

NATIONAL ENTRANCE SCREENING TEST (NEST – 2016)

SET A

Total Marks: 200

Time: 3 hours

General instructions

1. This question booklet contains five sections. Each section carries 50 marks.
2. Section 1 is the General section. Sections 2 to 5 are for the subjects Biology, Chemistry, Mathematics and Physics, respectively. Attempt the General section **and any three** of the four subject sections.
3. Read the instructions given at the beginning of each section carefully.
4. **Calculator, cell phone, log table, etc. are NOT permitted in the examination hall.**
5. Answers to the questions are to be marked on the **OMR sheet provided**.
6. **Please make sure that the question booklet code (A or B) matches with the OMR sheet code (A or B).** In case of any discrepancy, inform the invigilator immediately.
7. Return the OMR sheet to the invigilator at the end of the examination.

Instructions for writing on OMR sheet

1. Read and follow the instructions given on the OMR sheet.
2. As far as possible, fill in the answers only after you are sure that you do not need to change them. In case you do have to change the answer after filling, erase the mark completely so that no black spot is left inside the bubble.
3. **Check that you are filling the correct answers for the correct section on the OMR sheet.**
4. **Your examination number (as given in the admit card) MUST BE ENTERED CORRECTLY. If entered incorrectly or not entered at all, the OMR sheet shall be treated as invalid and shall not be assessed.**

Some useful constants

Planck constant	h	6.63×10^{-34} Js
Universal gas constant	R	8.31 Jmol ⁻¹ K ⁻¹
Permittivity of free space	ϵ_0	8.85×10^{-12} C ² N ⁻¹ m ⁻²
Magnitude of charge of an electron	e	1.60×10^{-19} C
Mass of the electron	m_e	9.11×10^{-31} kg
Speed of light in vacuum	c	3.00×10^8 ms ⁻¹
Rydberg constant	R_∞	1.10×10^7 m ⁻¹
Refractive index of water w.r.t. air	${}_a\mu_w$	$\frac{4}{3}$

SPACE FOR ROUGH WORK

Section 1: GENERAL

Marks for Section 1: 50

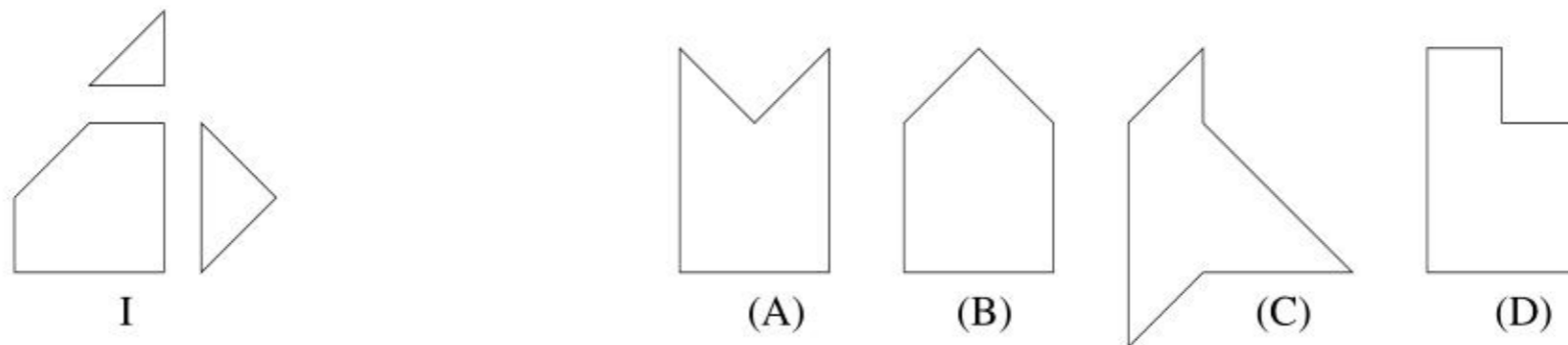
This section contains 15 questions.

For each question, **only one** of the four options is a correct answer. For questions, 1.1 to 1.10, a correct answer will earn 3 marks. For questions, 1.11 to 1.15, a correct answer will earn 4 marks. For this **GENERAL** section, a wrong answer or an unattempted question will earn 0 marks.

1.1 The lengths of all sides of triangle ABC are in integral units. If $l(AB) = 10$ units and $l(BC) = 15$ units, the number of distinct possible values for $l(AC)$ are

- (A) 5 (B) 19 (C) 21 (D) 25

1.2 Which of the following figures **cannot** be made from the pieces in figure I by arranging them side-by-side?



1.3 The recently published results of the LIGO experiment are related to

- (A) sequencing of the human genome. (B) discovery of gravitational waves.
(C) comprehensive model of geomagnetism. (D) identification of greenhouse gasses.

1.4 To which human activity is the poet, John Milton, referring in the lines below:

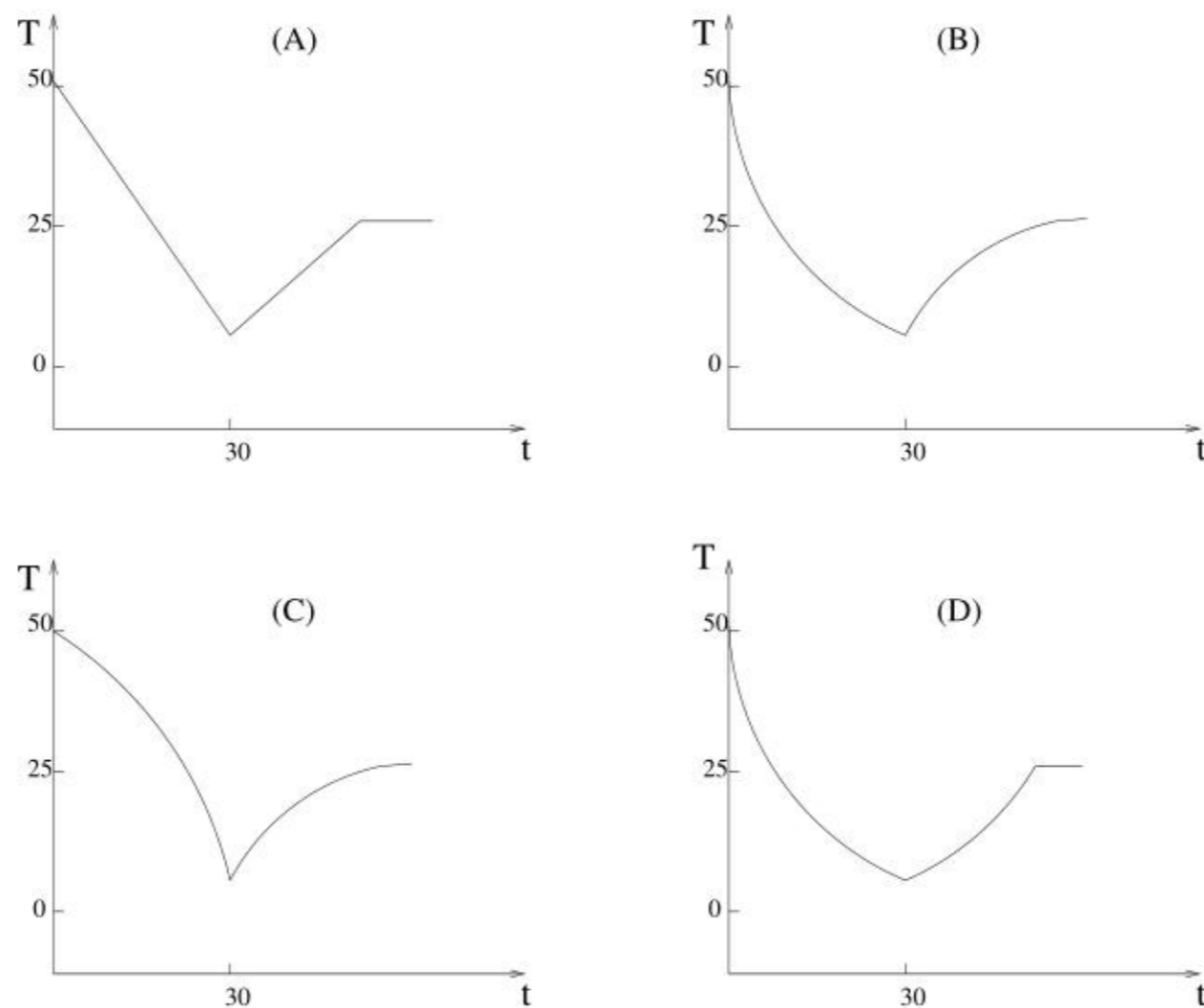
... By him first
Men also, and by his suggestion taught,
Ransacked the centre, and with impious hands
Rifled the bowels of their mother Earth
For treasures better hid. Soon had his crew
Opened into the hill a spacious wound
And digged out ribs of gold

- (A) Laying railway tracks. (B) Clearing forests.
(C) Guerilla warfare. (D) Mining.

1.5 In the product of two two-digit numbers ($_7$) \times ($7_$), there is same digit at both the blank spaces. A possible answer to this multiplication can be

- (A) 807 (B) 1844 (C) 2701 (D) 6686

1.6 A glass of water at 50°C is placed in a refrigerator (maintained at 0°C) for half an hour and then brought out and kept in a room at 25°C . Which of the panels in the figure below best describes the temperature variation of the water as a function of time (in minutes)? Note that the rate of change of temperature of a body is proportional to the difference between the temperature of the body and temperature of the surroundings.



1.7 There was a depository of gold bars at the centre of a bank vault, which was protected by seven concentric, concrete walls with exactly one door in each wall. Each door was protected by a guard. One day a man made a deal with the first guard, which allowed him to pass on the condition that he would give the guard half of the gold bars that he was carrying back with him. However, the guard was required to return one gold bar to him. He made the same deal with the remaining guards at each door. How many gold bars should he have originally taken if he ended up getting two for himself?

- (A) 2 (B) 6 (C) 66 (D) 130

1.8 A protective protein that is produced in humans in response to a bacterial infection is called

- (A) Haemoglobin. (B) Antigen. (C) Antibody. (D) Serum albumin.

1.9 Suppose you wanted to design your own temperature scale based on methyl alcohol (methanol). On the Celsius scale, methanol has a melting point of -96°C and a boiling point of 64°C , but on your new scale calibrated in units of degrees methanol, $^{\circ}\text{M}$, you define methanol to melt at 0°M and boil at 200°M . On the methanol scale, the melting and boiling points of water are

- (A) 0°M and 200°M (B) 60°M and 222.5°M
 (C) 76.8°M and 228.8°M (D) 120°M and 245°M

1.10 The Chicxulub meteor impact event, believed to have caused the mass-extinction of the Cretaceous dinosaurs, is estimated to have occurred

- (A) 100 million years ago. (B) 78 million years ago.
 (C) 65 million years ago. (D) 43 million years ago.

For rest of the questions in this section, each correct answer will earn 4 marks. Analyse the following data table carefully and answer questions 1.11 and 1.12.

In the game of cricket, a test match is played between teams of two countries, where the home team (H. T.) hosts the matches in its country. The table below gives the statistics of the teams that won the test matches after winning the toss, for the period January 2001 to December 2015. The table is grouped into two parts. Columns 2 and 3 list match won/ match lost record for the home team, when the home team won the toss. Columns 4 and 5 list match won/ match lost record for the guest team (G. T.), when the guest team won the toss. Drawn matches are not included in the win/loss ratio.

Hosts	Toss Won - H. T.		Toss Won - G. T.	
	H.T. Won	H. T. Lost	G. T. Won	G. T. Lost
Australia	28	6	4	27
Pakistan	12	3	7	10
India	15	4	5	20
England	28	9	13	24
Sri Lanka	14	5	12	15
South Africa	21	10	8	16
New Zealand	8	6	9	8
West Indies	7	14	14	6

1.11 When playing at home, the teams which performed equally well irrespective of the outcome of the toss are

- (A) India and South Africa. (B) South Africa and Sri Lanka.
 (C) India and New Zealand. (D) Australia and West Indies.

1.12 The team that seems to lose equally frequently regardless of whether it wins or loses the toss at home is

- (A) West Indies. (B) Australia. (C) India. (D) South Africa.

Read the following passage carefully and answer questions 1.13 to 1.15.

The 'Milgram experiment' was a series of psychology experiments conducted by Yale University psychologist Stanley Milgram. Milgram described results of his experiments in his book, 'Obedience to Authority: An Experimental View'. In each setup, three individuals were involved: the researcher who was running the experiment, a volunteer who was assigned the role of a teacher, and an actor pretending to be another volunteer, who would assume the role of a learner. The teacher was then given a list of word pairs that he was to teach the learner, who was sitting in another room. The teacher would read the first word of each pair and read four possible answers. The learner would press a button to indicate his response. The teacher was instructed to administer a shock to the learner, if the answer was incorrect. The level of shock was increased in steps of 15 V with each wrong answer.

The teacher believed that the learner was receiving actual shocks. In reality, no shocks were given. The 'learner' had set up a tape recorder, which played prerecorded sounds for each shock level. After the voltage was increased beyond 135 V, the actor would start to bang on the wall that separated him from the teacher and would complain about his heart condition. After several such high voltage shocks all responses by the learner would stop.

When the 'teachers' heard the screams of pain coming from the learner, many of them indicated their desire to stop the experiment and check on the learner. Some 'teachers' began to question the purpose of the experiment. If at any time the teacher indicated his desire to halt the experiment, he was ordered by the researcher to continue. Most teachers continued after being assured that they would not be held responsible, but they began to exhibit signs of extreme stress like sweating, stuttering, biting their lips, and even having nervous laughing fits. If after four orders to continue, the teacher still wished to stop, the experiment was halted. Otherwise, it was halted after the teacher had given the maximum shock of 450 V three times in succession.

Before conducting the experiment, Milgram asked many psychology students and professors to predict the behaviour of 100 teachers. All of the poll respondents believed that only a very small fraction of teachers (three or less out of 100) would be prepared to inflict the maximum voltage. In Milgram's first set of experiments, 26 out of 40 teachers administered the final 450 V shock. Later, Milgram and other psychologists performed variations of the experiment throughout the world, with similar results.

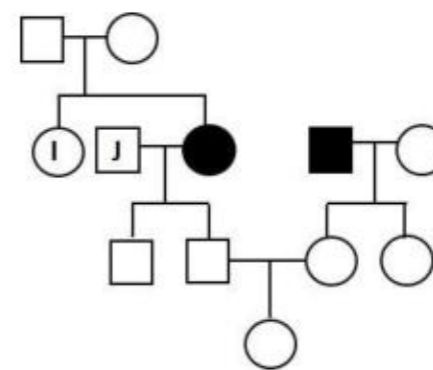
Section 2: Biology

Marks for Section 2: 50

This section contains 15 questions.

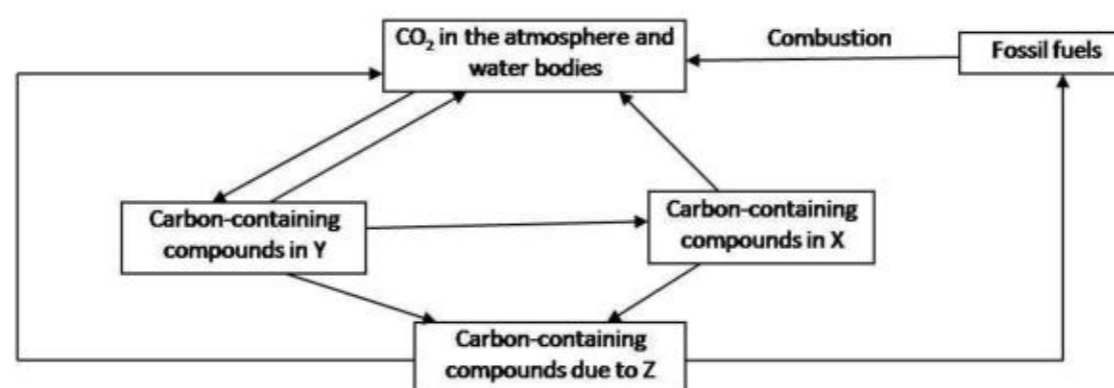
For questions 2.1 to 2.10, **only one** of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

- 2.1 A pedigree chart of a family with an autosomal recessive disease Z is given. Assume that inheritance is Mendelian, and that all individuals with homozygous recessive genotype have the disease. Circles in the pedigree represent females and squares represent males. Filled shapes indicate affected individuals, while unfilled shapes indicate unaffected individuals. If J does not have the recessive allele, the probability that I is a 'carrier of disease Z' is



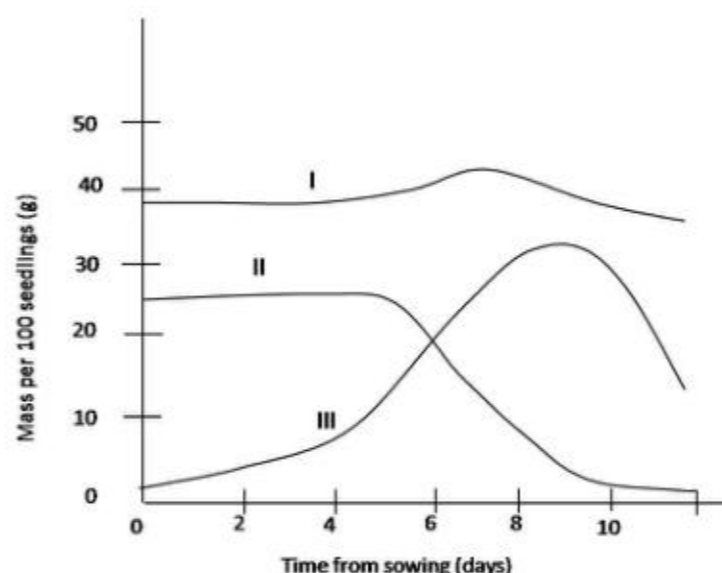
- (A) 0.67 (B) 0.5 (C) 0.25 (D) 0
- 2.2 Which of the following pairs is an **incorrect** match?
- (A) Transcriptional repression — gene regulation
(B) Post-translational modifications — phosphorylation
(C) Cytoplasm — exon splicing of eukaryotic pre-mRNA
(D) Enhancers — increased transcription

- 2.3 A simplified model of the carbon cycle is depicted below.



- X, Y and Z in the figure could represent
- (A) X: rice plants, Y: locusts and Z: bacteria.
(B) X: goats, Y: grass and Z: fungi.
(C) X: saprophytic bacteria, Y: horses and Z: shrubs.
(D) X: cows, Y: saprophytic fungi and Z: bacteria.

2.4 Changes in certain parameters of the Castor-oil seed during germination in the dark are graphically represented. The curves I, II and III could indicate



- (A) I: Embryo, II: sugar content and III: endosperm.
 (B) I: Dry mass of endosperm, II: sugar content and III: lipid content.
 (C) I: Total dry mass, II: lipid content and III: sugar content.
 (D) I: Embryo, II: lipid content and III: total dry mass.

2.5 Students in a biology laboratory were asked to view certain cells microscopically and measure the size of a single cell. They were given a compound microscope with an eyepiece of 15X magnification and an objective lens of 10X magnification. The eyepiece has a scale with a least count of $10\mu\text{m}$. When the students focused the cells under the given microscope, they found that one cell measured 60 divisions on the eyepiece scale. What is the approximate size of that cell?

- (A) $0.47\mu\text{m}$ (B) $2.8\mu\text{m}$ (C) $4.0\mu\text{m}$ (D) $24\mu\text{m}$

2.6 Three features of nitrogenous excretory products (M, N and O) of animals are tabulated below.

Features of excretory products	M	N	O
Toxicity	High	Very low	Low
Energy cost to produce	Low	Moderate	High
Water loss to excretion	High	Moderate	Low

The animals that secrete M, N and O as their major excretory product would be

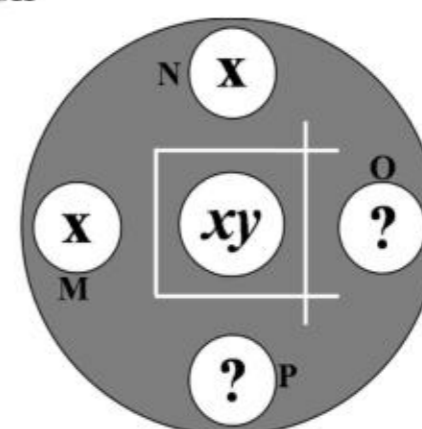
- (A) M: human, N: pigeon, O: tadpole.
 (B) M: cockroach, N: cat, O: snake.
 (C) M: crow, N: lizard, O: dog.
 (D) M: goldfish, N: rat, O: lizard.

2.7 For the peptide sequence (from N to C terminus) Lys-Arg-Ser-Cys-Tyr-Tyr-Trp-His-Lys (KRSCYYWHK), which of the following is the correct coding sequence of the gene encoding this peptide?

- (A) 3'-aaa tac ggt tat tat cgt cga cgc aaa-5'
- (B) 3'-aaa cgc agc tgc tat tat tgg cat aaa-5'
- (C) 3'-ttt gtagta gtc gta gca act gcg ctt-5'
- (D) 3'-ttc gcg tca acg ctg ctg att gcg ttc-5'

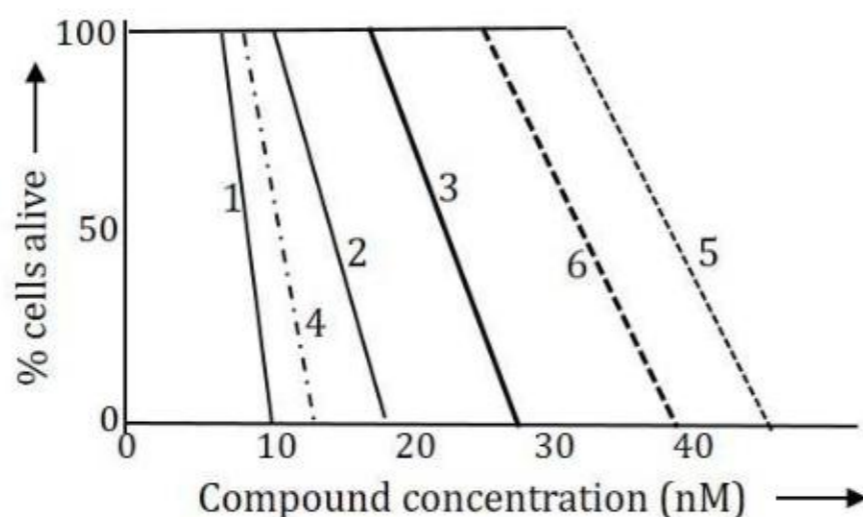
2.8 A glass plate is coated with a layer of agarose gel. A central well

and four surrounding wells (M, N, O, P) are cut on the agarose as shown in the figure on the right side. A mixture of x and y antibodies is added in the central well that react specifically with antigens X and Y, respectively. Wells M and N contain the antigen X. Wells P and O contain antigens of unknown identity. All the antigens and antibodies can diffuse through the agarose. The plate is then incubated for 48 hours. White lines of precipitates are formed in between the antigen and antibody wells as shown in the figure. Which of the following is likely to cause this?



- (A) antigen Y in both O and P
- (B) antigen X in both O and P
- (C) antigen X in O and Y in P
- (D) antigen Y in O and X in P

2.9 Three promising candidate compounds (X, Y and Z) were analysed in a drug-screening project to identify a novel potent compound against lung cancer. The following graph shows efficacies of these compounds in killing cancerous and normal cells.



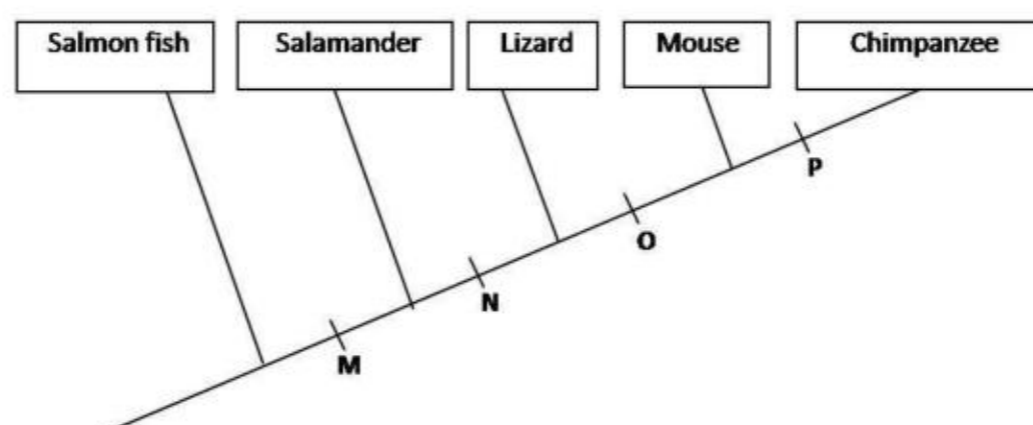
Conditions:

1. Cancer cells + Compound X
2. Cancer cells + Compound Y
3. Cancer cells + Compound Z
4. Normal cells + Compound X
5. Normal cells + Compound Y
6. Normal cells + Compound Z

Based on the above data, which of the following has the best prospect to be a useful drug?

- (A) Compound X
- (B) Compound Y
- (C) Mixture of compounds X and Y
- (D) Compound Z

2.10 Cladograms are branched diagrams representing evolutionary relationships. A cladogram with representative animals is shown below.



The traits M, N, O and P would represent:

- (A) M: Nails, N: Jaws, O: Fur, P: Opposable thumb
- (B) M: Lungs, N: Vertebral column, O: Bipedal locomotion, P: Mammary glands
- (C) M: Vertebral column, N: Jaws, O: Mammary glands, P: Nails
- (D) M: Lungs, N: Claws, O: Fur, P: Opposable thumb

For questions 2.11 to 2.15, one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 4 marks, a wrong answer or an unattempted question will earn 0 marks.

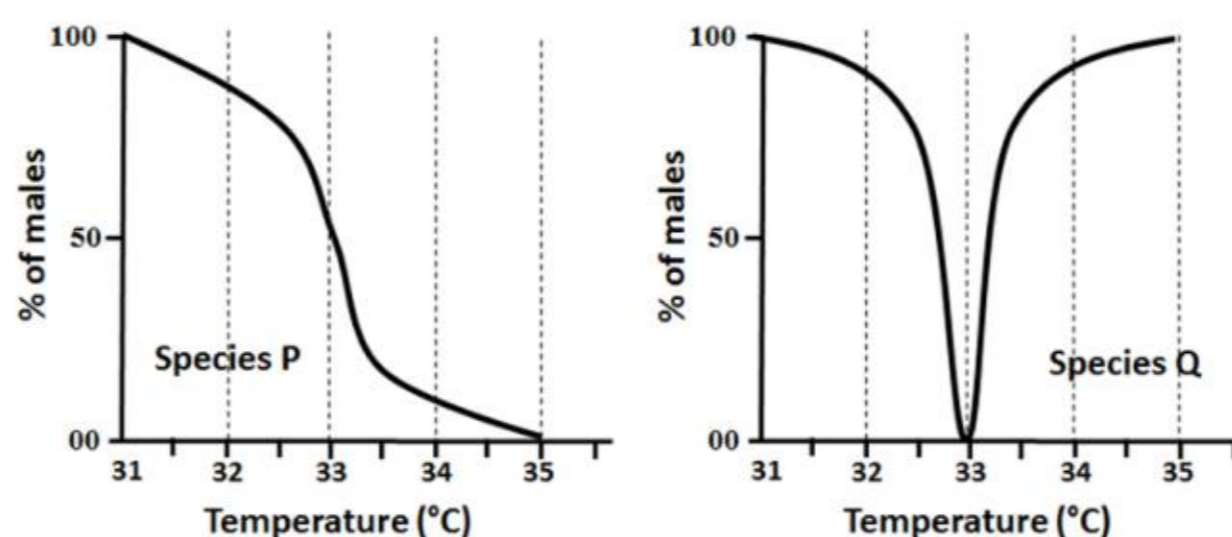
2.11 In order to study the lysine biosynthetic pathway in yeast, lysine auxotrophs (mutants that are unable to grow without lysine supplementation to the minimal growth medium) were screened. Four mutants (m1 to m4) were isolated, which were all recessive compared to the wild-type (WT). To find out the number of genes affected by these mutations, haploid mutants were mated and the ability of the resultant diploid progeny to grow on medium lacking lysine was tested. The results are shown in the table below.

	m1	m2	m3	m4	WT
m1	-	+	-	+	+
m2		-	+	-	+
m3			-	+	+
m4				-	+

(‘+’ indicates growth and ‘-’ indicates no growth) Based on these results, which of the following statement(s) is/are correct?

- (A) There are three genes affected by the mutations.
- (B) m1 and m3 are mutations on the same gene.
- (C) m2 and m4 are likely to display the same phenotypic defect.
- (D) m1 and m2 are mutations on different genes.

2.12 Two unique reptilian species (P and Q) reproduce in ambient temperature range (31 °C to 35 °C only). A group of scientists found a strange observation regarding the “male to female ratio” as obtained from fertilized eggs artificially hatched within laboratory incubators, as shown in the figure below.



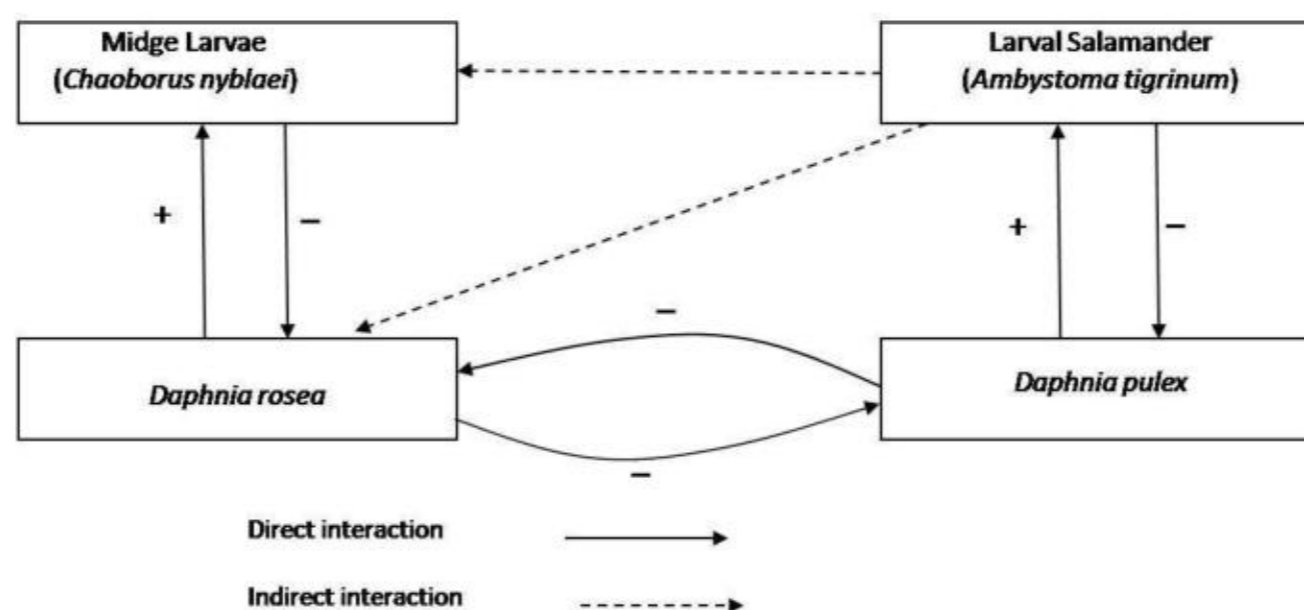
The maximum life span of these reptiles is 5 years. 50 male and 50 female adults each of species P and Q were used to start a 10 year long study. The eggs were always hatched in incubators during this study. In the absence of any other factors, which of the following possible outcome(s) is/are correct?

- (A) If the temperature of the incubator is strictly maintained at 33 °C for 10 consecutive years, the species P is expected to have more males than in species Q.
- (B) If the temperature of the incubator is strictly maintained at 31 °C for 10 consecutive years, there will be more males in species P than in species Q.
- (C) If the temperature of the incubator is strictly maintained at 31 °C for 10 consecutive years, then both P and Q will not survive in the lab.
- (D) If the temperature of the incubator is strictly maintained at 31 °C and 35 °C for every alternate year (for a total of 10 years), there will be more males in species P than species Q.

2.13 Over fifty percent of swallows living on cliffs in Nebraska were killed when a cold spell hit the area in 1996. Scientists collected nearly 2000 dead swallows from the base of the cliffs and captured about 1000 live ones. By measuring the body mass of these birds, they found that birds with larger than average body mass survived the cold spell better than the ones with smaller than average body mass. The data collected suggests that

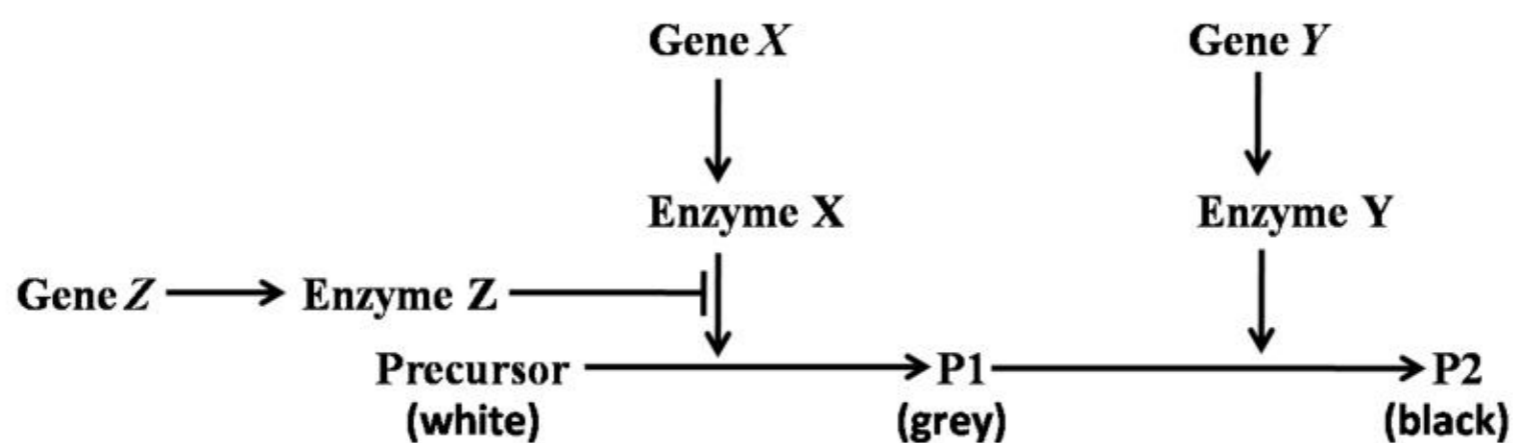
- (A) The population is undergoing natural selection.
- (B) The population is undergoing disruptive selection.
- (C) There is directional selection acting on the population.
- (D) Stabilizing selection is acting on the population.

2.14 The diagram below shows the relationships among the midge larvae (*Chaoborus nyblaei*), larval salamander (*Ambystoma tigrinum*) and two species of the crustacean *Daphnia* (*D. rosea* and the larger *D. pulex*) that inhabit certain pond communities. Study the relationships and deduce which of the following is/are correct.



- (A) Presence of high numbers of larval salamander has an indirect effect on *D. rosea* and it is a positive effect.
- (B) When larval salamanders are present, predation reduces the population growth rate of the larger *Daphnia*, facilitating the coexistence of the two species of *Daphnia*.
- (C) The two species of *Daphnia* compete for the same resources and hence increase in resource availability will reduce the number of midge larvae and larval salamander.
- (D) Increase in the number of *D. pulex* would lead to a decrease in the number of midge larvae.

2.15 Assume that the coat colour in a mammal is dictated by a genetically controlled biosynthetic pathway as depicted in the diagram below



The pigment gene 'X' makes an enzyme 'X' that catalyzes the conversion of the precursor to the grey pigment (P1). The dominant allele 'X' produces the enzyme necessary for this conversion. Similarly a dominant gene 'Y' produces an enzyme 'Y' that controls the

conversion of the P1 to a black pigment (P2). On the other hand, the dominant allele 'Z' produces an active enzyme 'Z' that can completely inhibit the conversion of precursor to P1. In all cases, the recessive alleles respectively produce defective enzymes with no activity. All the three genes assort independently with no interference from other genes. State the colour of a male with the genotype $XXyyZZ$ and a female with the genotype $xxYYzz$.

- (A) Male is black and female is white.
- (B) Both male and female are white.
- (C) Male is white and female is black.
- (D) Male is grey and female is white.

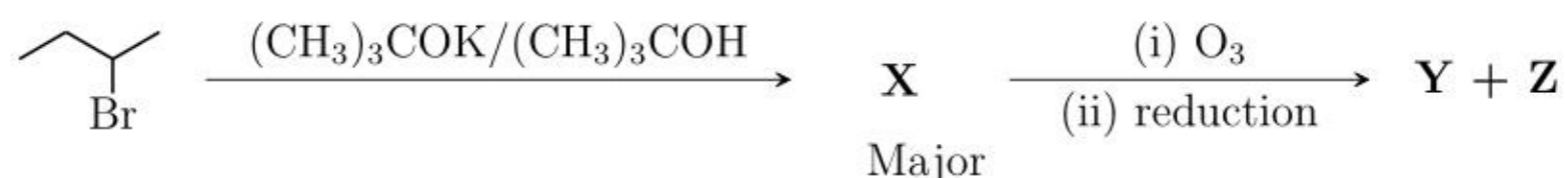
Section 3: Chemistry

Marks for Section 3: 50

This section contains 15 questions.

For questions 3.1 to 3.10, **only one** of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

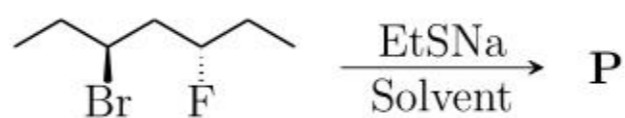
- 3.1** In a base catalyzed elimination reaction the base takes away a proton from the β position of the leaving group (the group that goes away with the bonding pair of electrons) and a double bond is formed. When the base is bulky it prefers to take away the proton from less hindered position.



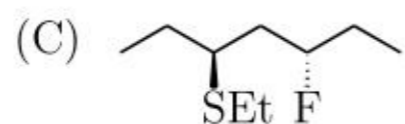
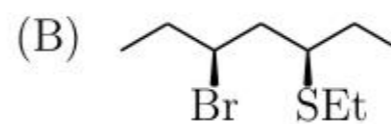
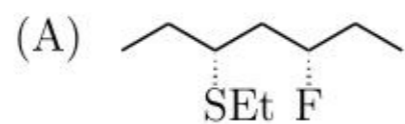
In the above process, **Y** and **Z** are

- (A) two molecules of ethanal.
 - (B) propanal and ethanol.
 - (C) propanoic acid and carbon dioxide.
 - (D) propanal and methanal.
- 3.2** Compound **X** ($\text{C}_4\text{H}_6\text{OCl}_2$) reacts with cold water to form **Y** ($\text{C}_4\text{H}_7\text{O}_2\text{Cl}$). **X** on heating with aq. KOH, followed by acidification, gives **Z** ($\text{C}_4\text{H}_8\text{O}_3$), which can be resolved into enantiomers. **X** is-
- (A) $\text{CH}_3-\text{CCl}_2-\text{CH}_2-\text{CHO}$
 - (B) $\text{ClCH}_2-\text{CH}_2-\text{CH}_2-\text{COCl}$
 - (C) $\text{Cl}-\underset{\text{CH}_3}{\text{CH}}-\text{CO}-\text{CH}_2\text{Cl}$
 - (D) $\text{CH}_3-\text{CH}_2-\text{CHCl}-\text{COCl}$
- 3.3** 4-Amino salicylic acid, called PAS, is an anti-tuberculosis drug. The proper method for its preparation involves a reaction of
- (A) 3-aminophenol with chloroform in an alkaline medium.
 - (B) 3-aminophenol with carbon dioxide in an alkaline medium under pressure.
 - (C) conc. HNO_3 and conc. H_2SO_4 on salicylic acid followed by catalytic reduction using H_2/Ni .
 - (D) chlorine (limited amount) with 4-aminobenzoic acid in the presence of FeCl_3 , followed by boiling the product in aq. alkali.

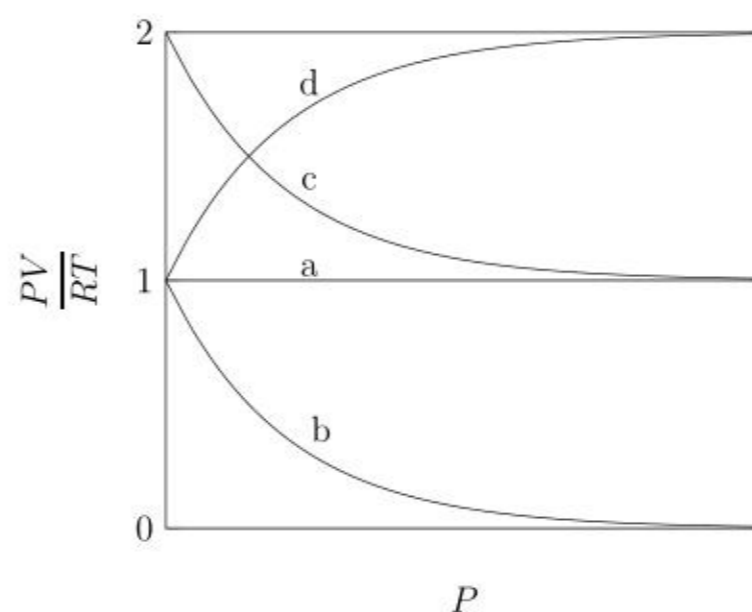
3.4 Consider the following reaction.



The major product (**P**) of the reaction is



3.5 Consider the equilibrium $\text{N}_2\text{O}_4(\text{g}) \rightleftharpoons 2\text{NO}_2(\text{g})$, established from pure $\text{N}_2\text{O}_4(\text{g})$, at a suitable temperature T . Keeping the temperature constant, the equilibrium is studied as a function of the total pressure P of the gas mixture. Let V be the volume of the gas mixture. For one mole as the initial amount of $\text{N}_2\text{O}_4(\text{g})$, the correct plot (among a, b, c, d) in the given figure, assuming ideal gas behaviour, is



- (A) *a* (B) *b* (C) *c* (D) *d*

3.6 An aqueous solution containing a nonvolatile solute is heated in an open vessel till the boiling point is reached. Thereafter, the boiling is continued and the temperature and the vapour pressure of the solution are recorded at 5 min, 10 min, and 15 min. The corresponding temperatures and pressures are T_5, T_{10}, T_{15} and P_5, P_{10}, P_{15} , respectively. The correct relationship among them is:

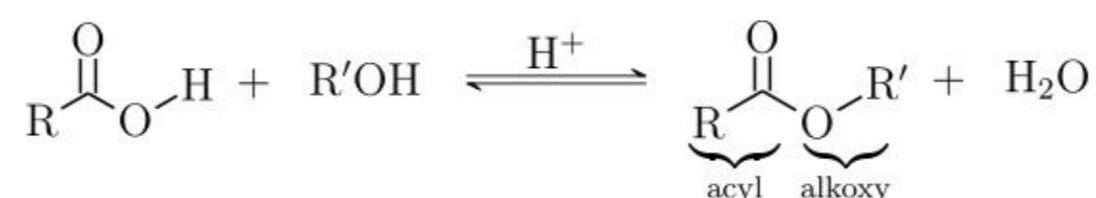
- (A) $T_5 < T_{10} < T_{15}$ and $P_5 = P_{10} = P_{15}$
 (B) $T_5 = T_{10} = T_{15}$ and $P_5 = P_{10} = P_{15}$
 (C) $T_5 = T_{10} = T_{15}$ and $P_5 < P_{10} < P_{15}$
 (D) $T_5 < T_{10} < T_{15}$ and $P_5 < P_{10} < P_{15}$

For questions 3.11 to 3.15, one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 4 marks, a wrong answer or an unattempted question will earn 0 marks.

3.11 Several octahedral complexes are possible from combinations of Co^{3+} , Cl^- and NH_3 . The correct statement(s) regarding the octahedral coordination entities having the formulae $[\text{Co}(\text{NH}_3)_n\text{Cl}_{6-n}]^{(n-3)+}$ with $n \geq 3$, is/are

- (A) At most six octahedral complexes are possible.
- (B) One of the complexes is homoleptic.
- (C) All the complexes are paramagnetic.
- (D) Some of the complexes dissociate in water to give Co^{3+} and Cl^- ions.

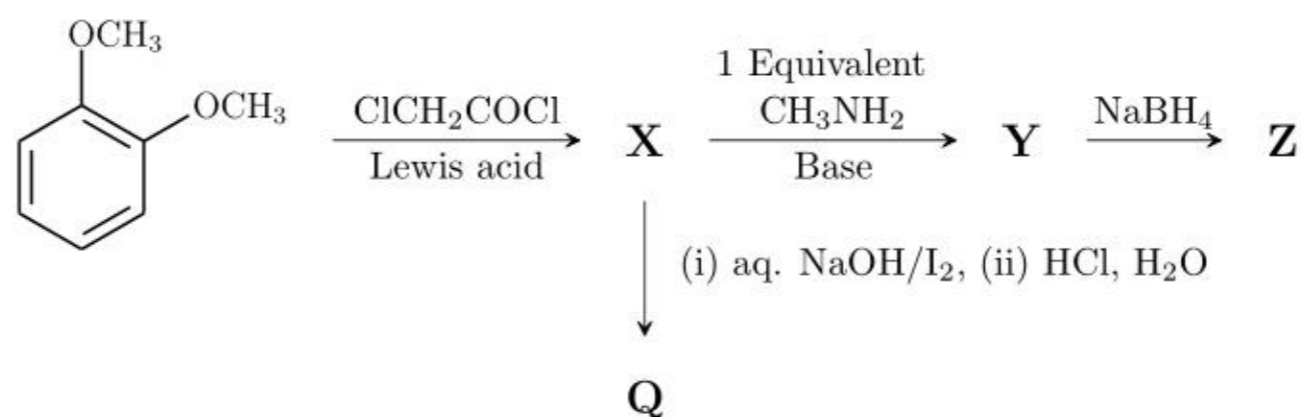
3.12 A carboxylic acid ester is prepared commonly by reacting a carboxylic acid with excess of an alcohol in the presence of a small quantity of a strong acid such as conc. H_2SO_4 as a catalyst. The reaction is reversible and generally the equilibrium constant for the esterification is close to 1. As it is a reversible reaction, different methods are used to increase the yield of ester. It is established that the mechanism of esterification under such conditions involves initial protonation of the acid followed by the attack of the alcohol as a nucleophile.



The correct statement(s) is/are

- (A) When esterification is carried out using molar ratio of acid and alcohol as 1:10, the yield of the ester with respect to the acid should be close to 90%.
- (B) The function of conc. H_2SO_4 is also to increase the free energy change of the reaction and thereby increasing the equilibrium constant.
- (C) When an ester is heated with acidified water the protonation will be more favourable at the acyl oxygen atom than at the alkoxy oxygen atom.
- (D) When butyl hexanoate is heated with a large excess of methanol in the presence of a small quantity of conc. H_2SO_4 , methyl hexanoate is the major product.

3.13 Consider the following sequence of reactions.



The correct statement(s) is / are

- (A) **X** is a mixture of almost equal amounts of two isomers.
- (B) **Y** is a basic compound.
- (C) **Z** is a racemic mixture.
- (D) **Q** is 3,4-dimethoxybenzoic acid.

3.14 The decomposition reaction $\text{X}(\text{g}) \rightarrow 3\text{Y}(\text{g})$ is studied at constant volume and temperature. For pure $\text{X}(\text{g})$ as the starting material, the first few readings of the observed total pressure P as a function of time are given below:

Time (min)	0	1	2	4
Total Pressure P (atm)	8	16	20	P_1

The correct statement(s), assuming ideal gas behaviour, is/are:

- (A) P_1 is 23 atm.
- (B) The total pressure, on completion of reaction, will be 24 atm.
- (C) The partial pressure of Y will exceed that of X in less than 1 min.
- (D) The partial pressures of X and Y will be equal when the total pressure is 12 atm.

3.15 One mole of an ideal gas is expanded irreversibly from an initial pressure of 2 atm to a final pressure of 1 atm at a temperature of 300 K. Let q_{irr} , ΔS_{sys} and ΔS_{surr} represent the magnitude of the heat transfer, the entropy change of the system, and the entropy change of the surrounding respectively. Assuming that the surrounding does not perform any work, the correct relation(s) is/are (R is the universal gas constant)

- (A) $\Delta S_{\text{surr}} = -q_{\text{irr}}/300$
- (B) $\Delta S_{\text{sys}} = R \ln 2$
- (C) $\Delta S_{\text{sys}} = q_{\text{irr}}/300$
- (D) $q_{\text{irr}} < 300 R \ln 2$

Section 4: Mathematics

Marks for Section 4: 50

This section contains 15 questions.

For questions 4.1 to 4.10, only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

4.1 Let A be a 3×2 matrix with real entries. Let $H = A(A^T A)^{-1} A^T$ where A^T is the transpose of A and let I be the identity matrix of order 3×3 . Then

- (A) $H^2 = I$.
- (B) $H^2 = -I$.
- (C) $H^2 = H$.
- (D) $H^2 = -H$.

4.2 The number of terms that are integers in the binomial expansion of $(\sqrt{7} + \sqrt[3]{5})^{35}$ is

- (A) 4
- (B) 5
- (C) 6
- (D) 7

4.3 Suppose $f : [0, \pi] \rightarrow \mathbb{R}$ satisfies $f(x) + f(\pi - x) = 1$ for all x . Then $\int_0^\pi f(x) \sin x \, dx$ is

- (A) $1/4$
- (B) $1/2$
- (C) $3/4$
- (D) 1

4.4 Let an acute-angled triangle ABC be inscribed in a circle whose centre is the origin. Let $B = (3, 4)$ and $C = (-4, 3)$. Then $\angle BAC$ is

- (A) $\pi/5$
- (B) $\pi/4$
- (C) $\pi/3$
- (D) $\pi/2$

4.5 Let positive real numbers x and y be such that $3x + 4y = 14$. The maximum value of $x^3 y^4$ is

- (A) 4
- (B) 128
- (C) 216
- (D) 432

4.6 Let A, G, H be the arithmetic mean, geometric mean and harmonic mean respectively of two distinct positive real numbers. The roots of the quadratic equation $Ax^2 - 2Gx + H = 0$ lie in the interval

- (A) (0,1)
- (B) (1,2)
- (C) (2,3)
- (D) (3,4)

4.7 The number of real solutions of $e^x = 1 + \sin x$ is

- (A) 0
- (B) 1
- (C) 2
- (D) not finite

4.12 Let \vec{a} and \vec{b} be two non-collinear vectors and $|\vec{a}| = |\vec{b}| = 1$. If $\vec{U} = \vec{a} - (\vec{a} \cdot \vec{b})\vec{b}$ then $\sqrt{\vec{U} \cdot \vec{a}}$ is

- (A) $|\vec{a} \times \vec{b}|$ (B) $|\vec{U}|$
 (C) $|\vec{U} \times \vec{a}|$ (D) $|\vec{U} \times \vec{b}|$

4.13 The real and imaginary part of the complex number $1 + \sqrt{i}$ where $i = \sqrt{-1}$ are

- (A) $1 - \frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$ respectively.
 (B) $1 - \frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$ respectively.
 (C) $1 + \frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$ respectively.
 (D) $1 + \frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$ respectively.

4.14 There are 100 students in a class. It is found that 60 students study Mathematics, 45 students study Physics and 35 students study Chemistry. Moreover, 20 students study Mathematics and Physics, 15 students study Physics and Chemistry, 25 students study Chemistry and Mathematics and 10 students study none of these subjects. Then

- (A) 95 students study at least one of Mathematics, Physics and Chemistry.
 (B) 10 students study each of Mathematics, Physics and Chemistry.
 (C) 70 students study either Mathematics or Chemistry.
 (D) 35 students study Mathematics along with either Physics or Chemistry.

4.15 For a given positive integer n let $f : (0, \infty) \rightarrow \mathbb{R}$ be defined as

$$f(x) = x^n \ln x.$$

Then

- (A) $f(x)$ is an increasing function on $(0, \infty)$.
 (B) f has only one zero in $(0, \infty)$.
 (C) $f(x)$ has a minimum at $x = 1/\sqrt[n]{e}$.
 (D) the line $y = x - 1$ is a tangent to the graph of $f(x)$ at the point $(1, 0)$.

Section 5: Physics

Marks for Section 5: 50

This section contains 15 questions.

For questions 5.1 to 5.10, **only one** of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

- 5.1 A certain quaternary star system consists of a central star of mass M and three smaller stars, each of mass m . The three smaller stars orbit the central star in the same circular orbit of radius r and have the same sense of revolution. They are positioned one-third of a revolution apart from one another in the orbit. The period of revolution of each smaller star is

(A) $2\pi\sqrt{\frac{r^3}{G(m+M)}}$

(B) $2\pi\sqrt{\frac{r^3}{G(m+\frac{M}{\sqrt{3}})}}$

(C) $2\pi\sqrt{\frac{r^3}{G(M-m)}}$

(D) $2\pi\sqrt{\frac{r^3}{G(M+\frac{m}{\sqrt{3}})}}$

- 5.2 A uniform block of mass m resting on a rough horizontal plane just starts moving when pulled by a string with a force of magnitude mg making an angle of 45° with the horizontal. Then the block will move if it is

- (A) *pulled* by a horizontal force of magnitude mg .
(B) *pulled* by a force of magnitude mg making an angle of 30° with the horizontal.
(C) *pushed* by a force of magnitude mg making an angle of 45° with the horizontal.
(D) *pulled* by a force of magnitude mg making an angle 60° with the horizontal.

- 5.3 A fan of moment of inertia I is set into rotation at time $t = 0$ with constant power P . Then

- (A) the angular speed of the fan is proportional to t .
(B) the angular acceleration of the fan remains constant.
(C) the magnitude of the angular displacement is proportional to t^2 .
(D) the magnitude of the torque is proportional to $t^{-1/2}$.

- 5.4 Two persons are playing *shehnai* made of the same material. *Shehnai* I is 1.5 times the length of *shehnai* II. Let $f_0^{(I)}$ and $f_0^{(II)}$ be the fundamental frequencies of *shehnai* I and *shehnai* II respectively. Then,

(A) $3f_0^{(II)} = f_0^{(I)}$
(C) $2f_0^{(II)} = 3f_0^{(I)}$

(B) $3f_0^{(II)} = 2f_0^{(I)}$
(D) $f_0^{(II)} = 3f_0^{(I)}$

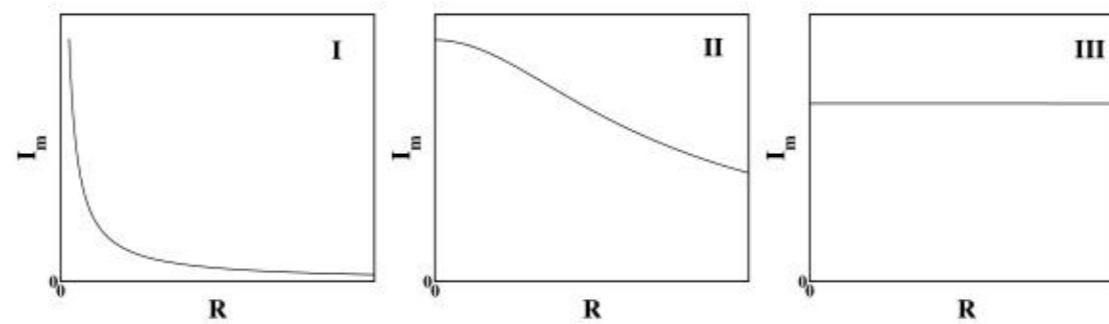
5.5 A hole of radius 2.12 cm is drilled in a metal sheet maintained at 30 °C. The coefficient of linear expansion of the metal is $1.7 \times 10^{-5} \text{ K}^{-1}$. The sheet is now heated to 230 °C. Then,

- (A) area of the hole decreases to $9.31 \times 10^{-2} \text{ cm}^2$.
- (B) the change in the radius of the hole is $7.00 \times 10^{-3} \text{ cm}$.
- (C) area of the hole increases by $6.65 \times 10^{-2} \text{ cm}^2$.
- (D) the change in the radius of the hole is $5.00 \times 10^{-3} \text{ cm}$.

5.6 A sphere, a cylinder and a very thin disk are made out of the same material. Each one of them carry the same amount of charge Q , spread uniformly over their surfaces. The radius of the cylinder is identical to its height. The radii of sphere, cylinder and disk are in the ratio 1:2:3. I, II and III are points in space, very close to the sphere, curved surface of the cylinder (away from the edges) and centre of flat surface of the disk, respectively. Then magnitudes of electric fields at the points I, II and III are approximately in the ratio

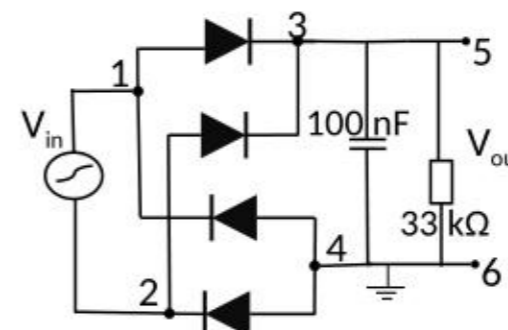
- (A) $1:\frac{1}{4}:\frac{2}{9}$
- (B) $1:\frac{1}{2}:\frac{2}{3}$
- (C) 1:2:3
- (D) 1:4:9

5.7 A series LCR circuit is built with a variable resistor. The resonant frequency of the circuit is ω_0 . The circuit response (I_m - R profile, where, I_m is peak current and R is resistance) is recorded for three different values of frequencies (ω) of input signal, and these are depicted schematically in the accompanying figures, I, II and III. The figures respectively correspond to



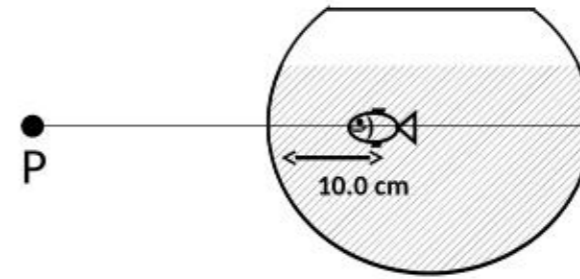
- (A) $\omega = \omega_0$, $\omega \approx \omega_0$ and $\omega \gg \omega_0$.
- (B) $\omega \approx \omega_0$, $\omega = \omega_0$ and $\omega \gg \omega_0$.
- (C) $\omega \approx \omega_0$, $\omega = \omega_0$ and $\omega \ll \omega_0$.
- (D) $\omega = \omega_0$, $\omega \ll \omega_0$ and $\omega \gg \omega_0$.

5.8 Four ideal diodes are connected to form a circuit as shown in the figure. An AC signal $V_{in} = 10 \sin(100\pi t)$ volt is applied across points 1 and 2 and the output is measured across points 5 and 6. The output voltage (V_{out}) is



- (A) $10 \sin(100\pi t) \text{ V}$
- (B) 10 V
- (C) 6.4 V
- (D) 3.2 V

5.9 A fish is 10.0 cm from the front surface of a spherical fish bowl of radius 20.0 cm, as shown in the figure. The walls of the bowl are thin, and their effects can be neglected. As observed from the point P (see figure), the fish is at a distance of



- (A) 6.7 cm from the surface and appears magnified in size.
- (B) 8.6 cm from the surface and appears magnified in size.
- (C) 10.3 cm from the surface and appears diminished in size.
- (D) 11.4 cm from the surface and appears diminished in size.

5.10 In an experiment on photoelectric effect photons of wavelength 300 nm eject electrons from a metal of work function 2.25 eV. A photon of energy equal to that of the most energetic electron corresponds to the following transition in the hydrogen atom:

- (A) $n = 2$ to $n = 1$ state.
- (B) $n = 3$ to $n = 1$ state.
- (C) $n = 3$ to $n = 2$ state.
- (D) $n = 4$ to $n = 3$ state.

For questions 5.11 to 5.15, one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 4 marks, a wrong answer or an unattempted question will earn 0 marks.

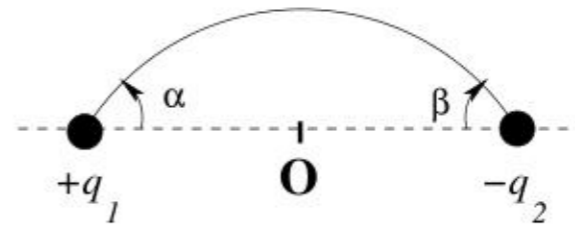
5.11 A particle performs uniform circular motion of radius R and speed v about the origin O . Then the angular speed of the particle about

- (A) O is v/R .
- (B) a point close to the circumference and inside the circle exhibits a sharp maximum.
- (C) a point on the circumference is $2v/R$.
- (D) a point outside the circle is zero at two instants during the motion of the particle around the circle.

5.12 Two bodies each of mass M and temperature independent specific heat C are initially at temperatures 100 K and 400 K. A reversible engine is used to extract heat with the hotter body as the source and cooler body as the sink. Both the bodies attain the same final temperature. Then,

- (A) the final temperature of the two bodies is 250 K.
- (B) the work done by the engine is $100 MC$
- (C) total change in entropy of the system is zero.
- (D) the total heat extracted from the hotter body is $150 MC$

5.13 An electrostatic field line leaves at an angle α from point charge q_1 , and connects with point charge $-q_2$ at an angle β (q_1 and q_2 are positive), as shown schematically in the figure. We have $q_1 = 2q_2$ and $\alpha = 60^\circ$. Then,



- (A) a test charge placed at the midpoint O will be in equilibrium
- (B) $\beta = 90^\circ$
- (C) the magnitude of the electric field at a distance r far away from O (see figure) is $q_2/4\pi\epsilon_0 r^2$.
- (D) for a negative test charge, the electrostatic potential energy is lower near q_1 as compared to that near q_2 .

5.14 A student did an experiment to determine the Young's modulus (Y) of a nylon thread, 2.00 m in length, using Searle's method. A Vernier callipers having a least count of 0.01 mm was used to measure the diameter of the thread. The length and extensions were measured by using a scale with a least count of 1 mm. The data obtained by the student are shown in the table below. Assume the uncertainty in force to be 0.01 N.

Length of the thread	1994 mm
Force	0.32 N
Extension of the thread	95 mm
Diameter of the thread	0.04 mm

Choose the correct alternative(s).

- (A) The student reports Y to be about 5 GPa.
- (B) The reported uncertainty in Y is 54%.
- (C) Measuring the extension using a travelling microscope with a least count of 0.1 mm would improve the accuracy of the result by at most 1%.
- (D) The stress is 1.8 GPa.

5.15 Two speakers are driven by the same oscillator of frequency f . They are located a distance d from each other on a vertical pole. An observer far away walks slowly, straight towards the lower speaker and his ears are at the same level as the lower speaker. Let v represent the speed of sound and assume that there is no reflection from the ground. Let n_{max} be the number of times the observer hears the minimum in sound intensity and let $\lambda = v/f$ be wavelength of the sound emitted. Choose the correct alternative(s).

- (A) $n_{max} \leq \frac{d}{\lambda} + 0.5$.
- (B) A minimum occurs at $\frac{4d^2 + 9\lambda^2}{12\lambda}$.
- (C) $n_{max} \leq \frac{d}{\lambda} - 0.5$.
- (D) A minimum occurs at $\frac{4d^2 - \lambda^2}{4\lambda}$.

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK