JEE-Main-29-07-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: The value of $\sum_{r=1}^{20}\left(r^2+1\right)\cdot r!$ is:

Options:

(a) $22!-2\cdot(20)!$

(b) $(22)!-2(21)!$

(c) $(22)!$

(d) $2(21)!$

Answer: (b)

Solution:

$$\sum_{r=1}^{20}\left(r^2+1\right)r!=\sum_{r=1}^{20}\left((r+1)(r+2)-3(r+1)+2\right)r!$$

$$=\sum_{r=1}^{20}\left((r+2)!-3(r+1)!+2r!\right)$$

$$=\sum_{r=1}^{20}\left((r+2)!-(r+1)!\right)-2\sum_{r=1}^{20}\left((r+1)!-r!\right)$$

$$=\left(22!-2!\right)-2\left(21!-1!\right)$$

$$=22!-2\times 2!-2+2$$

$$=(22)!-2(21)!$$

Question: If $|\vec{a}||\vec{b}||\vec{c}|=14$ and $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\cdot\left(\vec{c}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\cdot\left(\vec{a}\times\vec{b}\right)=168$ and $\vec{a},\vec{b},\vec{c}$ are coplanar, concurrent and make equal angles with each other, then $|\vec{a}|+|\vec{b}|+|\vec{c}|$ is equal to:

Options:

(a) 14

(b) 16

(c) 10

(d) 12

Answer: (b)

Solution:

$\because$ $\vec{a},\vec{b},\vec{c}$ are coplanar and make equal angle with each other (say θ)

So, $\theta=60°$

$\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}\right)=\left|\vec{a}\times\vec{b}\right|\left|\vec{b}\times\vec{c}\right|$ (a $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{c}$ will be parallel)

$= |\vec{a}||\vec{b}|^2 |\vec{c}|\sin^2\theta = 14\sin^2\theta|\vec{b}|$

So, $14\times\frac{3}{4}\left(|\vec{a}|+|\vec{b}|+|\vec{c}|\right)=168$

$\Rightarrow |\vec{a}|+|\vec{b}|+|\vec{c}|=16$

Question: A perpendicular drawn from (1, 2, 3) to the plane $x+2y+z=14$ and intersect plane at Q. R be a point on plane such that PR makes an angle $60°$ with the plane, then area of ΔPQR is:

Options:

(a) $\sqrt{3}$ sq. units

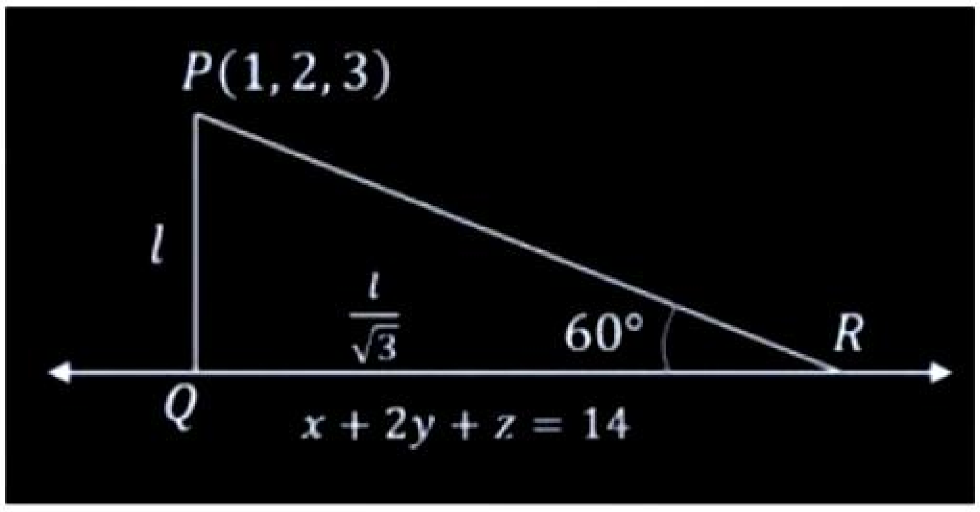
(b) 3 sq. units

(c) $\frac{\sqrt{3}}{2}$ sq. units

(d) 4 sq. units

Answer: (a)

Solution:



$\because QR = PQ\cdot\cot 60° = \frac{l}{\sqrt{3}}$

Also, $l=\left|\frac{1+4+3-14}{\sqrt{1+4+1}}\right|=\sqrt{6}$

Area of $\Delta PQR = \frac{1}{2}l\cdot\frac{l}{\sqrt{3}}=\frac{6}{2\sqrt{3}}=\sqrt{3}$

Question: The number of solution of the equation $2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}$ is/are:

Options:

(a) 1

(b) 0

(c) 3

(d) Infinite

Answer: (a)

Solution:

$$2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}$$

Equality holds when $4^x+4^{-x}=2$ and $\cos\left(\frac{x^2+x}{6}\right)=1$

$4^x+4^{-x}=2$ gives $x=0$ for which $\cos\left(\frac{x^2+x}{6}\right)=1$

So, there exist only one solution $x=0$.

Question: Let $\vec{a},\vec{b}$ are two vectors and $\vec{a}\cdot\vec{b}=3$, $\left|\vec{a}\times\vec{b}\right|^2=75$, and $\left|\vec{a}+\vec{b}\right|^2=\left|\vec{a}\right|^2+2\left|\vec{b}\right|^2$, then $\left|\vec{a}\right|^2$ is equal to ____.

Answer: 14.00

Solution:

$$\left|\vec{a}+\vec{b}\right|^2=\left|\vec{a}\right|^2+2\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2=\left|\vec{a}\right|^2+2\left|\vec{b}\right|^2$$

$$\Rightarrow\left|\vec{b}\right|^2=2\left(\vec{a}\cdot\vec{b}\right)=6$$

Also,

$$\Rightarrow\left|\vec{a}+\vec{b}\right|^2+\left|\vec{a}\cdot\vec{b}\right|^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2$$

$$\Rightarrow 75+9=6\left|\vec{a}\right|^2$$

$$\Rightarrow\left|\vec{a}\right|^2=\frac{84}{6}=14$$

Question: If sum and product of mean and variance in a binomial distribution are 82.5 and 1350 respectively, then n is equal to ___.
(where n is number of trial in binomial distribution).

Answer: 96.00

Solution:

$\because$ Mean and variance are the roots of

$x^2-82.5x+1350=0$

So, mean $=np=60$

and variance $=npq=22.5$

$$\Rightarrow q=\frac{22.5}{60}=\frac{3}{8}$$

So, $p=\frac{5}{8}$ and $n=\frac{60}{\frac{5}{8}}=96$

Question: The number of numbers lying between 1024 and 23146 which are divisible by 55 and made from 2, 3, 4, 5, 6 without repetition, is ____.

Answer: 6.00

Solution:

We will solve this in two cases:

Case I:

When number has 4 digits (say $\overline{abcd}$)

Here d is fixed as 5.

So, a,b,c can be

$(6,4,3),(3,4,6),(2,3,6),(6,3,2),(3,2,4)$ or $(4,2,3)$ only

Number of numbers possible = 6

Case II:

When number has 5 digits.

No such number is possible because even last number formed is greater than 23146.

Total number of such number = 6