**65/6/1**

# QUESTION PAPER CODE 65/6/1
# EXPECTED ANSWER/VALUE POINTS
## SECTION A

*Question numbers 1 to 6 carry 2 marks each.*

**1.** Evaluate:

$$\int_0^{\pi/2} \frac{1}{1+\cot^{5/2} x} dx$$

**Ans.** $I = \int_0^{\pi/2} \frac{1}{1+\cot^{5/2} x} dx = \int_0^{\pi/2} \frac{\sin^{5/2} x}{\sin^{5/2} x + \cos^{5/2} x} dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^{5/2} x}{\sin^{5/2} x + \cos^{5/2} x} dx$ ...(ii) 1

adding (i) and (ii)

$2I = \int_0^{\pi/2} 1 \cdot dx \Rightarrow 2I = x]_0^{\pi/2} = \frac{\pi}{2}$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{4}$ $\frac{1}{2}$

**2.** If $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$ are three vectors, then find a vector perpendicular to both the vectors $(\vec{a} + \vec{b})$ and $(\vec{b} - \vec{c})$.

**Ans.** $\vec{a} + \vec{b} = 3\hat{j}$, $\vec{b} - \vec{c} = 3\hat{k}$ $\frac{1}{2} + \frac{1}{2}$

Vector perpendicular to $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$

$= (3\hat{j}) \times (3\hat{k}) = 9\hat{i}$ 1

**3.** A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, without replacement. Find the probability that the number on each card is a multiple of 7.

**Ans.** Multiples of 7 from 1 to 2 are 7, 14, 21 $\frac{1}{2}$

P (number on each card is a multiple of 7)

$= \frac{3}{25} \times \frac{2}{24} = \frac{1}{100}$ $1\frac{1}{2}$

**4.** One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.

**Ans.** **Case I:** White ball is transferred from bag I to bag II

P(white ball from bag II) = $\frac{4}{9} \times \frac{7}{14}$ $\frac{1}{2}$

**Case II:** Black ball is transferred from bag I to bag II

P(white ball from bag II) = $\frac{5}{9} \times \frac{6}{14}$ $\frac{1}{2}$

Total Probability = $\frac{4}{9} \times \frac{7}{14} + \frac{5}{9} \times \frac{6}{14}$ $\frac{1}{2}$

$= \frac{29}{63}$ $\frac{1}{2}$

**5.** If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$.

**Ans.** $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ 1

$\Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ $\frac{1}{2}$

$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-3}{2}$ $\frac{1}{2}$

**6.** (a) Find the general solution of the differential equation $x \cos y \, dy = (x \log x + 1) e^x \, dx$.

OR

(b) Find the value of (2a – 3b), if a and b represent respectively the order and degree of the differential equation $x\left[y\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}\frac{dy}{dx}\right] = 0$.

**Ans.** $x \cos y \, dy = (x \log x + 1)e^x \, dx$

$\Rightarrow \int \cos y \, dy = \int \left(\log x + \frac{1}{x}\right) \cdot e^x \, dx$ $\frac{1}{2}$

$\Rightarrow \sin y = \log x.e^x + C$ $\left(\because \int [f(x) + f'(x)] \cdot e^x dx = f(x) \cdot e^x + C\right)$ $1+\frac{1}{2}$

OR

order = 2, degree = 3 $1+\frac{1}{2}$

$\therefore$ 2a – 3b = 4 – 9 = –5 $\frac{1}{2}$

## SECTION B

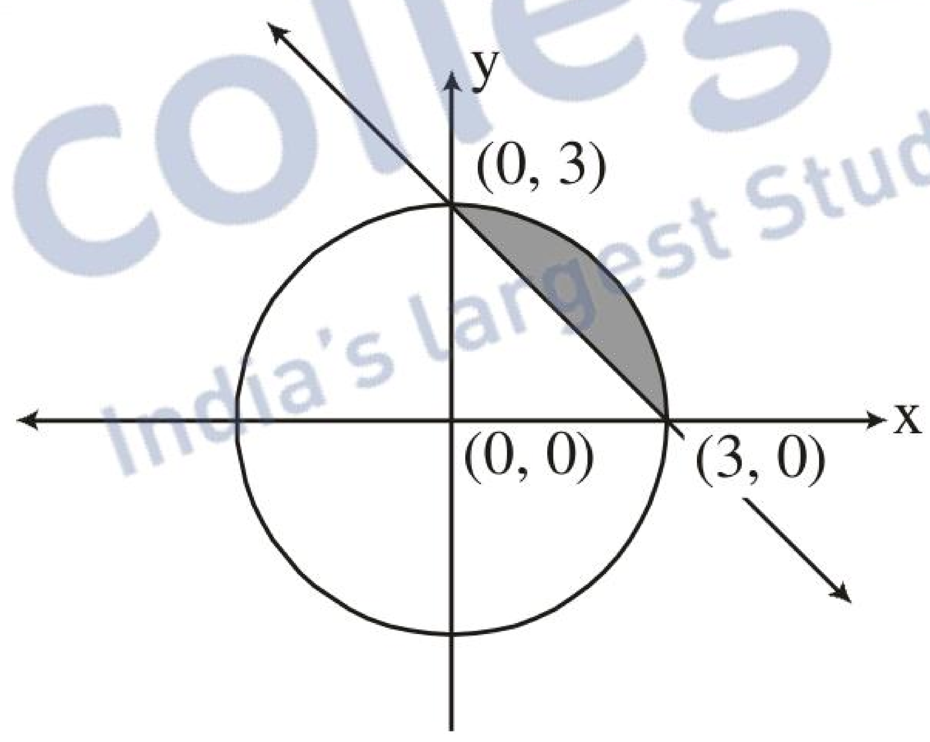
*Question numbers 7 to 10 carry 3 marks each.*

**7.** (a) Find the area of the region $\{(x, y) : x^2 + y^2 \le 9, x + y \ge 3\}$, using integration.

OR

(b) Using integration, find the area of the region bounded by the parabola $y^2 = 4x$, the lines x = 0 and x = 3 and the x-axis.

**Ans.** Point of intersection (3, 0) and (0, 3)




Correct figure $\frac{1}{2}$

Required Area

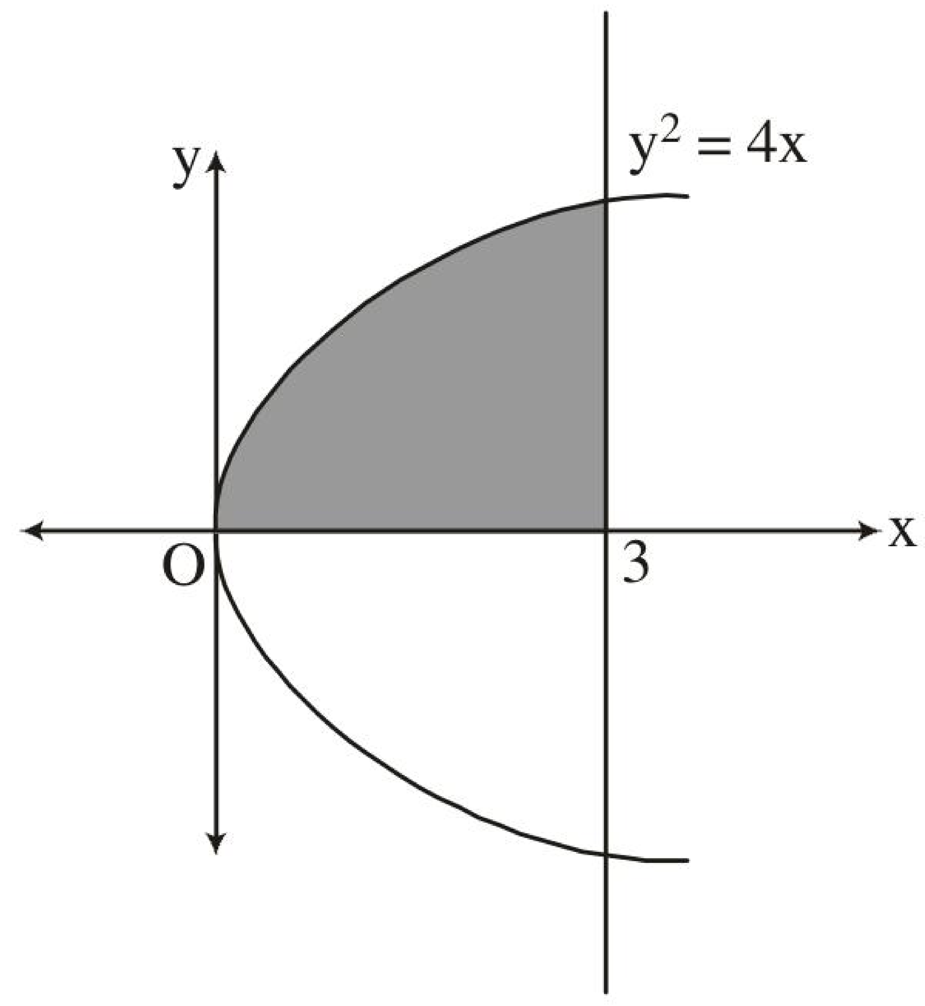
$$= \int_0^3 \sqrt{9 - x^2}\, dx - \int_0^3 (3 - x) dx$$ 1

$$= \left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3 - \left[\frac{(3 - x)^2}{-2}\right]_0^3$$ 1

$$= \frac{9}{2}\sin^{-1} 1 - \frac{9}{2} = \frac{9}{2}\left(\frac{\pi}{2} - 1\right)$$ $\frac{1}{2}$

OR

Required Area




Correct Figure 1

$$= \int_0^3 2\sqrt{x}\, dx$$ 1

$$= 2 \times \frac{2}{3}[x^{3/2}]_0^3$$ $\frac{1}{2}$

$$= \frac{4}{3} \times 3^{3/2} = 4\sqrt{3}$$ $\frac{1}{2}$

**8.** Find:

$$\int \frac{\sin x}{\sin (x - 2a)} dx$$

**Ans.** $I = \int \frac{\sin x}{\sin (x - 2a)} dx$

$$= \int \frac{\sin [(x - 2a) + 2a]}{\sin (x - 2a)} dx$$ $\frac{1}{2}$

$$= \int \frac{\sin (x - 2a) \cos 2a + \cos (x - 2a) \sin 2a}{\sin (x - 2a)} dx$$ 1

$= \int \cos 2a \, dx + \sin 2a \int \cot(x - 2a) \, dx$ $\frac{1}{2}$

$= x \cos 2a + \sin 2a \cdot \log|\sin(x - 2a)| + C$ 1

**9.** Find the equation of the plane passing through three points whose position vectors are $-\hat{j}, 3\hat{i} + 3\hat{j}$ and $\hat{i} + \hat{j} + \hat{k}$.

**Ans.** Given points are (0, –1, 0), (3, 3, 0) and (1, 1, 1)

Equation of required plane is

$$\begin{vmatrix} x & y+1 & z \\ 3 & 4 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 0$$ $1\frac{1}{2}$

$\Rightarrow 4x - 3y + 2z = 3$ $1\frac{1}{2}$

**10.** (a) Find the distance between the following parallel lines:

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$$

OR

(b) Find the coordinates of the point where the line through the points (–1, 1, –8) and (5, –2, 10) crosses the ZX-plane.

**Ans.** (a) $\vec{a}_2 - \vec{a}_2 = -\hat{i} - 3\hat{j} + 2\hat{k}, \ \vec{b} = \hat{i} + \hat{j} - \hat{k}$ $\frac{1}{2}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 2 \\ 1 & 1 & -1 \end{vmatrix} = \hat{i} + \hat{j} + 2\hat{k}$$ 1

Required distance $= \dfrac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$

$= \dfrac{\sqrt{1+1+4}}{\sqrt{1+1+1}} = \sqrt{\dfrac{6}{3}} = \sqrt{2}$ $1\frac{1}{2}$

OR

(b) Equation of line through (–1, 1, –8) and (5, –2, 10)

is $\frac{x+1}{5-(-1)} = \frac{y-1}{-2-1} = \frac{z+8}{10-(-8)}$

i.e. $\frac{x+1}{6} = \frac{y-1}{-3} = \frac{z+8}{18} = \lambda$ 1

Any point on this line is (6λ – 1, –3λ + 1, 18λ – 8) $\frac{1}{2}$

Line crosses ZX-plane i.e. y = 0

$\Rightarrow -3\lambda + 1 = 0 \Rightarrow \lambda = \frac{1}{3}$ 1

Required point is (1, 0, –2) $\frac{1}{2}$

## SECTION C

*Question numbers 11 to 14 carry 4 marks each.*

**11.** Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i}+2\hat{j}-3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i}+5\hat{j}+3\hat{k}) = 9$ and through the point (2, 1, 3).

**Ans.** Equation of required plane is

$\vec{r} \cdot [(2\hat{i}+2\hat{j}-3\hat{k}) + \lambda(2\hat{i}+5\hat{j}+3\hat{k})] = 7 + 9\lambda$ 1

or $\vec{r} \cdot [(2+2\lambda)\hat{i} + (2+5\lambda)\hat{j} + (-3+3\lambda)\hat{k}] = 7 + 9\lambda$

As the plane passes through (2, 1, 3), we have

$(2\hat{i}+\hat{j}+3\hat{k}) \cdot [(2+2\lambda)\hat{i} + (2+5\lambda)\hat{j} + (-3+3\lambda)\hat{k}] = 7 + 9\lambda$ 1

$\Rightarrow 2(2 + 2\lambda) + 1(2 + 5\lambda) + 3(-3 + 3\lambda) = 7 + 9\lambda$

$\Rightarrow 9\lambda = 10 \Rightarrow \lambda = \frac{10}{9}$ 1

Required plane is $\vec{r} \cdot \left(\frac{38}{9}\hat{i} + \frac{68}{9}\hat{j} + \frac{3}{9}\hat{k}\right) = 17$ 1

or $\vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$

**12.** (a) Find:

$$\int \cos x \bullet \tan^{-1} (\sin x)\, dx$$

OR

(b) Find:

$$\int \frac{e^x}{(e^x+1)(e^x+3)} dx$$

**Ans.** $I = \int \cos x \cdot \tan^{-1}(\sin x)\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$ 1

$\therefore\ I = \int \tan^{-1} t \cdot 1\, dt$

$= \tan^{-1} t \cdot t - \frac{1}{2}\int \frac{2t}{1+t^2} dt$ $1\frac{1}{2}$

$= t \cdot \tan^{-1} t - \frac{1}{2} \log|1+t^2| + C$ 1

$= \sin x \cdot \tan^{-1}(\sin x) - \frac{1}{2}\log|1+\sin^2 x| + C$ $\frac{1}{2}$

OR

$I = \int \frac{e^x}{(e^x+1)(e^x+3)} dx$

Put $e^x = t \Rightarrow e^x\, dx = dt$ 1

$I = \int \frac{dt}{(t+1)(t+3)}$

$= \frac{1}{2}\int\left(\frac{1}{t+1} - \frac{1}{t+3}\right) dt$ $1\frac{1}{2}$

$= \frac{1}{2}[\log|t+1| - \log|t+3|] + C$ 1

$= \frac{1}{2}[\log|e^x+1| - \log|e^x+3|] + C$ $\frac{1}{2}$

or $\frac{1}{2}\log\left|\frac{e^x+1}{e^x+3}\right| + C$

**13.** Find the particular solution of the different equation $x\frac{dy}{dx} + 2y = x^2 \log x$, given y(1) = 1.

**Ans.** $x\frac{dy}{dx} + 2y = x^2 \log x$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x} \cdot y = x \log x$ $\frac{1}{2}$

I.F. $= e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$ $\frac{1}{2}$

Solution is given by

$y \cdot x^2 = \int x^2 (x \log x) dx$ 1

$\Rightarrow yx^2 = \int x^3 \log x \, dx$

$= \log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx$

$= \frac{x^4}{4} \log x - \frac{1}{16} \cdot x^4 + C$ 1

$\Rightarrow y = \frac{x^2}{4} \log x - \frac{1}{16} x^2 + \frac{C}{x^2}$

when x = 1, y = 1, we get $C = \frac{17}{16}$ $\frac{1}{2}$

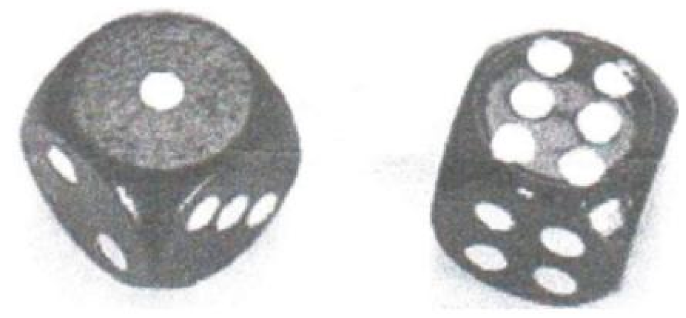
Required particular solution is

$y = \frac{x^2}{4} \log x - \frac{1}{16} x^2 + \frac{17}{16x^2}$ $\frac{1}{2}$

## Case-Study Based Question

**14.** A biased die is tossed and respective probabilities for various faces to turn up are the following:

| Face | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Probability | 0.1 | 0.24 | 0.19 | 0.18 | 0.15 | K |



Based on the above information, answer the following questions:

(a) What is the value of K?

(b) If a face showing an even number has turned up, then what is the probability that it is the face with 2 or 4?

**Ans.** (a) $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$ 1

$\Rightarrow 0.1 + 0.24 + 0.19 + 0.18 + 0.15 + K = 1$

$\Rightarrow K = 0.14$ 1

(b) A: face shows 2 or 4

B: even face have turned up

Required probability $= P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

here $P(A \cap B) = P(2) + P(4)$

$= 0.24 + 0.18 = 0.42$ 1

$P(B) = P(2) + P(4) + P(6) = 0.56$ $\frac{1}{2}$

$\therefore P(A/B) = \dfrac{0.42}{0.56} = \dfrac{3}{4}$ $\frac{1}{2}$