QUESTION PAPER CODE 65/2/A

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\overrightarrow{OB} = \dfrac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ ½ m

$\overrightarrow{OC} = 2\vec{b} - \vec{a}$ ½ m

2. Vector Perpendicular to $\vec{a}$ and $\vec{b}$ $= \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|}$

[Finding or using] ½ m

Required Vector $= \hat{i} - 11\,\hat{j} - 7\,\hat{k}$ ½ m

3. Writing standard form

$\dfrac{x}{3} = \dfrac{y}{2} = \dfrac{z}{-6}$ and $\dfrac{x}{2} = \dfrac{y}{-12} = \dfrac{z}{-3}$ ½ m

Finding $\theta = \dfrac{\pi}{2}$ ½ m

4. getting $|A| = 1$ ½ m

$|A^n| = 1$ ½ m

5. Order 2 or degree = 1 ½ m

sum = 3 ½ m

6. Writing $\displaystyle\int \frac{y}{\sqrt{1+y^2}}\,dy = -\int \frac{x\,dx}{\sqrt{1+x^2}}$ ½ m

Getting $\sqrt{1+y^2} + \sqrt{1+x^2} = c$ ½ m

SECTION - B

7. $A = IA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1 m

Using elementary row trans formations to get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} A$$ 2 m

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$$ 1 m

OR

$$AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$$ 1 m

$$BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$$ 1 m

$$AC + BC = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ ½ m

$$(A + B)\ C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$ ½ m

$$= \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ 1 m

Yes, $(A + B)\ C = AC + BC$

8. $$f(x) = \begin{cases} -2x + 1 & \text{if} \quad x < 0 \\ 1 & \text{if} \quad 0 \le x < 1 \\ 2x - 1 & \text{if} \quad x \ge 1 \end{cases}$$ 1½ m

Only possible discontinuties are at $x = 0,\ x = 1$

at $x = 0$: at $x = 1$

L. H. limit = 1 : L. H. limit = 1 1 m

$f(0)$ = R. H. limit = 1 : $f(1)$ = R. H. limit = 1

$\therefore$ $f(x)$ is continuous in the interval $(-1, 2)$ ½ m

At $x = 0$

L. H. D $= -2 \ne$ R. H. D $= 1$ 1 m

$\therefore$ $f(x)$ is not differentiable in the interval $(-1, 2)$

9. $x = a\ (\cos 2t + 2t \sin 2t)$

$y = a\ (\sin 2t - 2t \cos 2t)$

$\Rightarrow \dfrac{dx}{dt} = 4\,at \cos 2\,t$ 1 m

$\Rightarrow \dfrac{dy}{dt} = 4\,at \sin\ 2\,t$ 1 m

$\Rightarrow \dfrac{dy}{dx} = \tan 2\,t$ ½ m

$$\Rightarrow \frac{d^2y}{dx^2} = 2\sec^2 2t \cdot \frac{dt}{dx}$$ 1 m

$$\frac{d^2y}{dx^2} = \frac{1}{2\,at\cos^3 2t}$$ ½ m

10. $$\frac{y}{x} = \log x - \log\ (ax+b)$$

differentiating w.r.t. x, 1 m

$$= \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{a}{ax+b} = \frac{b}{x\,(ax+b)}$$

$$= x \cdot \frac{dy}{dx} - y = \frac{bx}{(ax+b)} \text{ .... (1)}$$ 1 m

differentiating w.r.t. x again

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(ax+b)\,b - abx}{(ax+b)^2}$$

$$x\frac{d^2y}{dx^2} = \frac{b^2}{(ax+b)^2}$$ 1 m

Writing $\Rightarrow x^3\frac{d^2y}{dx^2} = \left(\frac{bx}{ax+b}\right)^2$ .... (2) ½ m

From (1) and (2) $\Rightarrow$

$$x^3\frac{d^2y}{dx^2} = \left(x \cdot \frac{dy}{dx} - y\right)^2$$ ½ m

11. $I = \int \frac{x + \sin x - x(1+\cos x)}{x(x+\sin x)}\, dx$ 1 m

$= \int \frac{1}{x}\, dx - \int \frac{1+\cos x}{x+\sin x}\, dx$ put $x + \sin x = t$ $\Rightarrow (1 + \cos x)\, dx = dt$ 2 m

$= \log|x| - \log|x + \sin x| + c$ 1 m

OR

$I = \int \frac{(x-1)(x^2+x+1)+1}{(x-1)(x^2+1)}\, dx$ ½ m

$= \int \frac{x^2+x+1}{x^2+1}\, dx + \int \frac{dx}{(x-1)(x^2+1)}$ 1 m

$= \int \left(1 + \frac{x}{x^2+1} + \frac{1}{2}\, \frac{1}{x-1} - \frac{1}{2}\, \frac{x}{x^2+1} - \frac{1}{2}\, \frac{1}{x^2+1}\right) dx$ 1½ m

$= x + \frac{1}{4} \log|x^2+1| + \frac{1}{2} \log|x-1| - \frac{1}{2} \tan^{-1}x + c$ 1 m

12. $\begin{array}{l} \text{Family A} \Rightarrow \\ \text{Family B} \Rightarrow \end{array} \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{array}{c} \begin{array}{cc} C & \quad P \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$ 2 m

Writing Matrix Multiplication as $\begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix}$ 1 m

Writing about awareness of balanced diet 1 m

Alt : Method

Taking the given data for all Men, all Women, all Children
for each family, the solution must be given marks
accordingly

13. $$\tan\left\{\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\} = \tan\left\{\tan^{-1}\left(\frac{2/5}{1-\frac{1}{25}}\right)+\frac{\pi}{4}\right\}$$ 1 m

$$= \tan\left\{\tan^{-1}\left(\frac{5}{12}\right)+\frac{\pi}{4}\right\}$$ 1 m

$$= \frac{\frac{5}{12}+1}{1-\frac{5}{12}} = \frac{17}{7}$$ 1+1 m

14. Writing $C_1 \leftrightarrow C_2$

$$A = -2\begin{vmatrix} 1 & a^3 & a \\ 1 & b^3 & b \\ 1 & c^3 & c \end{vmatrix}$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$A = -2\begin{vmatrix} 0 & a^3-b^3 & a-b \\ 0 & b^3-c^3 & b-c \\ 1 & c^3 & c \end{vmatrix}$$ 1+1 m

$$A = -2\,(a-b)\,(b-c)\begin{vmatrix} 0 & a^2+ab+b^2 & 1 \\ 0 & b^2+c^2+bc & 1 \\ 1 & c^3 & c \end{vmatrix}$$ 1 m

$$= -2\,(a-b)\;(b-c)\;\left\{a^2+ab+b^2-b^2-bc-c^2\right\}$$ ½ m

$$= \;2\;(a-b)\;(b-c)\;(c-a)\;(a+b+c)$$ ½ m

15. Writing $\vec{d} = \lambda\left(\vec{a}\times\vec{b}\right)$

$$= \lambda\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 4 & 2\\ 3 & -2 & 7\end{vmatrix}$$ 1 m

$$= \lambda\left(32\,\hat{i}-\hat{j}-14\,\hat{k}\right) \text{ .... } (1)$$ 1 m

$$\vec{c}\cdot\vec{d} = 27$$

$$\left(2\hat{i}-\hat{j}+4\hat{k}\right)\cdot\lambda\left(32\hat{i}-\hat{j}-14\hat{k}\right) = 27$$

$$9\lambda = 27$$ 1 m

$$\lambda = 3$$

$$\therefore\ \vec{d} = 96\hat{i}-3\hat{j}-42\hat{k}$$ 1 m

16. Lines are parallel ½ m

$$\therefore\ \text{S.D} = \left|\frac{\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}}{\left|\vec{b}\right|}\right|$$ 1 m

$$\vec{a}_2-\vec{a}_1 = \hat{i}+2\hat{j}+2\hat{k} \text{ and } \rightarrow b = 2\hat{i}+3\hat{j}+4\hat{k}$$

$$\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 2 & 3 & 4\end{vmatrix},\quad \left|\vec{b}\right| = \sqrt{29}$$ 1½ + ½ m

$\therefore\ \text{S.D} = \left| \dfrac{2\hat{i} - \hat{k}}{\sqrt{29}} \right| = \dfrac{\sqrt{5}}{\sqrt{29}}$ or $\dfrac{\sqrt{145}}{29}$ ½ m

OR

Required equation of plane is

$2x + y - z - 3 + \lambda(5x - 3y + 4z + 9) = 0 \quad \rightarrow \quad (1)$ 1 m

$x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) + 9\lambda - 3 = 0$ 1 m

(1) is parallel to $\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-5}{5}$

$\therefore\ 2(2 + 5\lambda) + 4(1 - 3\lambda) + 5(-1 + 4\lambda) = 0$

$\Rightarrow\ \lambda = -\dfrac{1}{6}$ 1 m

(1) $\Rightarrow$ $7x + 9y - 10z - 27 = 0$ 1 m

17. P (step forward) $= \dfrac{2}{5}$, P (step backword) $= {}^{3}\!/_{5}$ ½ m

He can remain a step away in either of the

ways : 3 steps forward & 2 backwards 1 m

or 2 steps forward & 3 backwards

$\therefore$ required possibility $= {}^{5}C_{3}\left(\dfrac{2}{5}\right)^{3}\left(\dfrac{3}{5}\right)^{2} + {}^{5}C_{2}\left(\dfrac{2}{5}\right)^{2}\left(\dfrac{3}{5}\right)^{3}$ 2 m

$= \dfrac{72}{125}$ ½ m

OR

A die is thrown

Let E_1 be the event of getting 1 or 2

Let E_2 be the event of getting 3, 4, 5 or 6

Let A be the event of getting a tail

$$P(E_1) = 1/3, \; P(E_2) = 2/3$$ 1 m

$$\Rightarrow \; P\left(A/E_1\right) = \frac{3}{8}, \; \& \; P\left(A/E_2\right) = \frac{1}{2}$$ 1 m

$$P\left(E_2/A\right) = \frac{P(E_2)\times P\left(A/E_2\right)}{P(E_1)\times P\left(A/E_1\right) + P(E_2)\times P\left(A/E_2\right)}$$ 1 m

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}}$$

$$= \frac{8}{11}$$ 1 m

18. $$I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+4\tan^2 x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{\left(1+\tan^2 x\right)\left(1+4\tan^2 x\right)}\,dx$$ 1 m

Put $\tan x = t$

$$I = \int_0^{\infty} \frac{dt}{\left(1+t^2\right)\left(1+4t^2\right)} = -\frac{1}{3}\int_0^{\infty}\frac{dt}{1+t^2} + \frac{4}{3}\int_0^{\infty}\frac{dt}{1+(2t)^2}$$ 1 m

$$= -\frac{1}{3}\tan^{-1}t\Big]_0^{\infty} + \frac{4}{3\times 2}\tan^{-1}(2t)\Big]_0^{\infty}$$ 1 m

$$= -\frac{1}{3}\left(\frac{\pi}{2}\right) + \frac{2}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{6}$$ 1 m

19. $I = -\int_{0}^{\frac{\pi}{4}} \frac{\sin\,x + \cos\,x}{(\sin\,x - \cos x)^2 - 2^2}\,dx$ 1½ m

Put $\sin x - \cos x = t \quad \Rightarrow \quad t = -1 \text{ to } 0$ 1 m

$(\cos x + \sin x)\,dx = dt$

$$I = -\int_{-1}^{0} \frac{dt}{t^2 - 2^2}$$

$= -\frac{1}{4}\,\log\left|\frac{t-2}{t+2}\right|\Bigg]_{-1}^{0}$ 1 m

$= -\frac{1}{4}\,\{0 - \log 3\}$

½ m

$= \frac{1}{4}\,\log 3$

SECTION - C

20.

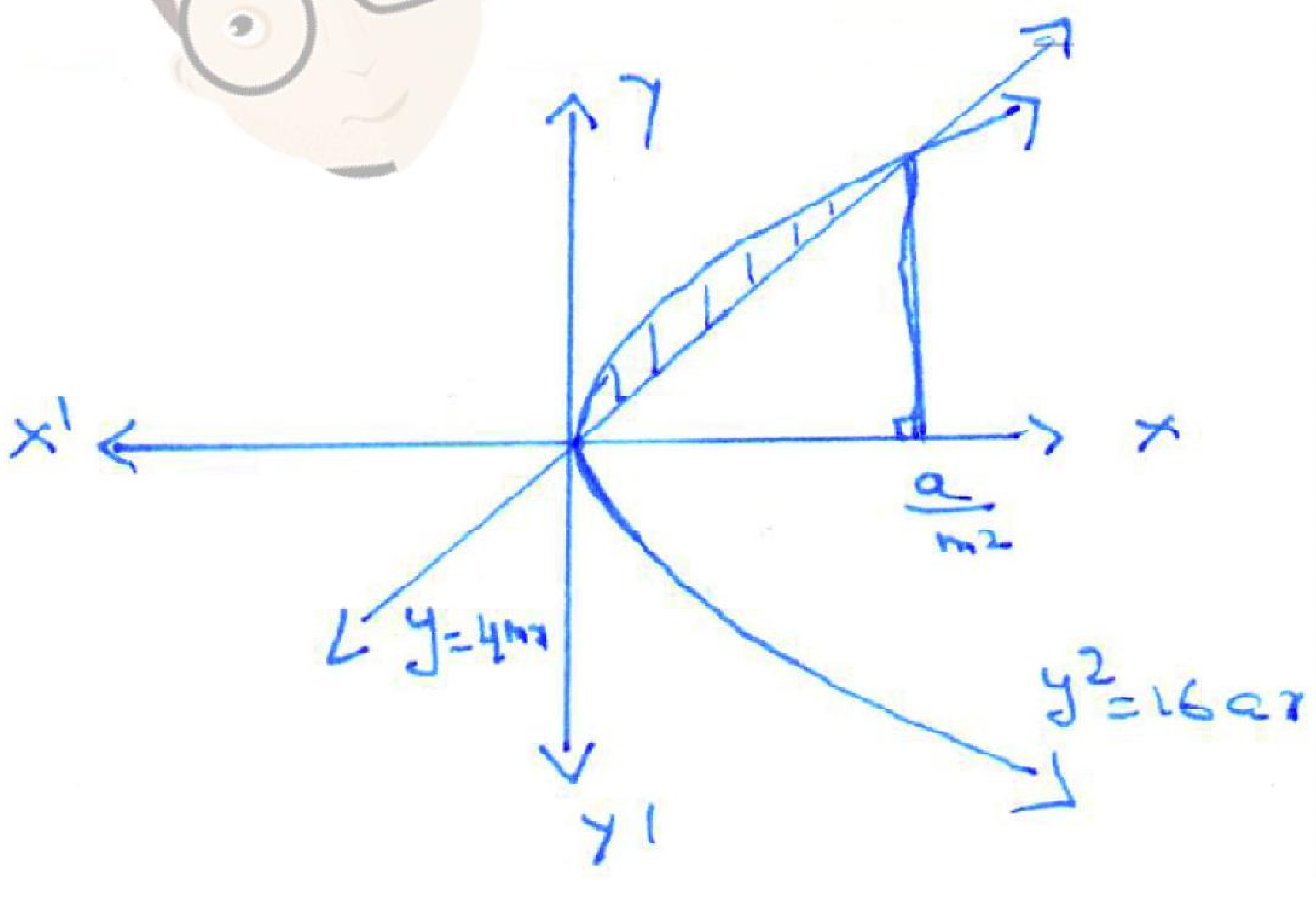


Figure .... ½ m

$y = 4mx \rightarrow (1) \quad \text{and} \quad y^2 = 16\,ax \rightarrow (2)$ 1 m

$$\Rightarrow x = \frac{a}{m^2}$$

Required area $= 4\sqrt{a}\int_{0}^{\frac{a}{m^2}}\sqrt{x}\;dx - 4m\int_{0}^{\frac{a}{m^2}} x\;dx$ 2 m

$$= \frac{8}{3}\sqrt{a}\;x^{3/2}\Big]_{0}^{\frac{a}{m^2}} - 2m\;x^2\Big]_{0}^{\frac{a}{m^2}}$$

$$= \frac{8}{3}\;\frac{a^2}{m^3} - \frac{2a^2}{m^3} = \frac{2}{3}\;\frac{a^2}{m^3}$$ 2 m

$$\Rightarrow \frac{2}{3}\cdot\; a^2\!/\!_{m^3} = \frac{a^2}{12} \text{ given}$$

$m^3 = 8$

$m = 2$ ½ m

21. $(x - y)\;\frac{dy}{dx} = x + 2y$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

$$\frac{dy}{dx} = \frac{1+2\frac{y}{x}}{1-\frac{y}{x}} = f\left(y\!/\!_x\right) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(1)$$

$\therefore$ differential equation is homogeneous Eqn. 1 m

$y = vx$ to give

$$v + x\cdot\frac{dv}{dx} = \frac{1+2v}{1-v}$$ ½ m

$$\Rightarrow \int \frac{1-v}{1+v+v^2}\,dv = \int \frac{dx}{x}$$ 1 m

$$\Rightarrow -\frac{1}{2}\int \frac{2v+1}{1+v+v^2}\,dv + \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \int \frac{dx}{x}$$ 1½ m

$$-\frac{1}{2}\log\left|1+v+v^2\right| + \sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = \log|x|+c$$ 1 m

$$-\frac{1}{2}\log\left|\frac{x^2+xy+y^2}{x^2}\right| + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{x\sqrt{3}}\right) = \log|x|+c$$ 1 m

OR

$$(x-h)+(y-k)\frac{dy}{dx} = 0$$ 1 m

and $1+(y-k)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 0$ 1 m

$$\Rightarrow (y-k) = \frac{-\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$ 1 m

(1) $\Rightarrow (x-h) = \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\,\frac{dy}{dx}$ 1 m

Putting in the given eqn.

$$\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2}\cdot\left(\frac{dy}{dx}\right)^2+\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2}=r^2$$ 1 m

or $$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=r^2\left(\frac{d^2y}{dx^2}\right)^2$$ 1 m

22. Eqn. of a plane through

and Points A (6, 5, 9), B (5, 2, 4) & C (– 1, – 1, 6) is

$$\Rightarrow\begin{vmatrix}x-6 & y-5 & z-9\\ 2 & 3 & 2\\ -6 & -3 & 2\end{vmatrix}=0$$ 2½ m

$\Rightarrow$ 3x – 4y + 3z – 25 = 0 $\rightarrow$ (2) 1½ m

distance from (3, – 1, 2) to (2)

$$d=\left|\frac{9+4+6-25}{\sqrt{9+16+9}}\right|=\frac{6}{\sqrt{34}}\text{ units}$$ 2 m

23. Here R = {(a, b) : a, b $\in \Re$ and a – b + $\sqrt{3} \in$ S, where

S is the set of all irrational numbers.}

(i) $\forall$ a $\in \Re$, (a, a) $\in$ R as a – a + $\sqrt{3}$ is irrational

$\therefore$ R is reflexive 1½ m

(ii) Let for a, b $\in \Re$, (a, b) $\in$ R i. e. a – b + $\sqrt{3}$ is irrational

a – b + $\sqrt{3}$ is irrational $\Rightarrow$ b – a + $\sqrt{3} \in$ S $\therefore$ (b, a) $\in$ R

Hence R is symmetric 2 m

(iii) Let $(a, b) \in R$ and $(b, c) \in R$, for $a, b, c \in \Re$

$\therefore \quad a - b + \sqrt{3} \in S$ and $b - c + \sqrt{3} \in S$

adding to get $a - c + 2\sqrt{3} \in S$ Hence $(a, c) \in R$ 2½ m

$\therefore$ R is Transitive

OR

$\forall\, a, b, c, d, e, f \in \Re$

$((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f)$ 1 m

$= (a + c + e, b + d + f) \quad \rightarrow \quad (3)$

$(a, b) * ((c_{,}\ d) * (e, f)) = (a, b) * (c + e, d + f)$ 1 m

$= (a + c + e, b + d + f) \quad \rightarrow \quad (4)$

$\therefore$ * is Associative

Let (x, y) be on identity element in $\Re \times \Re$

$\Rightarrow \quad (a, b) * (x, y) = (a, b) = (x, y) * (a, b)$

$\Rightarrow \quad a + x = a,\ b + y = b$

$x = 0,\ y = 0$ 2 m

$\therefore$ (0, 0) is identity element

Let the inverse element of (3, – 5) be $(x_1, y_1,)$

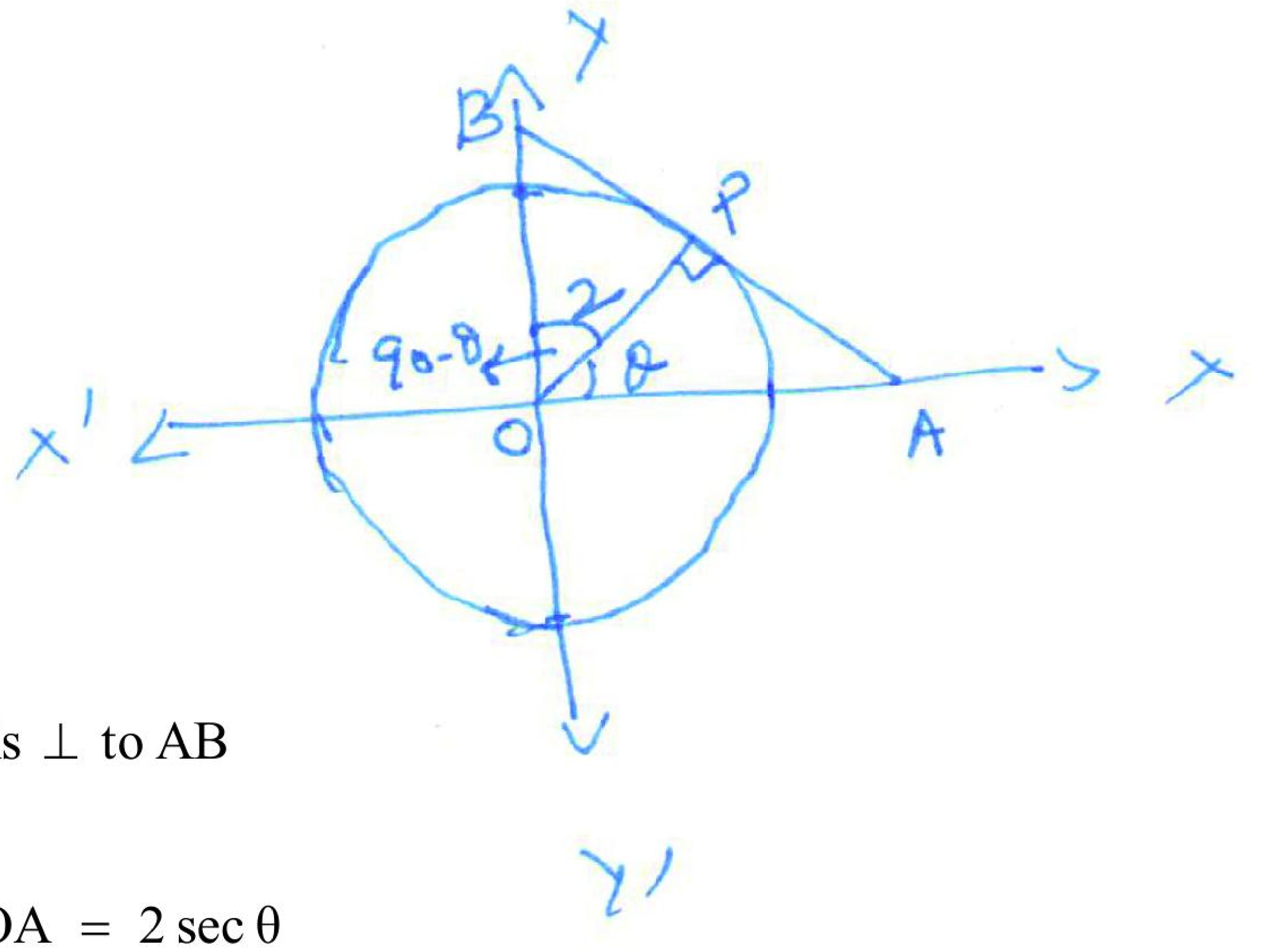
$\Rightarrow \quad (3, -5) * (x_1, y_1) = (0, 0) = (x_1, y_1) * (3, -5)$

$3 + x_1 = 0, \quad -5 + y_1 = 0$

$x_1 = -3, \quad y_1 = 5$

$\Rightarrow$ (– 3, 5) is an inverse of (3, – 5) 2 m

24. Fig. ½ m



$x^2 + y^2 = 4$. OP is ⊥ to AB

$\cos\theta = \frac{2}{OA}$; $OA = 2\sec\theta$ ½ m

$\cos(90^\circ - \theta) = \frac{2}{OB}$

$OB = 2\ \text{cosec}\,\theta$ ½ m

Let $S = OA + OB = 2(\sec\theta + \text{cosec}\,\theta)$ .... (1) 1 m

$\frac{dS}{d\theta} = 2(\sec\theta\tan\theta - \text{cosec}\,\theta\cdot\cot\theta)$ 1m

$= 2\left(\frac{\sin^3\theta - \cos^3\theta}{\sin^2\theta\cdot\cos^2\theta}\right)$ .... (2)

for maxima or minima $\frac{dS}{d\theta} = 0$

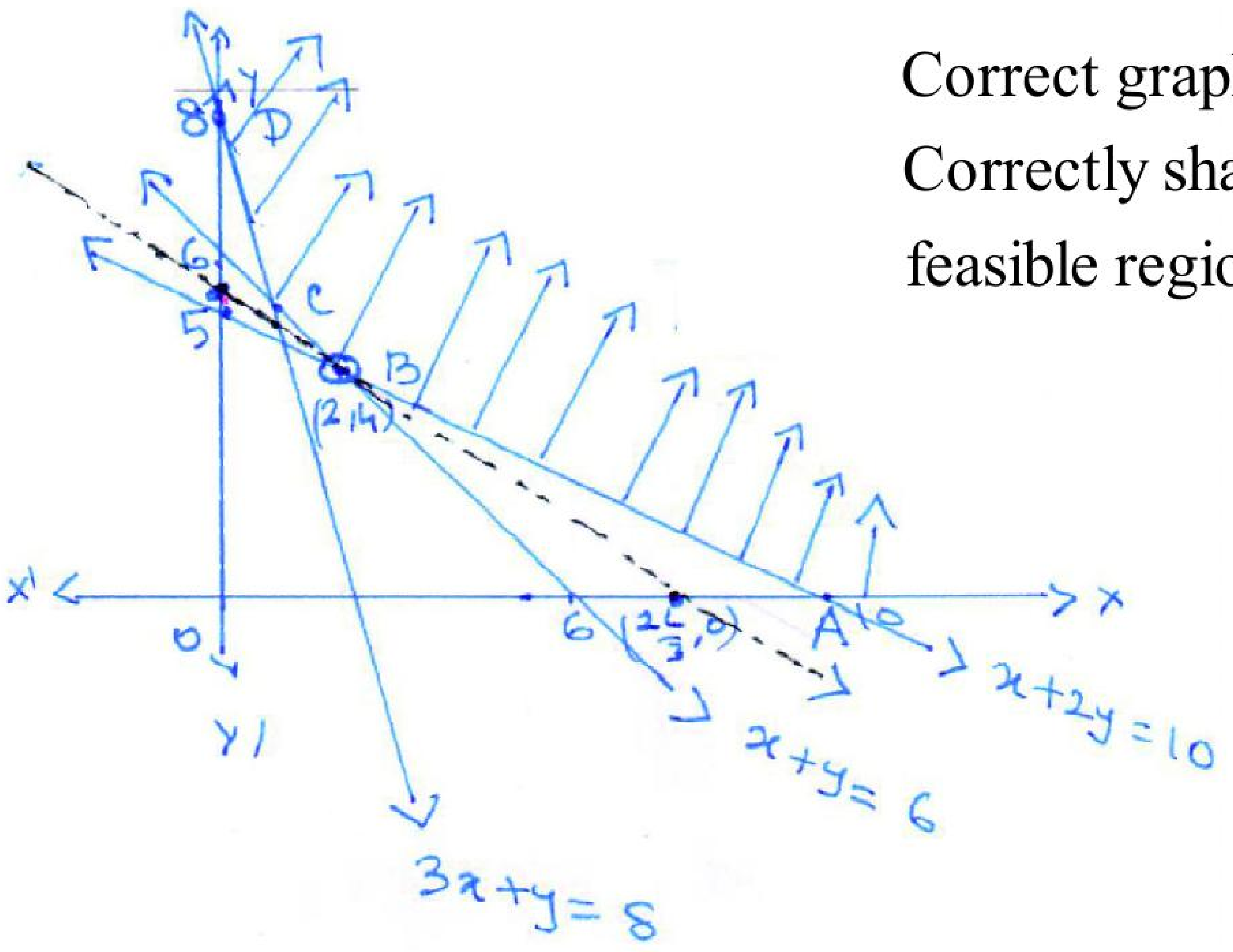
$\Rightarrow \theta = \frac{\pi}{4}$, 1 m

(2) $\Rightarrow \frac{d^2S}{d\theta^2} > 0$ when $\theta = \frac{\pi}{4}$ 1 m

$\therefore$ OA + OB is minimum

$\Rightarrow OA + OB = 4\sqrt{2}$ unit ½ m

25.



Correct graphs of 3 lines 3 m

Correctly shading feasible region ½

Vertices are A (10, 0), B (2, 4), C (1, 5) & D (0, 8) 1 m

Z = 3x + 5y is minimum

at B (2, 4) and the minimum Value is 26. 1 m

on Plotting $(3x + 5y < 26)$

since these it no common point with the feasible

region, Hence, $x = 2$, $y = 4$ gives minimum Z ½ m

26. Possible values of x are 0, 1, 2 and x is a random variable 1½ m

x :	P(x)	x P(x)	x^2 P(x)		
0	$\frac{{}^2C_0 \times {}^5C_2}{{}^7C_2} = \frac{20}{42}$	0	0	For P (x)	1½ m
1	$\frac{{}^2C_1 \times {}^5C_1}{{}^7C_2} = \frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$	For x P (x)	½ m
2	$\frac{{}^2C_2 \times {}^5C_0}{{}^7C_2} = \frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$	For x^2 P (x)	½ m

$$\sum x\ P(x) = \frac{24}{42};\ \sum x^2\ P(x) = \frac{28}{42}$$ 1 m

$$\text{Mean} = \sum x\ P(x) = \frac{4}{7};\ \text{var}\,iance = \sum x^2\ P(x) - \left[\sum x\ P(x)\right]^2$$ 1 m

$$\text{Variance} = \frac{50}{147} \qquad = \frac{2}{3} - \frac{16}{49} = \frac{50}{147}$$