QUESTION PAPER CODE 65/2/P

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\vec{a}\cdot\left(\vec{b}\times\vec{a}\right)=\left[\vec{a}\ \ \vec{b}\ \ \vec{a}\right]=0$ 1 m

2. $\vec{a}+\vec{b}=3\hat{i}+3\hat{j}$ ½ m

$\left(\vec{a}+\vec{b}\right)\cdot\vec{c}=3$ ½ m

3. $\frac{x+3}{0}=\frac{y-4}{3}=\frac{z-2}{-1}$ ½ m

D.Rs are 0, 3, –1 ½ m

4. $|A|=-19$ ½ m

$A^{-1}=-\frac{1}{19}\begin{pmatrix}-2 & -5\\ -3 & 2\end{pmatrix}$ ½ m

5. $\frac{dy}{dx}=c$ ½ m

$y=x\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^2$ ½ m

6. $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$ ½ m

I.F. $= e^{\tan^{-1}y}$ ½ m

SECTION - B

7. Let y $=\cos^{-1}\left(\frac{x-x^{-1}}{x-x^{-1}}\right)=\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$ 1 m

$= \pi - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ 1 m

$= \pi - 2\tan^{-1}x$ 1 m

$\therefore \frac{dy}{dx} = -\frac{2}{1+x^2}$ 1 m

8. Let $y = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} + x^x$

Let $u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\}$; $v = x^x$

$\therefore y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ½ m

$u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} = \cos^{-1}\left[\cos\cdot\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right]$ ½ m

$= \frac{\pi}{2} - \sqrt{\frac{1+x}{2}}$

$\therefore \frac{du}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}}$ .... (i) ½ m

$v = x^x$

$\therefore \log v = x \log x$

$\frac{1}{v}\frac{dv}{dx} = x\cdot\frac{1}{x} + 1\log x = 1 + \log x$

$$\frac{dv}{dx} = x^x (1 + \log x) \text{ ....(ii)}$$ 1½ m

$$\therefore \frac{dy}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}} + x^x (1 + \log x)$$ ½ m

$$\left(\frac{dy}{dx}\right)_{at\ x=1} = -\frac{1}{4} + 1 = \frac{3}{4}$$ ½ m

9. $$I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx \text{ .... (i)}$$

$$= \int_0^{\pi/2} \frac{2^{\sin\left(\frac{\pi}{2}-x\right)}}{2^{\sin\left(\frac{\pi}{2}-x\right)} + 2^{\cos\left(\frac{\pi}{2}-x\right)}} dx \left[\text{using} \int_0^a (x)\,dx = \int_0^a f(a-x)\,dx\right]$$ 1½ m

$$= \int_0^{\pi/2} \frac{2^{\cos x}}{2^{\sin x} + 2^{\cos x}} dx \text{ ....(ii)}$$ 1 m

Adding (i) and (ii),

$$2I = \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$ 1 m

$$\Rightarrow I = \frac{\pi}{4}$$ ½ m

OR

$$I = \int_0^{3/2} |x \cos(\pi x)|\,dx$$

$$= \int_0^{1/2} x \cos \pi x\,dx - \int_{1/2}^{3/2} x \cos \pi x\,dx$$ 1 m

$$= \left[\frac{x \sin \pi x}{\pi}\right]_0^{1/2} - \int_0^{1/2} \frac{\sin \pi x}{\pi} dx - \left[\frac{x \sin \pi x}{\pi}\right]_{1/2}^{3/2} + \int_{1/2}^{3/2} \frac{-\sin \pi x}{\pi} dx$$ 1½ m

$$= \frac{1}{2\pi} + \frac{1}{\pi^2}\left[\cos \pi x\right]_0^{1/2} + \frac{3}{2\pi} + \frac{1}{2\pi} + \frac{1}{\pi^2}\left[\cos \pi x\right]_{1/2}^{3/2}$$

$$= \frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{3}{2\pi} + \frac{1}{2\pi} + 0$$ 1 m

$$= \frac{5}{2\pi} - \frac{1}{\pi^2}$$ ½ m

10. $\begin{matrix} A \\ B \\ C \end{matrix}\begin{pmatrix} 25 & 12 & 34 \\ 22 & 15 & 28 \\ 26 & 18 & 36 \end{pmatrix}\begin{pmatrix} 20 \\ 15 \\ 5 \end{pmatrix}$ 1½ m

$$= \begin{pmatrix} 850 \\ 805 \\ 970 \end{pmatrix}$$ 1½ m

Any relevant value 1 m

11. $\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left\{\frac{1-\frac{a-b}{a+b}\tan^2\frac{x}{2}}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}\right\}$ 1½ m

$$= \cos^{-1}\left\{\frac{a+b-a\tan^2\frac{x}{2}+b\tan^2\frac{x}{2}}{a+b+a\tan^2\frac{x}{2}-b\tan^2\frac{x}{2}}\right\}$$ 1 m

$$= \cos^{-1}\left\{\frac{a\left(1-\tan^2\frac{x}{2}\right)+b\left(1+\tan^2\frac{x}{2}\right)}{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}\right\}$$ ½ m

$$= \cos^{-1}\left\{\frac{a\,\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+b}{a+b\,\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\right\}$$ ½ m

$$= \cos^{-1}\left\{\frac{a\cos x+b}{a+b\cos x}\right\}$$ ½ m

OR

$$\tan^{-1}\left(\frac{x-2}{x-3}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-2}{x-3}+\frac{x+2}{x+3}}{1-\frac{x-2}{x-3}\cdot\frac{x+2}{x+3}}\right) = \frac{\pi}{4}$$ 1½ m

$$\Rightarrow \tan^{-1}\left(\frac{2x^2-12}{-5}\right) = \frac{\pi}{4}$$ 1½ m

$$\Rightarrow \frac{2x^2-12}{-5} = 1 \Rightarrow x^2 = \frac{7}{2}$$ ½ m

$$\Rightarrow x = \sqrt{\frac{7}{2}}$$

For writing no solution as $|x| < 1$ ½ m

12. $$A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$ 2 m

$$A^2 - 5A + 16I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix} + \begin{pmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{pmatrix}$$ 1 m

$$= \begin{pmatrix} 11 & -1 & -3 \\ -1 & 9 & -10 \\ -5 & 4 & 14 \end{pmatrix}$$ 1 m

13. Taking x from R_2, x (x – 1) from R_3 and (x + 1) from C_3

$$\Delta = x^2 (x-1)(x+1) \begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ -3 & x-2 & 1 \end{vmatrix}$$ 2 m

$C_2 \to C_2 - x\, C_1$; $C_3 \to C_3 - C$,

$$= x^2 (x^2 - 1) \begin{vmatrix} 1 & 0 & 0 \\ 2 & -1-x & -1 \\ -3 & 4x-2 & 4 \end{vmatrix}$$ 1 m

$$= x^2 (x^2 - 1) \begin{vmatrix} -1(1+x) & -1 \\ 4x-2 & 4 \end{vmatrix}$$ ½ m

$$= 6x^2 (1 - x^2)$$ ½ m

14. $$\frac{dx}{dt} = \alpha\left[-2 \sin 2t \sin 2t + 2 \cos 2t\,(1 + \cos 2t)\right]$$ 1 m

$$\frac{dy}{dt} = \beta\left[2 \sin 2t \cos 2t - (1 - \cos 2t) \cdot 2 \sin 2t\right]$$ 1 m

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) \Big/ \left(\frac{dx}{dt}\right) = \frac{\beta\,(2 \sin 4t - 2 \sin 2t)}{\alpha\,(2 \cos 4t + 2 \cos 2t)}$$ ½+1 m

$$= \frac{\beta}{\alpha} \cdot \frac{2 \cos 3t \sin t}{2 \cos 3t \cos t} = \frac{\beta}{\alpha} \tan t$$ ½ m

15. Here $\begin{vmatrix} b-c-(a-d) & b-a & b+c-(a+d) \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-\gamma & \beta & \beta+\gamma \end{vmatrix}$ 2½ m

$$= 2\begin{vmatrix} b-a & b-a & b+c-a-d \\ \alpha & \alpha & \alpha+\delta \\ \beta & \beta & \beta+\gamma \end{vmatrix} \quad C_1 \to C_1 + C_3$$ ½ m

$= 0 \qquad (\because\ C_1 \text{ and } C_2 \text{ are identical})$ ½ m

Hence given lines are coplanar ½ m

OR

D.R's of normal to the plane are $5, -4, 7$ 1 m

D.R's of y – axis : $0, 1, 0$ ½ m

If θ is the angle between the plane and y–axis, then

$$\sin\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}$$ 1 m

$$= \frac{-4}{3\sqrt{10}}$$ 1 m

$$\therefore\ \theta = \sin^{-1}\left(\frac{-4}{3\sqrt{10}}\right)$$

$$\therefore\ \text{Acute angle is } \sin^{-1}\left(\frac{4}{3\sqrt{10}}\right)$$ ½ m

16. Let E be the event of getting number greater than 4

$\therefore\ P(E) = \frac{1}{3}$ and $P(\bar{E}) = \frac{2}{3}$ ½+½ m

Required Probability $= P\ (\bar{E}\ E \text{ or } \bar{E}\ \bar{E}\ \bar{E}\ E \text{ or } \bar{E}\ \bar{E}\ \bar{E}\ \bar{E}\ \bar{E}\ E \text{ or } ....)$ 1 m

$$=\frac{2}{3}\cdot\frac{1}{3}+\left(\frac{2}{3}\right)^3\cdot\frac{1}{3}+\left(\frac{2}{3}\right)^5\cdot\frac{1}{3}+\ldots\ldots\ldots\ldots\infty$$ 1 m

$$=\frac{2}{9}\left[1+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^4+\ldots\ldots\ldots\ldots\infty\right]$$ ½ m

$$=\frac{2}{9}\times\frac{9}{5}=\frac{2}{5}$$ ½ m

OR

A = {(5, 6, 1), (5, 6, 2), (5, 6, 3), (5, 6, 4), (5, 6, 5), (5, 6, 6),}

$P(A)=\frac{6}{6\times6\times6}=\frac{1}{36}$, P (B) = P (getting 3 or 4 on the third throw) 1½ m

$A\cap B=\{(5,6,3),(5,6,4)\}\ \Rightarrow\ P(A\cap B)=\frac{2}{6\times6\times6}=\frac{1}{108}$ 1½ m

$P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{1}{3}$ 1 m

17. $I=\int\left(\sqrt{\cot x}+\sqrt{\tan x}\right)dx$

$$=\int\frac{\cos x+\sin x}{\sqrt{\sin x\cos x}}dx$$ 1 m

$$=\sqrt{2}\int\frac{(\cos x+\sin x)}{\sqrt{1-(1-2\sin x\cot x)}}dx$$ 1 m

$$=\sqrt{2}\int\frac{\cos x+\sin x}{\sqrt{1-(\sin x-\cos x)^2}}dx$$ ½ m

Put $\sin x-\cos x=t\ \Rightarrow\ (\cos x+\sin x)dx=dt$ ½ m

$$\therefore\ I=\sqrt{2}\int\frac{dt}{\sqrt{1-t^2}}=\sqrt{2}\sin^{-1}t+c$$ ½ m

$$=\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$$ ½ m

18. $I = \int \frac{x^3 - 1}{x(x^2+1)} dx = \int \left(1 - \frac{x+1}{x(x^2+1)}\right) dx$ 1 m

$= x - \int \frac{x+1}{x(x^2+1)} dx$ ½ m

$= x - I_1$

Let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx + C}{x^2+1} = \frac{1}{x} + \frac{1-x}{x^2+1}$ 1 m

$\therefore \; I_1 = \int \frac{1}{x} + \frac{(1-x)}{x^2+1} dx = \log x - \frac{1}{2} \log|x^2+1| + \tan^{-1} x$ 1 m

$\therefore \; I = x - \log|x| + \frac{1}{2} \log|x^2+1| - \tan^{-1} x + c$ ½ m

19. Here $\left.\begin{aligned} \vec{AB} &= -4\hat{i} - 6\hat{j} - 2\hat{k} \\ \vec{AC} &= -\hat{i} + 4\hat{j} + 3\hat{k} \\ \vec{AD} &= -8\hat{i} - \hat{j} + 3\hat{k} \end{aligned}\right\}$ 1½ m

For them to be coplanar, $\left[\vec{AB} \;\; \vec{AC} \;\; \vec{AD}\right] = 0$ 1½ m

i.e. $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -60 + 126 - 66 = 0$ ½ m

$\therefore$ Points A, B, C and D are coplanar ½ m

SECTION - C

20. Let the equation of line be $y = mx + c$ 1½ m

the line is at unit distance from the origin i.e. $\left|\dfrac{0+c}{\sqrt{1+m^2}}\right| = 1 \Rightarrow c = \sqrt{1+m^2}$ 1½ m

$\therefore \quad y = m\,x + \sqrt{1+m^2}$ .... (i) 1 m

$\dfrac{dy}{dx} = m$ 1 m

$\therefore \quad y = x\dfrac{dy}{dx} + \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$ 1 m

OR

$\dfrac{dy}{dx} = \dfrac{x^2+3y^2}{2\,xy} = \dfrac{1+3\left(\dfrac{y}{x}\right)^2}{2\left(\dfrac{y}{x}\right)}$ ....(i) 1 m

Differential equation is homogeneous

Put $y = v\,x$

$\Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ 1½ m

$\therefore \quad v + x\dfrac{dv}{dx} = \dfrac{1+3v^2}{2v}$ 1 m

$\Rightarrow \quad x\dfrac{dv}{dx} = \dfrac{1+v^2}{2v}$

$\Rightarrow \int\left(\dfrac{2v}{1+v^2}\right) dv = \int \dfrac{dx}{x}$ 1 m

$\Rightarrow \log\left|1+v^2\right| = \log|x| + \log c$ 1 m

$\Rightarrow 1+v^2 = c\,x$

$\Rightarrow 1+\left(\dfrac{y}{x}\right)^2 = c\,x \quad \text{or} \quad x^2+y^2 = c\,x^3$ ½ m

21. Equation of plane passing through (1, 0, 0)

$a(x-1)+b(y-0)+c(z-0)=0$

or $ax+by+cz-a=0$ .... (i) 1 m

Plane (i) passes throgh (0, 1, 0)

$b-a = 0$ ....(ii) ½ m

Angle between plane (i) and plane $x+y=3$ is $\frac{\pi}{4}$ ½ m

$\therefore \quad \cos\frac{\pi}{4}=\frac{a+b}{\sqrt{a^2+b^2+c^2}\,\sqrt{2}}$ 1 m

$\Rightarrow \frac{1}{\sqrt{2}}=\frac{a+b}{\sqrt{a^2+b^2+c^2}\,\sqrt{2}}$ 1 m

$\Rightarrow a+b=\sqrt{a^2+b^2+c^2}$

$\Rightarrow 2a=\sqrt{2a^2+c^2}$ (using ii)

$\Rightarrow c= \pm\sqrt{2}\,a$ .... (iii) 1 m

$\therefore$ Equation (i) becomes

$a(x-1)+a(y-0)\pm\sqrt{2}\,a(z-0)=0$

$\Rightarrow x+y\pm\sqrt{2}\,z-1 = 0$ ½ m

D.R's of the normal is $1, 1, \pm\sqrt{2}$ ½ m

22. Let $y=(fog)(x)$ [say $y=h(x)$]

$= f[g(x)] = f(x^3+5)$ 2½ m

$= 2(x^3+5)-3$

$= 2x^3+7$ 2½ m

$\therefore \quad x = \sqrt[3]{\dfrac{y-7}{2}} = h^{-1}(y)$ ½ m

$\therefore \quad (fog)^{-1} = \sqrt[3]{\dfrac{x-7}{2}}$ ½ m

OR

Let (x, y) be the identity element in Q × Q, then

$(a, b) * (x, y) = (a, b) = (x, y) * (a, b) \ \forall \ (a, b) \in Q \times Q$ 1½ m

$\Rightarrow \quad (ax, b + ay) = (a, b)$

$\Rightarrow \quad a = ax \ \text{ and } \ b = b + ay$

$\Rightarrow \quad x = 1 \ \text{ and } \ y = 0$ 1 m

$\therefore$ (1, 0) is the identity element in Q × Q ½ m

Let (a, b) be the invertible element in Q × Q, then

there exists $(\alpha, \beta) \in Q \times Q$ such that

$(a, b) * (\alpha, \beta) = (\alpha, \beta) * (a, b) = (1, 0)$ 1½ m

$\Rightarrow \quad (a\alpha, b + a\beta) = (1, 0)$ 1 m

$\Rightarrow \quad \alpha = \dfrac{1}{a}, \ \beta = -\dfrac{b}{a}$

$\therefore$ the invertible element in A is $\left(\dfrac{1}{a}, -\dfrac{b}{a}\right)$ ½ m

23. $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1, m > 0$

$f'(x) = 6x^2 - 18mx + 12m^2$ 1 m

$f''(x) = 12x - 18m$ 1 m

For Max. or minimum, $f'(x) = 0 \Rightarrow 6x^2 - 18mx + 12m^2 = 0$

$\Rightarrow (x - 2m)(x - m) = 0$

$\Rightarrow x = m \text{ or } 2m$ 1 m

At $x = m,\ f''(x) = 12m - 18m = -\text{ve} \Rightarrow x = m$ is a maxima 1 m

At $x = 2\,m,\ f''(x) = 24m - 18m = +\text{ve} \Rightarrow x = 2m$ is manimum 1 m

$\therefore\quad p = m$ and $q = 2\,m$ ½ m

Given $p^2 = q \Rightarrow m^2 = 2\,m \Rightarrow m^2 - 2\,m = 0$

$\Rightarrow m = 0, 2$

$\Rightarrow m = 2$ as $m > 0$ ½ m

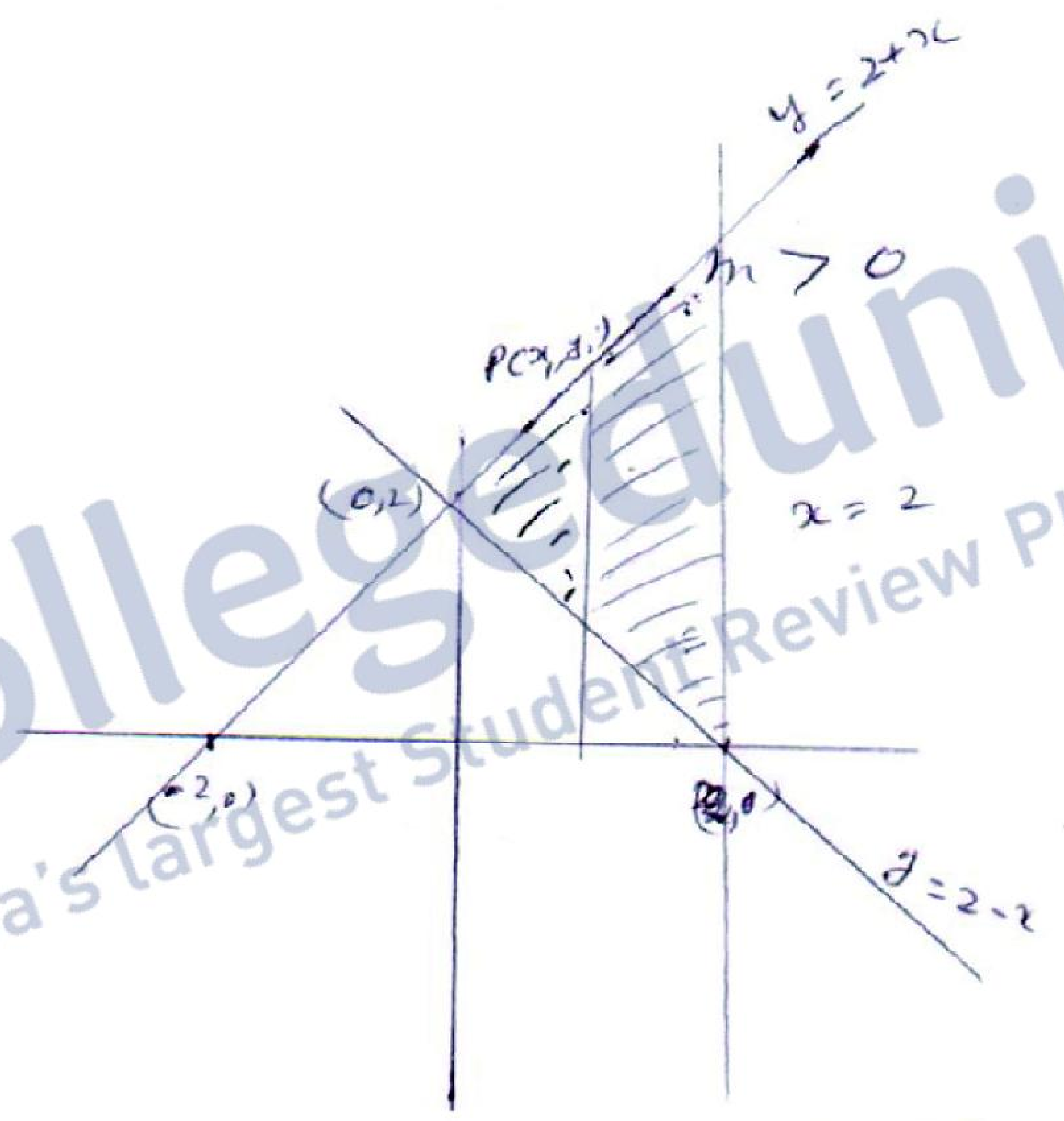
24. $y = 2 + x$ (i)

$y = 2 - x$ (ii)

$x = 2$ (iii),

y_1 is the value of y from (i)

and y_2 is the value of y from (ii)



Required Area $= \int_0^2 (y_1 - y_2)\,dx$ 1 m

correct graph 1+1+1 m

$= \int_0^2 \{(2 + x) - (2 - x)\}\,dx$ correct shading 1 m

$= 2\int_0^2 x\,dx = 2\left[\frac{x^2}{2}\right]_0^2$ ½ m

$= 4$ sq. units ½ m

25. Let x be the man helpers and y be the woman helpers

Pay roll : $Z = 225\,x + 200\,y$ 1 m

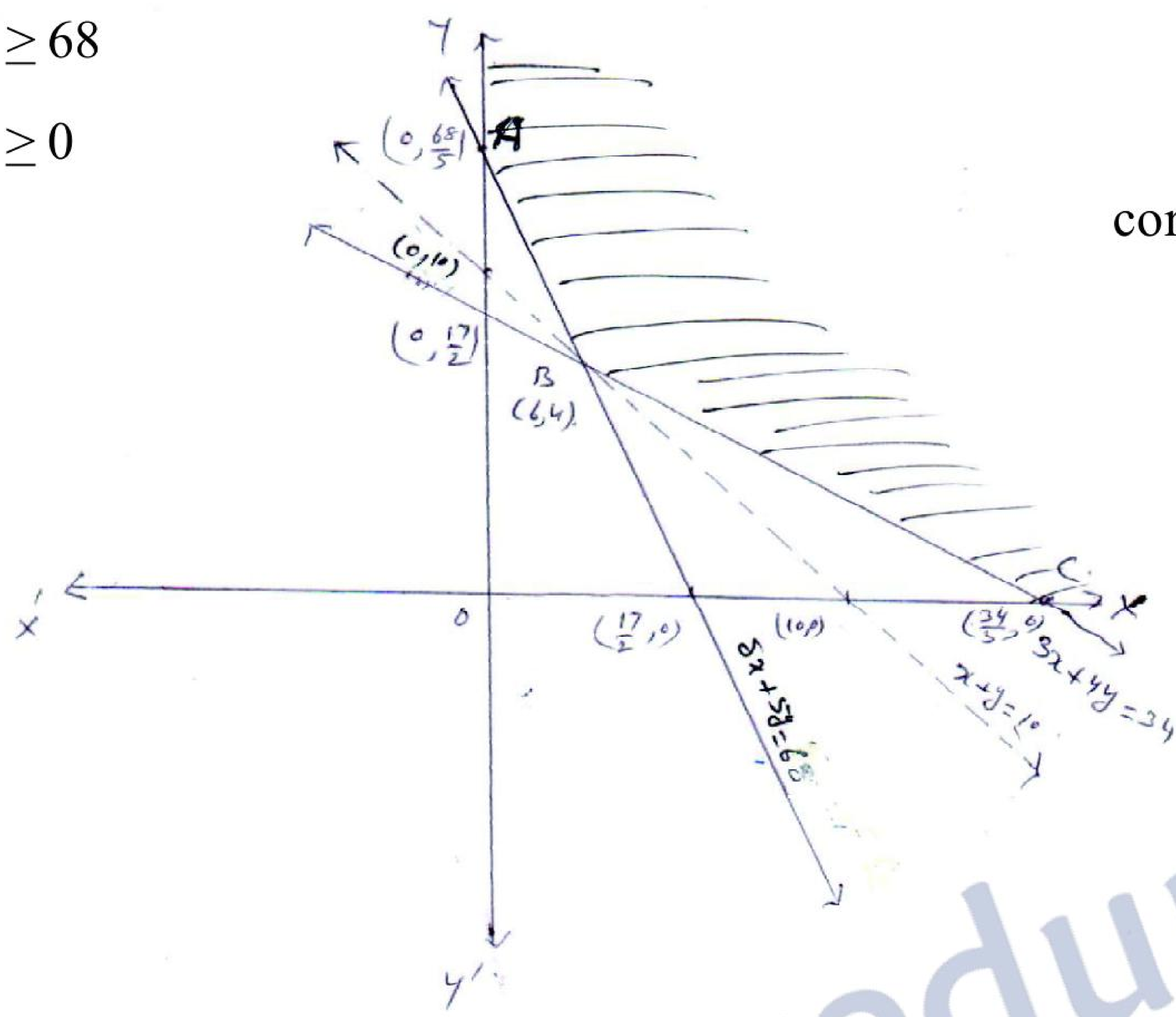
Subject to constraints :

$x + y \leq 10$

$3x + 4y \geq 34$ ½×4 =2 m

$8x + 5y \geq 68$

$x \geq 0, y \geq 0$



correct graph : 2 m

At A $\left(0, \frac{68}{5}\right)$, Z (A) = Rs. 2720

At B (6, 4), Z (B) = Rs. 2150 Minimum ½ m

At C $\left(\frac{34}{5}, 0\right)$, Z (C) = Rs. 2550

Minimum Z = Rs. 2150 at (6, 4) ½ m

[Feasible region is unbounded and to check minimum

of Z, $225x + 200y < 2150$

corresponding line is outside of the shaded region]

26. Let E_1, E_2 and E be the events such that

E_1 : students residing in hostel

E_2 : students residing outside hostel 1½ m

E_3 : students getting 'A' grade

$\therefore \quad P(E_1) = \frac{40}{100}, \quad P(E/E_1) = \frac{50}{100}$

2 m

$P(E_2) = \frac{60}{100}, \quad P(E/E_2) = \frac{30}{100}$

$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \ P(E/E_2)}$ 1 m

$= \frac{\frac{40}{100} \times \frac{50}{100}}{\frac{40}{100} \times \frac{50}{100} + \frac{30}{100} \times \frac{60}{100}}$ 1 m

$= \frac{10}{19}$ ½ m