + 4\hat{k}, \overrightarrow{CA} = \hat{i} - 3\hat{k}, \overrightarrow{DA}\, 3\hat{i} + 3\hat{j} - 2\hat{k}$ 1½ m

$$\left[\overrightarrow{BA}, \overrightarrow{CA}, \overrightarrow{DA}\right] = 0$$ 1 m

$$\begin{vmatrix} 1 & x-1 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$ 1 m

$x = 4$ ½ m

13. $\vec{r} = (4\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})\ \vec{a} + \lambda\vec{b}$ 1 m

Let L be the foot of perpendicular

Position vector of L is $(2\lambda + 4)\hat{i} + (3\lambda + 2)\hat{j} + (6\lambda + 2)\hat{k}$ ½ m

$\overrightarrow{PL} = (2\lambda + 3)\hat{i} + 3\lambda\hat{j} + (6\lambda - 1)\hat{k}$ ½ m

$\overrightarrow{PL} \cdot \vec{b} = 2(2\lambda + 3) + 3(3\lambda) + 6(6\lambda - 1) = 0$

$\Rightarrow \lambda = 0$ 1 m

$\overrightarrow{PL} = 3\hat{i} - \hat{k}$

$\left|\overrightarrow{PL}\right| = \sqrt{10}$ units 1 m

14. $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$(1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$ 1 m

$1-x = \cos\left(2\sin^{-1}x\right)$ 1 m

$1-x = 1-2x^2$ 1 m

$\Rightarrow x = 0, \frac{1}{2}$ ½ + ½ m

$x = \frac{1}{2}$ is rejected

OR

L.H.S $= 2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$

$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$ 1 m

$= \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$ 1 m

$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7}\cdot\frac{17}{31}}\right)$ 1 m

$= \tan^{-1}\left(\frac{625}{625}\right) = \frac{\pi}{4}$ 1 m

15. $y = e^{ax} \cos bx$

$y_1 = ae^{ax} \cos bx - b\, e^{ax} \sin b\, x$ 1 m

$y_1 = ay - b\, e^{ax} \sin b\, x$ 1 m

$y_2 = ay_1 - b\, [ae^{ax} \sin bx + b\, e^{ax} \cos b\, x]$ 1 m

$y_2 = ay_1 - a\, be^{ax} \sin bx - b^2 e^{ax} \cos b\, x$

$y_2 = a\, y_1 - a\, (ay - y_1) - b^2 y$

$y_2 - 2\, a\, y_1 + (a^2 + b^2)\, y = 0$ 1 m

16. $x^x + x^y + y^x = a^b$

Let $u = x^x$, $v = x^y$, $w = y^x$, $\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ½ m

$\frac{du}{dx} = x^x \left(1 + \log x\right)$ 1 m

$\frac{dv}{dx} = x^y \left(\frac{y}{x} + \frac{dy}{dx} \log x\right)$ 1 m

$\frac{dw}{dx} = y^x \left(\frac{x}{y} \cdot \frac{dy}{dx} + \log y\right)$ 1 m

$\frac{dy}{dx} = -\left(\frac{x^x \left(1 + \log x\right) + y\, x^{y-1} + y^x \log y}{x^y \log x + x\, y^{x-1}}\right)$ ½ m

17. $\frac{dx}{at} = a\ [\sin 2t\,(-2\sin 2t) + (1+\cos 2t)(2\cos 2t)]$ 1 m

$\frac{dy}{dt} = b\ [2\sin 2t\cos 2t - 2\sin 2t\,(1-\cos 2t)]$ 1 m

$\frac{dy}{dx} = \frac{b\ [2\sin 2t\cos 2t - 2\sin 2t\,(1-\cos 2t)]}{a\ [\sin t\,(-2\sin 2t) + (1+\cos 2t)(2\cos 2t)]}$ 1 m

$= \frac{4b\cos 3t\sin t}{4a\cos 3t\cos t} = \frac{b}{a}\tan t = \frac{b}{2}\times 1 = \frac{b}{a}$ 1 m

18. $\int \frac{x+3}{(x+5)^3}\ e^x\,dx$

$\int \frac{1}{(x+5)^2} - \frac{2}{(x+5)^3}\ e^x\,dx$ 1 m

$\int \frac{1}{(x+5)^2}\ e^x\,dx - \int \frac{2}{(x+5)^3}\ e^x\,dx$ ½ m

$= \frac{1}{(x+5)^2}\ e^x + \int \frac{2}{(x+5)^3}\ e^x\,dx - \int \frac{2}{(x+5)^3}\ e^x\,dx$ 2 m

$= \frac{e^x}{(x+5)^2} + c$ ½ m

19. $\begin{matrix} & F & M & T \\ x \\ y \\ z \end{matrix}\begin{pmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{pmatrix}\begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 5450 \\ 5250 \\ 6625 \end{pmatrix}$ 1½ m

Funds collected by school x : ₹ 5450, school y = ₹ 5250

school z = ₹ 6625 1 m

Total collected funds = ₹ 17325 ½ m

For writing any value 1 m

## SECTION - C

20. (i) Let $(e, e')$ be the identity element in A

$(a, b) * (e, e') = (a, b) = (e, e') * (a, b)$

$(a\,e, b + a\,e') = (a, b)$

$$\left.\begin{array}{l} ae = a \Rightarrow e = 1 \\ b + a\,e' = b \Rightarrow e' = 0 \end{array}\right] \Rightarrow \text{identity} : (1, 0)$$ 2 ½ m

(ii) Let $(x, y)$ is inverse of $(a, b) \in A$

$(a, b) * (x, y) = (1, 0) = (x, y) * (a, b)$

$(a\,x, b + a\,y) = (1, 0)$

$$\left.\begin{array}{l} ax = 1 \Rightarrow x = \frac{1}{a} \\ b + a\,y = 0 \Rightarrow y = \frac{-b}{a} \end{array}\right] \Rightarrow \text{inverse of } (a, b) = \left(\frac{1}{a}, \frac{-b}{a}\right)$$ 2 ½ m

Inverse of $(5, 3) = \left(\frac{1}{5}, \frac{-3}{5}\right)$ ½ m

Inverse of $\left(\frac{1}{2}, 4\right) = (2, -8)$ ½ m

OR

One – One : - Case I : when x and y are even

$f(x) = f(y) \Rightarrow x + 1 = y + 1 \Rightarrow x = y$

Case II : when x and y are odd

$f(x) = f(y) \Rightarrow x - 1 = y - 1 \Rightarrow x = y$

Case III : one of them is even and one of them is odd

$f(x) \neq f(y) \Rightarrow x + 1 \neq y - 1 \Rightarrow x \neq y$ 2 ½ m

Onto : Let $y \in W$

$$f(y-1) = y \text{ if y is odd}$$

$$f(y+1) = y \text{ if y is even}$$

So $\forall\ y \in W$, there exist some element in domain of f

$\Rightarrow$ f is invertible 2½ m

$$f^{-1}(x) = \begin{cases} x-1, & x \text{ is odd} \\ x+1, & x \text{ is even} \end{cases}$$ 1 m

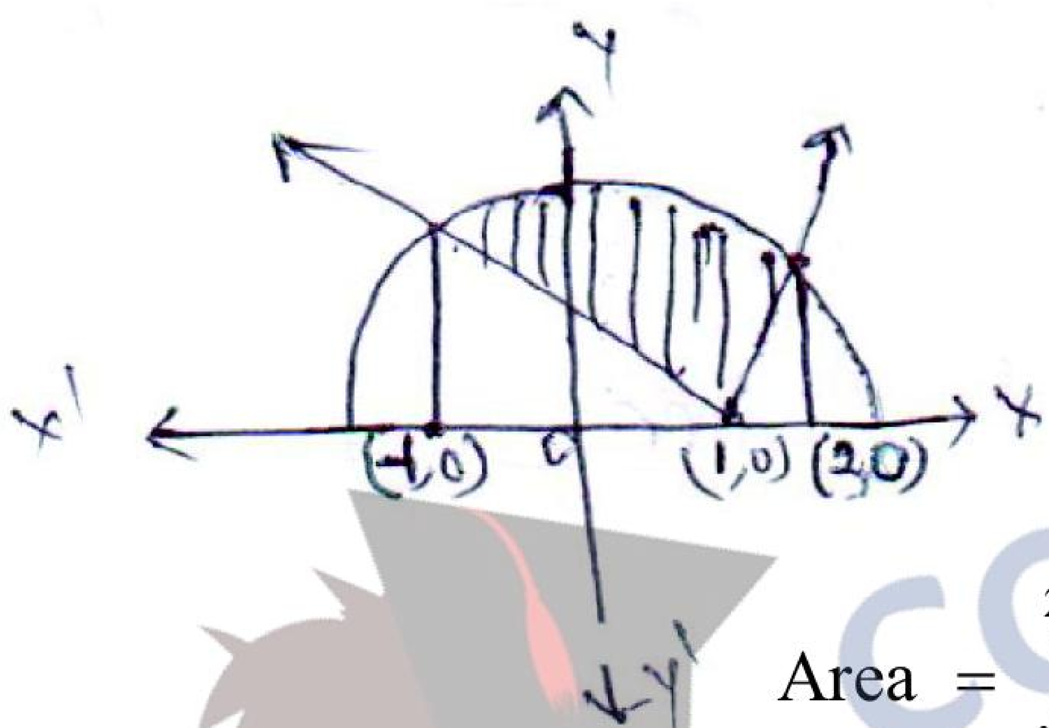
21. 


Figure 1 m

For finding (– 1, 0) , (1, 0) (2, 0) 1½ m

$$\text{Area} = \int_{-1}^{2}\sqrt{5-x^2}\,dx - \int_{-1}^{1}-(x-1)\,dx - \int_{-1}^{2}(x-1)\,dx$$ 1½ m

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} + \left[\frac{(x-1)^2}{2}\right]_{-1}^{1} - \left[\frac{(x-1)^2}{2}\right]_{1}^{2}$$ 1 ½ m

$$= \left(1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right) + \left(1+\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2}\times 4 - \frac{1}{2}\times 1$$

$$= \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2} \text{ sq. units}$$ ½ m

22. $x^2\,dy = (2\,xy + y^2)\,dx$

$$\frac{dy}{dx} = \frac{2\,xy + y^2}{x^2}$$ ½ m

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ 1 m

$$v + x\frac{dv}{dx} = 2v + v^2 \Rightarrow \int\frac{1}{v^2 + v}dv = \int\frac{1}{x}dx$$ 2 m

$$\Rightarrow \log\left|\frac{v}{v+1}\right| = \log x + \log c$$ 1 m

$$\Rightarrow \log\left|\frac{y}{y+x}\right| = \log cx \Rightarrow \frac{y}{y+x} = cx$$ 1 m

$$x = 1,\ y = 1 \Rightarrow c = \frac{1}{2}$$

$$x^2 + xy - 2y = 0$$ ½ m

OR

Given differential equation can be written as

$$\frac{dy}{dx} + \frac{1}{1+x^2}y = \frac{e^{m\tan^{-1}x}}{1+x^2}$$ 1 m

Integrating factor is $e^{\tan^{-1}x}$ 1 m

Solution is $y \cdot e^{\tan^{-1}x} = \int\frac{e^{m\tan^{-1}x}}{1+x^2} \cdot e^{\tan^{-1}x}dx$ 1½ m

$$\Rightarrow y\,e^{\tan^{-1}x} = \int e^{(m+1)t}dt,\ \text{where } \tan^{-1}x = t$$ 1 m

$$= \frac{e^{(m+1)t}}{m+1} = \frac{e^{(m+1)\tan^{-1}x}}{m+1} + c$$ 1 m

$$y = 1, x = 0 \Rightarrow c = \frac{m}{m+1}$$

$$y\,e^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{m+1} + \frac{m}{m+1}$$ ½ m

23. $f(x) = \sin^2 x - \cos x$

$f'(x) = \sin x\,(2\cos x + 1)$ 1 m

$f'(x) = 0 \Rightarrow \sin x = 0$ and $2\cos x + 1 = 0 \Rightarrow x = 0, 2\frac{\pi}{3}, \pi$ 2½ m

$f(0) = -1,\ f\left(\frac{2\pi}{3}\right) = \frac{5}{4},\ f(\pi) = 1$ 1½ m

Absolute maximum value is $\frac{5}{4}$ ½ m

Absolute minimum value is – 1 ½ m

24. Two lines $\vec{r} = \vec{a_1} + \lambda\,\vec{b_1}$ and $r = \vec{a_2} + \mu\,\vec{b_2}$ are coplanar

if $\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = 0$ 1 m

Here $\left(-\hat{i} + 3\hat{j} + \hat{k}\right)\cdot\left[\left(\hat{i} - \hat{j} + \hat{k}\right)\times\left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right] = 0$ 2 m

Equation of plane is

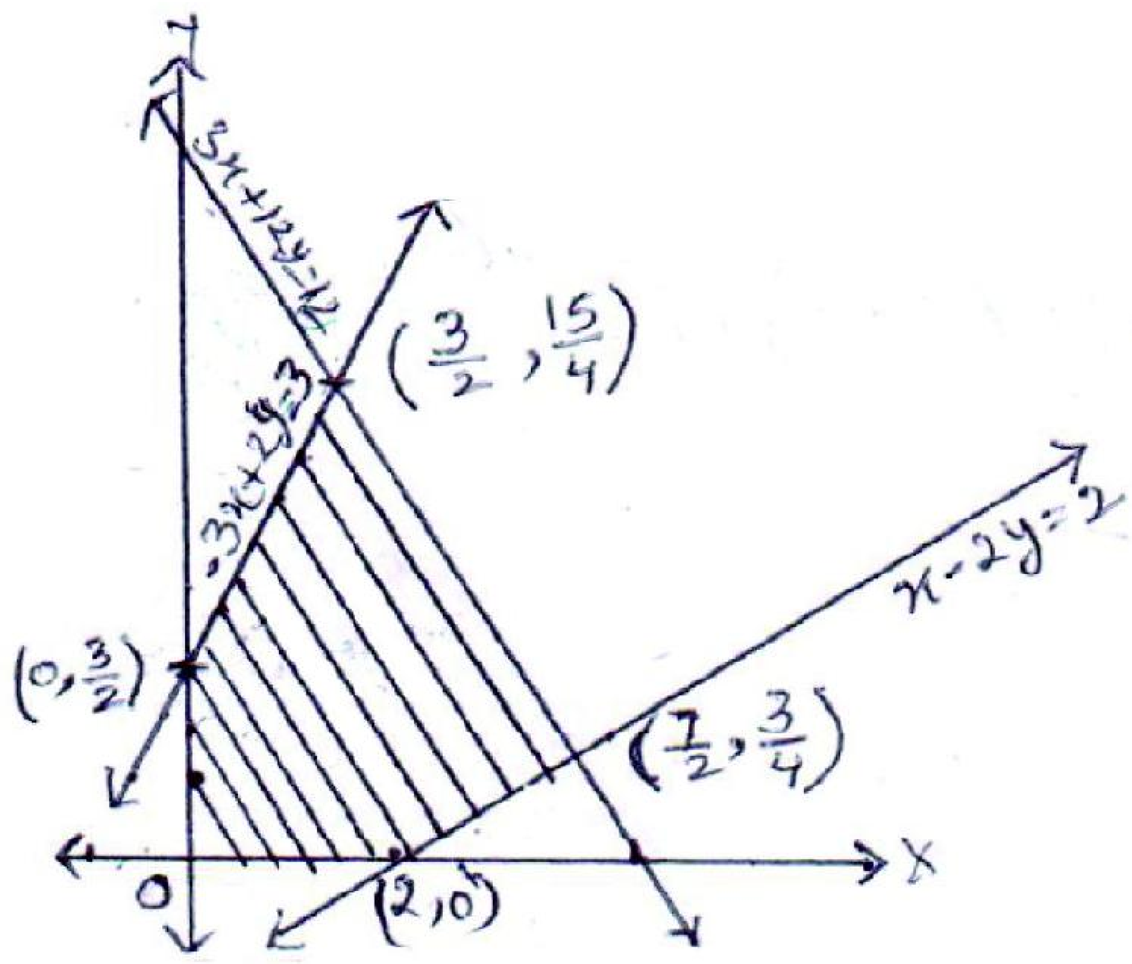
$\left(\vec{r} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = 0$ 1 m

$\left[\vec{r} - \left(\hat{i} + \hat{j} + \hat{k}\right)\right]\cdot\left[\left(\hat{i} - \hat{j} + \hat{k}\right)\times\left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right] = 0$

$\vec{r}\cdot\left(-2\hat{i} - \hat{j} + \hat{k}\right) + 2 = 0$ 2 m

25. Correct graph of three lines 1×3 m

correct shading of feasible region 1 m




vertices are $\left(0, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{15}{4}\right),$

$\left(\frac{7}{2}, \frac{3}{4}\right), (2, 0)$ 1 m

$z = 5x + 2y$ is maximum

at $\left(\frac{7}{2}, \frac{3}{4}\right) = 19$ and

minimum at $\left(0, \frac{3}{2}\right) = 3$ 1 m

26.

| | | | | | | |
|---|---|---|---|---|---|---|
| x : | 2 | 3 | 4 | 5 | 6 | 1 m |
| P (x) : | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{4}{15}$ | $\frac{5}{15}$ | 2 m |
| x . P (x) : | $\frac{2}{15}$ | $\frac{6}{15}$ | $\frac{12}{15}$ | $\frac{20}{15}$ | $\frac{30}{15}$ | ½ m |
| $x^2$ P (x) : | $\frac{4}{15}$ | $\frac{18}{15}$ | $\frac{48}{15}$ | $\frac{100}{15}$ | $\frac{180}{15}$ | ½ m |

Mean $= \sum x \cdot P(x) = \frac{70}{15} = \frac{14}{3}$ 1 m

Variance $= \sum x^2 \, P(x) = (\text{Mean})^2 = \frac{350}{15} - \frac{196}{9} = \frac{14}{9}$ 1 m