MATHEMATICS (PG)
(Final)

1. The number of 4 digit numbers with no two digits common is

(A) 5040 (B) 4823
(C) 4536 (D) 3024

2. Let $S = \left\{ A : A = \left[a_{ij} \right]_{7\times 7},\ a_0 = 0 \text{ or } 1,\ \forall i, j,\ \sum a_{ij} = 1,\ \forall\ i \text{ and } \sum a_{ij} = 1,\ \forall\ j \right\}$. Then the number of elements in S is

(A) 7! (B) 7^2
(C) 7^7 (D) 77

3. The number of elements in the set { m : $1 \le m \le 1000$, m and 1000 are relatively prime } is

(A) 400 (B) 300
(C) 250 (D) 100

4. The unit digit of 2^{100} is

(A) 2 (B) 4
(C) 6 (D) 8

5. The number of multiples of 10^{44} that divide 10^{55} is

(A) 11 (B) 12
(C) 121 (D) 144

6. The number of primitive divisors of 50000 is

(A) 50 (B) 40
(C) 30 (D) 20

7. The number $\sqrt{2}\, e^{i\pi}$ is

(A) a transcendental number
(B) a rational number
(C) an imaginary number
(D) an irrational number

8. The number of divisors of 360 is

(A) 36 (B) 48
(C) 24 (D) 52

9. The smallest number with 18 divisors is

(A) 90 (B) 18
(C) 60 (D) 180

10. The remainder obtained when dividing 2^{46} by 47

(A) 0 (B) 1
(C) 2 (D) 3

11. The value of $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!}$

(A) $e+1$ (B) $e-1$
(C) e^2+1 (D) e^2-1

12. The expansion of $(2x-3y)^4$ is

(A) $16x^4+96x^3y-216x^2y^2+216xy^3-81y^4$
(B) $16x^4-96x^3y+216x^2y^2-216xy^3+81y^4$
(C) $16x^4-96x^3y+216x^2y^2+216xy^3+81y^4$
(D) None of the above

13. The value of $8C_0+8C_2+8C_4+...+8C_8$

(A) 2^8 (B) 2^4
(C) 2^7 (D) 2^5

14. The sum of the divisors of 140 is

(A) 236 (B) 216
(C) 336 (D) 440

15. $e^{\log m}=?$

(A) $-m$ (B) 0
(C) m (D) $\frac{1}{m}$

16. The function $f(x)=e^x,\ x\in R$ is

(A) onto but not one-one (B) one-one onto
(C) one-one but not onto (D) neither one-one nor onto

17. The set of all limit points of the set $S = \left\{ \frac{1}{n} : n \in N \right\}$ is

(A) ϕ (B) $\{0\}$
(C) N (D) None of the above

18. A set contains $2n+1$ elements. The number of subsets of this set containing more than 'n' elements is equal to

(A) 2^{n-1} (B) 2^{n}
(C) 2^{n+1} (D) 2^{2n}

19. Which one of the following sequences is convergent?

(A) $\langle 2^n \rangle$ (B) $\langle 3^n \rangle$
(C) $\left\langle \left(\frac{1}{3}\right)^n \right\rangle$ (D) None of the above

20. The series $\sum \sin \frac{1}{n}$ is

(A) convergent (B) uniformly convergent
(C) divergent (D) None of the above

21. The sequence $\left\{ \frac{1}{n} \right\}$ is

(A) unbounded and convergent (B) bounded and convergent
(C) bounded and divergent (D) unbounded and divergent

22. I. Every convergent sequence is bounded
II. Every bounded sequence is convergent

(A) I is true, II is false
(B) I is false, II is true
(C) Both I and II are true
(D) Both I and II are false

23. A series $\sum_{n=1}^{\infty} a_n$ converges, then sequence $\{a_n\}_{n=1}^{\infty}$

(A) diverges (B) converges to any number
(C) converges to zero (D) None of the above

24. A function $f: R \to R$ satisfies the equation $f(x+y) = f(x).f(y), \forall x, y \in R$. If $f(x)$ is differentiable at 0 and $f'(0) = 2$, then $f'(x)$ is equal to

(A) $2f(x), \forall x \in R$ (B) $4f(x), \forall x \in R$
(C) $0, \forall x \in R - \{0\}$ (D) None of the above

25. Let $f: R \to R$ be defined by $f(x) = [x^2]$, where $[x]$ is greatest integer function. The points of discontinuity of 'f' are

(A) only the integral points (B) all rational numbers
(C) $\{\pm\sqrt{n} : n \text{ is positive integer}\}$ (D) all real number

26. Let $f: R \to R$ be given by $f(x) = [x]$, the greatest integer less than or equal to x. Then

(A) the points at which f is not continuous is countable
(B) the points at which f is not continuous is R
(C) f is strictly increasing
(D) f is strictly decreasing

27. One of the solution for the equation $15x \equiv 6 (\bmod 21)$

(A) 5 (B) 6
(C) 7 (D) 8

28. The solution of ordinary differential equation of order n contains

(A) n-arbitrary constants
(B) more than n-arbitrary constants
(C) no arbitrary constants
(D) None of the above

29. What is the order and degree of the differential equation $\frac{d^2y}{dx^2} + \sqrt{\left(1 + \left(\frac{dy}{dx}\right)^3\right)} = 0$?

(A) first order, second degree (B) first order, first degree
(C) second order, second degree (D) second order, first degree

30. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$, has the solution

(A) $y = C_1e^{-2x} + C_2e^{x}$ (B) $y = C_1e^{-2x}$
(C) $y = C_1e^{-2x} + C_2e^{-x} + C_3$ (D) None of the above

31. Let A be a square matrix of order $n > 1$ such that $A \neq I$ and the sum of each row is 1. Then the sum of each row of the matrix A^n is

(A) n (B) 1
(C) n^n (D) None of the above

32. The eigen values of the matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ are

(A) 1, 0, 1 (B) 2, −2, 0
(C) 2, −1, −1 (D) 0, 0, 0

33. The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{pmatrix}$ is

(A) 1 (B) 2
(C) 3 (D) 4

34. Let A be a 3×3 matrix with eigen values 1, -1 and 3. Then

(A) $A^2 + A$ is non-singular (B) $A^2 - A$ is non-singular
(C) $A^2 + 3A$ is non-singular (D) $A^2 - 3A$ is non-singular

35. The value of the determinant $\begin{bmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{bmatrix}$ is

(A) $(a-b)(b-c)(c-a)$ (B) $-(a-b)(b-c)(c-a)$
(C) $(b-a)(c-b)(c-a)$ (D) $-(b-a)(c-b)(c-a)$

36. The solution of the system of equations
$10x + y + z = 12,\ x + 10y + z = 12,\ x + y + 10z = 12$ is

(A) $1, -1, 1$ (B) $-1, -1, -1$
(C) 1, 1, 1 (D) $-1, 1, -1$

37. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, then $A^2-5A+7I=$

(A) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

(B) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

(C) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

(D) None of the above

38. The inverse of the matrix $\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ is

(A) $\begin{bmatrix}-\sin\theta & \cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

(B) $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

(C) $\begin{bmatrix}-\cos\theta & \sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

(D) None of the above

39. Let A be a 4×4 matrix with eigen values $1, -1, 5, 2$. Then the determinant of A^2-I is

(A) -10
(B) 100
(C) 99
(D) 0

40. $\tan^{-1}x$ can be expressed as

(A) $x+\frac{x^3}{3!}-\frac{x^5}{5!}+\ldots$

(B) $x-\frac{x^3}{3}+\frac{x^5}{5}+\ldots$

(C) $1+x+\frac{x^2}{2!}+\ldots$

(D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots$

41. If $\cos(A-B)=\frac{1}{2}$ and $\sin(A+B)=\frac{1}{2}$, then the smallest positive values of A and B are respectively

(A) $\frac{\pi}{4}, \frac{\pi}{3}$

(B) $\frac{7\pi}{12}, \frac{\pi}{4}$

(C) $\frac{5\pi}{12}, \frac{\pi}{4}$

(D) $\frac{\pi}{4}, \frac{5\pi}{12}$

42. If $x^3 - 11x^2 + ax - 36 = 0$ has a positive root which is the product of the other two roots, then the value of a is

(A) 36 (B) 6
(C) 24 (D) 64

43. The equation with rational coefficients, whose roots are $1 \pm \sqrt{2}, 3$ is

(A) $x^3 + 5x^2 + 5x + 3 = 0$ (B) $x^3 - 5x^2 + 5x + 3 = 0$
(C) $x^3 - 5x^2 - 5x + 3 = 0$ (D) None of the above

44. If α, β and γ are roots of the equation $x^3 + px^2 + qx + r = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

(A) $p^2 - 2q$ (B) $p^2 + 2q$
(C) $2p^2 + q^2$ (D) $2p - q^2$

45. Given that $2 + i\sqrt{3}$ is one root of $x^3 - 5x^2 + 11x - 7 = 0$. Then the other roots are

(A) $2 - i\sqrt{3}, -1$ (B) $2 - i\sqrt{3}, 1$
(C) $2 + i\sqrt{3}, 1$ (D) None of the above

46. $\lim_{x \to \infty} \dfrac{x^2 + 2x + 3}{3x^2 + 2x + 1} =$

(A) ∞ (B) $\dfrac{1}{3}$
(C) 3 (D) does not exist

47. The derivative of $\log_{10} x$ with respect to x is

(A) $\dfrac{1}{x \log_{10} x}$ (B) $\dfrac{1}{x}$
(C) $\dfrac{\log_e 10}{x}$ (D) $\dfrac{\log_{10} e}{x}$

48. If $\log_{27} x = \log_3 27$, then x equals

(A) 27 (B) 3
(C) 3^{27} (D) 27^3

49. The derivative of e^t with respect to $\sqrt{t}$ is

(A) $\frac{e^t}{2\sqrt{t}}$ (B) $\frac{2\sqrt{t}}{e^t}$

(C) $2\sqrt{t}e^t$ (D) $2\sqrt{te^t}$

50. The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x-\sin x$ is an increasing function for

(A) all x in $\mathbb{R}$
(B) all x such that $\cos x>0$
(C) all x such that $\cos x<0$
(D) all x such that $\sin x\geq 0$

51. The maximum value for the function xe^{-x} is

(A) e (B) $-e$

(C) $\frac{1}{e}$ (D) $\frac{-1}{e}$

52. A function $f:(0,1)\to\mathbb{R}$ is defined as $f(x)=\frac{1}{2^{n-1}}$ for $\frac{1}{2^n}<x\leq\frac{1}{2^{n-1}}$. Then the integral $\int_0^1 f(x)\,dx$ equals

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{4}{3}$ (D) $\frac{2}{3}$

53. $\int_0^{\frac{\pi}{2}}\sin^2 x\cos^2 x dx$ equals

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{16}$

(C) $\frac{\pi}{32}$ (D) 1

54. The area of the region $A=\{(x,y)\in\mathbb{R}^2:|x|+|y|\leq 1\}$ is

(A) 2 (B) $\sqrt{2}$

(C) 4 (D) $4\sqrt{2}$

55. Suppose for every integer m, $\int_m^{m+1} f(x)\,dx = m^2$. Then the value of $\int_{-2}^{4} f(x)\,dx$ is

(A) 16
(B) 14
(C) 19
(D) 35

56. $\int \frac{dx}{\sqrt{x^2-1}}$ is equal to

(A) $\cos h^{-1} x + c$
(B) $\sin h^{-1} x + c$
(C) $\cos^{-1} x + c$
(D) $\sin^{-1} x + c$

57. $\int \sqrt{a^2+x^2}\,dx$ is equal to

(A) $\frac{a^2}{2}\cos h^{-1}\frac{x}{a} + x\frac{\sqrt{a^2+x^2}}{2}$
(B) $\frac{a^2}{2}\tan h^{-1}\frac{x}{a} + x\frac{\sqrt{a^2+x^2}}{2}$
(C) $\frac{a^2}{2}\sin h^{-1}\frac{x}{a} + x\frac{\sqrt{a^2+x^2}}{2} + \text{constant}$
(D) None of the above

58. $\int_0^{\pi} \frac{dx}{(5+4\sin x)}$ is equal to

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$
(D) None of the above

59. The area bounded by one arch of the curve $y = \sin ax$ and the x-axis is

(A) a
(B) $\frac{a}{2}$
(C) $\frac{2}{a}$
(D) None of the above

60. The volume of revolution obtained by revolving the loop of the curve $y^2 = x(2x-1)^2$ about the x-axis is

(A) $\frac{\pi}{48}$
(B) $\frac{\pi}{24}$
(C) $\frac{\pi}{12}$
(D) None of the above

61. The length of complete arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is

(A) $6a$ (B) $8a$
(C) $4a$ (D) None of the above

62. The condition for the point (x, y) to lie on the straight line joining the points $(0, b)$ and $(a, 0)$ is

(A) $\frac{x}{a} + \frac{y}{b} = 1$ (B) $\frac{x}{a} - \frac{y}{b} = 1$
(C) $\frac{x}{a^2} + \frac{y}{b^2} = 1$ (D) None of the above

63. The centroid of the triangle whose vertices are $(2, 4, -3)$, $(-3, 3, -5)$ and $(-5, 2, -1)$ is

(A) $(-2, -3, -3)$ (B) $(-3, 3, -2)$
(C) $(3, -2, -3)$ (D) $(-2, 3, -3)$

64. The equation to the plane which passes through the point (–1, 3, 2) and parallel to the plane $x - y + z = 3$

(A) $x - y + z = 2$ (B) $x - y + z = -2$
(C) $x + y - z = 2$ (D) $x + y - z = -2$

65. The distance between the parallel planes $4x + 3y - 12z + 6 = 0$ and $4x + 3y - 12z - 9 = 0$ is

(A) $\frac{13}{15}$ (B) $\frac{14}{15}$
(C) $\frac{15}{13}$ (D) $\frac{15}{14}$

66. The coordinates of the point at which the line joining the points $(4, 3, 1)$ and $(1, -2, 6)$ meets the plane $3x - 2y - z + 3 = 0$

(A) $(-2, -7, 11)$ (B) $(-2, 7, 11)$
(C) $(-2, -7, -11)$ (D) $(2, 7, 11)$

67. The centre of the sphere $x^2+y^2+z^2-6x+8y-10z+1=0$ is

(A) $(5,-4,3)$ (B) $(3,4,-5)$
(C) $(-5,-4,-3)$ (D) $(3,-4,5)$

68. $\nabla\times(\nabla\times A)$ equals

(A) 0 (B) $-\nabla^2 A+\nabla(\nabla.A)$
(C) $\nabla^2 A+\nabla(\nabla.A)$ (D) $(\nabla\times A)\times A$

69. The directional derivative of $\phi(x,y,z)=xy^2+yz^3$ at the point $(2,-1,1)$ in the direction of the vector $\vec{i}+2\vec{j}+2\vec{k}$ is

(A) $-\frac{3}{11}$ (B) $\frac{3}{11}$
(C) $-\frac{11}{3}$ (D) $\frac{11}{3}$

70. The unit normal to the surface $x^2+2y^2+z^2=7$ at $(1,\,-1,\,2)$ is

(A) $\frac{1}{3}(\vec{i}-2\vec{j}+2\vec{k})$ (B) $\frac{1}{3}(\vec{i}+2\vec{j}+2\vec{k})$
(C) $\frac{1}{3}(\vec{i}-2\vec{j}-2\vec{k})$ (D) $\frac{1}{3}(-\vec{i}+2\vec{j}+2\vec{k})$

71. The divergence of $\vec{F}=xyz\vec{i}+3x^2y\vec{j}+(xz^2-y^2z)\vec{k}$ at $(1,2,-1)$ is

(A) 5 (B) – 5
(C) 6 (D) –6

72. If $A=(3x^2-6yz)i+(2y+3xz)j+(1-4xyz^2)k$, then $\int_C A\,dr$ from origin to $(1,1,1)$ along the path C given by $x=t,\ y=t^2,\ z=t^3$ is

(A) 0 (B) 1
(C) 2 (D) 4

73. Let $P(x,y)$ and $Q(x,y)$ be continuous and have continuous first partial derivatives at each point of a region R. If $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$ then, for every closed path C in R, $\oint_C (Pdx-Qdy)$ equals

(A) 0 (B) 2
(C) 3 (D) 4

74. The integral $\iint_S r.n\ dS$, where S is a closed surface and V is the volume enclosed by S, equals

(A) $2V$ (B) $3V$
(C) $6V$ (D) $12V$

75. The equation of the right circular cone with its vertex at the origin, axis along z-axis and semi-vertical angle α is

(A) $x^2+y^2=z^2\tan^2\alpha$ (B) $x^2-y^2=z^2\tan^2\alpha$
(C) $x^2+y^2=z\tan^2\alpha$ (D) $x^2-y^2=z\tan^2\alpha$

76. The probability of an element of order 2 in the symmetric group S_3 is

(A) 0 (B) $\frac{1}{2}$
(C) 1 (D) $\frac{1}{6}$

77. If 3 balls are randomly drawn from a bowl containing 5 white and 6 black balls, what is the probability that one of the drawn ball is black and the other two white?

(A) $\frac{5}{22}$ (B) $\frac{4}{11}$
(C) $\frac{5}{11}$ (D) $\frac{6}{11}$

78. The probability mass function or probability density function for which the mean in units and the variance in square units are same is

(A) binomial (B) Poisson
(C) standard normal (D) geometric

79. A random variable X has a probability density function $f(x) = \frac{C}{1+x^2}, -\infty < x < \infty$. Then the value of C is

(A) π (B) 1
(C) $\frac{1}{\pi}$ (D) $\frac{2}{\pi}$

80. How many different batting orders are possible for a cricket team consisting of 11 players?

(A) 11 (B) 11!
(C) 11^2 (D) 11^{11}

81. Let the cost of each pen is Rs.12 and the cost of each notebook is Rs.21. In how many different ways one can buy both pens and notebooks such that the total sum is Rs.502?

(A) 2 (B) 7
(C) 3 (D) 0

82. Two events A and B have probabilities 0.25, 0.5 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occur is

(A) 0.39 (B) 0.25
(C) 0.11 (D) None of the above

83 If $y = e^{-2x}\cos 3x$ and $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$, then a and b are

(A) 4, 13 (B) 13, 4
(C) 4, 4 (D) 13, 13

84. Let $g:\mathbb{R} \to \mathbb{R}$ be a continuous function and ϕ be a solution of the differential equation $y' = g(y)$. Then

(A) $y(x) = \phi(x+c)$ is also a solution for any $c \in \mathbb{R}$
(B) $y(x) = \phi(x-c)$ is not a solution for some $c \in \mathbb{R}$
(C) ϕ' is not a continuous function
(D) $\phi(x) = kc^{cx}, k, c$ are some constants

85. The solution of the IVP $\frac{dy}{dx} = x^2 y - 3x^2, y(0) = 1$ is $y =$

(A) $3 + ce^{x^3/3}, c$ is a constant
(B) $3 - 2e^{x^3/3}$
(C) $3 + 3e^{x^3/3}$
(D) $3 - 2e^{x^3}$

86. One of the integrating factors of the differential equation $x\frac{dy}{dx} + y\log x = e^x$

(A) $x^{\log x}$
(B) $x^{\frac{\log x}{2}}$
(C) e^x
(D) None of the above

87. A particular integral of the differential equation $\frac{d^2y}{dx^2} + 16y = \cos 4x$ is

(A) $\frac{x}{4}\sin 4x$
(B) $\frac{x}{8}\cos 4x$
(C) $\frac{x}{4}\cos 4x$
(D) $\frac{x}{8}\sin 4x$

88. Which one of the following differential equations is exact?

(A) $(3x^2 + 2xy)dx + (2y - x^2)dy = 0$
(B) $(3x^2 - 2xy)dx + (2y + x^2)dy = 0$
(C) $(3x^2 - 2y)dx + (2y - x^2)dy = 0$
(D) $(3x^2 - 2xy)dx + (2y - x^2)dy = 0$

89. Let $f(x, y) = x^5 y^2 \tan^{-1}\frac{y}{x}$. Then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ equals

(A) $2f$
(B) $3f$
(C) $5f$
(D) $7f$

90. The differential equation that represents parabolas which have a latus rectum $4a$ and whose axes are parallel to the x-axis is

(A) $4a\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$
(B) $2a\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$
(C) $2a\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^3 = 0$
(D) None of the above

91. The general solution of the equation $\frac{dy}{dx} + y\cos x = 0$ is

(A) $ce^{-\sin x}$
(B) $ce^{\sin x}$
(C) $ce^{-\cos x}$
(D) $ce^{\cos x}$

92. The solution of the partial differential equation $u_t + cu_x = 0$ is $u(x,t) =$

(A) $\sin(x-t)$
(B) $\cos(x-ct)$
(C) $\cos(cx-t)$
(D) $\cos xt$

93. The differential equation obtained from the equation of all circles passing through the origin and having their centers on the x-axis is

(A) $x^2 - y^2 - 2xy\frac{dy}{dx} = 0$
(B) $y^2 - x^2 + 2xy\frac{dy}{dx} = 0$
(C) $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$
(D) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

94. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x+2y^3}$ with the condition that $x(1) = 1$ is

(A) $y = x^3$
(B) $x = y^3$
(C) $y = x^2$
(D) $x = y^2$

95. The general solution of the partial differential equation $\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = xy$ is

(A) $z = a\frac{x^2}{2} - \frac{y^2}{2a} + b$
(B) $z = a\frac{x^2}{2} + \frac{y^2}{2a} - b$
(C) $z = a\frac{x^2}{2} + \frac{y^2}{2a} + b$
(D) $z = a\frac{x^2}{2} - \frac{y^2}{2a} - b$

96. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two continuous functions such that $f(a) < g(a)$ and $f(b) > g(b)$ for some $a, b \in \mathbb{R}$. Then

(A) $p(f(a)) \le p(g(a))$ for any polynomial $p(x) \in \mathbb{R}[x]$
(B) there exists $t \in \mathbb{R}$ such that $p(f(t)) = p(g(t))$ for any $p(x) \in \mathbb{R}[x]$
(C) $p(f(b)) \ge p(g(b))$ for all $p(x) \in \mathbb{R}[x]$
(D) for each $t \in \mathbb{R}$, there exists $p(x) \in \mathbb{R}[x]$ such that $p(f(t)) \ne p(g(t))$

97. The function $f:(-1,1)\to\mathbb{R}$ defined by $f(x)=\dfrac{x}{1-|x|}$ is

(A) one-one but not onto
(B) not onto
(C) one-one and onto
(D) neither one-one nor onto

98. Let $f:\mathbb{R}\to\mathbb{R}$ be the function $f(x)=\begin{cases} e^{\frac{-1}{x}} & \text{if } x\neq 0 \\ 0 & \text{otherwise}\end{cases}$

Then at $x=0$, f is

(A) not continuous
(B) differentiable
(C) continuous but not differentiable
(D) neither continuous nor differentiable

99. For each $n\in\mathbb{N}$, let $a_n=\sum_{k=1}^{n}\dfrac{(-1)^{k-1}}{k}$. Then the sequence (a_n) is

(A) not a Cauchy sequence
(B) a convergent sequence
(C) not a bounded sequence
(D) convergent to 0

100. The set of all polynomials with rational coefficients

(A) is not countable
(B) is finite
(C) does not contain $\mathbb{Q}$
(D) is countable

101. If the graph of the function $f:\mathbb{R}\to\mathbb{R}$ intersects with the line $y=x$, then the $\inf\{|x-f(x)|:x\in\mathbb{R}\}$ is

(A) 0
(B) greater than 0
(C) $|f(0)|$
(D) None of the above

102. The real valued function $f(x)=\min\{1,x,x^3\}$ on $\mathbb{R}$ is

(A) continuous on $\mathbb{R}$ but not differentiable at $x=1$
(B) differentiable at $x=1$
(C) differentiable at all reals
(D) None of the above

103. Let $f:\mathbb{R}\to\mathbb{R}$ be such that $f(x+y)=f(xy)$ for all $x,y\in\mathbb{R}$. The f is

(A) a one-one function
(B) an onto function
(C) a constant function
(D) a bijection

104. Let the sequence (x_n) converge to 0. Then the sequence $(x_n y_n)$ converges to 0 if the sequence (y_n) is

(A) not bounded (B) bounded
(C) monotone (D) None of the above

105. The sequence $\left(\frac{(-1)^n}{n}\right)$

(A) converges to 0 (B) is not bounded
(C) is monotone (D) None of the above

106. The series $\sum_{n=1}^{\infty}\frac{5^n}{(n-1)!}$ converges to

(A) $5e^5$ (B) e^5
(C) $5e$ (D) $e^{\frac{1}{5}}$

107. $\lim\limits_{n\to\infty}\frac{2^{n+1}+3^{n+1}}{2^n+3^n}$ equals

(A) 3 (B) 2
(C) 1 (D) 0

108. Let $f:(0,1)\cup(2,3)\to □$ be a function with $f'(x)=0$ for all x. Then

(A) f need not be constant (B) f is constant
(C) $f(x)=0$ for all x (D) f is constant on (0,1) but not in (2,3)

109. Let $f: □ \to □$ be a continuous function such that $f(□)\subseteq □$. Then

(A) f is constant (B) f need not be constant
(C) such f doesn't exist (D) $f(□)=□$

110. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n^2}\right)^{n^2}$ equals

(A) e (B) $\frac{1}{e}$
(C) e^2 (D) $\frac{1}{e^2}$

111. The Diophantine equation $4x+5y=8$ has

(A) a unique solution
(B) an infinite number of solutions
(C) no solution
(D) only finitely many solutions

112. The gcd and the lcm of the natural numbers n and $n+1$ are

(A) $1, n(n+1)$
(B) $n, n(n-1)$
(C) $n+1,\ n(n+1)$
(D) None of the above

113. $\int_{|z|=1/3} \frac{2}{2z-1} dz =$

(A) $2\pi i$
(B) 1
(C) 0
(D) 2π

114. $\left(\frac{1+i}{\sqrt{2}}\right)^4$ is equal to

(A) 1
(B) 0
(C) $\sqrt{2}$
(D) -1

115. Let the function $f:\mathbb{C}\to\mathbb{C}$ be defined by $f(z)=z^3+z+1$. Then f is

(A) one-one
(B) onto
(C) bijection
(D) None of the above

116. The Cauchy-Riemann equations are

(A) $u_x=-v_y,\ u_y=v_x$
(B) $u_x=v_y,\ u_y=-v_x$
(C) $u_x=v_x,\ u_y=v_y$
(D) $u_x=v_x,\ u_y=-v_y$

117. Let $\omega i, 1\le i\le 6$ denote the sixth root of unity. Then the product of ωi is

(A) 1
(B) -1
(C) $\frac{1}{2}(1+i)$
(D) None of the above

118. The residue of $\dfrac{1}{\left(z^2+a^2\right)^2}$ at $z = ai$ is

(A) $\dfrac{i}{4a^2}$ (B) $\dfrac{1}{4a^3}$

(C) $\dfrac{i}{4a^3}$ (D) None of the above

119. The number of elements in the set $\{a \in \mathbb{Z}_{18} : ab \equiv 1 (\text{mod } 18) \text{ for some } b \in \mathbb{Z}_{18}\}$

(A) 18 (B) 9
(C) 6 (D) 2

120. Let $C[0,1] := \{f : [0,1] \to \mathbb{R} \mid f \text{ is continuous}\}$ with addition and multiplication defined as $(f+g)(x) = f(x) + g(x)$ and $(f.g)(x) = f(x)g(x)$ respectively. Then $C[0,1]$ is

(A) an integral domain (B) is not an integral domain
(C) is not closed under addition (D) is not closed under multiplication

121. If $a \in G$ such that the order of a is 7, then the order of bab^{-1} for any b

(A) is 3 (B) is 7
(C) need not be 7 (D) need not be finite

122. Let G be a group with $a^2 = e$ for every $a \in G$. Then G is

(A) abelian (B) not abelian
(C) such a group not exists (D) cyclic

123. A polynomial of degree 5 has

(A) no real root (B) all its roots real
(C) at least one real root (D) at most four real roots

124. Which one of the following is not a group?

(A) $(□,+)$ (B) $(□,+)$

(C) $(□,+)$ (D) $(□,+)$

125. If G is a group and x is a non-identity element of G such that $x^{10} = \text{identity}$ and $x^{15} = \text{identity}$, then the order of x is

(A) 5 (B) 10
(C) 15 (D) 150

126. The group of order 19 is

(A) cyclic
(B) not abelian
(C) not cyclic
(D) None of the above

127. The order of $(143)(25)$ in S_5 is

(A) 5
(B) 12
(C) 6
(D) 3

128. Let n be a natural number. Which one of the following is not a vector space over the field ?

(A) The set of polynomials of degree less than or equal to n.
(B) The set of polynomials of degree less than n.
(C) The set of polynomials of degree greater than n.
(D) None of the above

129. Let W be the sub space spanned by
$S = \{(1,0,0,0),(0,1,0,0),(1,1,0,0),(1,1,1,0),(2,0,3,0)\}$.
Then the dimension of W is

(A) 4
(B) 5
(C) 2
(D) 3

130. The number of subsets (including the empty subset and the whole set) for a set of n elements is

(A) n
(B) n^2
(C) n^n
(D) 2^n

131. Let G be the complete graph on n vertices. Then the number of edges in G is

(A) n
(B) n^2
(C) $2n$
(D) $\frac{n(n-1)}{2}$

132. Let T be a tree with n vertices. Then the trace of the adjacency matrix of T is

(A) 0
(B) n
(C) $n-1$
(D) $2(n-1)$

133. A graph in which all the vertices are of equal degree is

(A) complete graph
(B) multi graph
(C) Hamiltonian graph
(D) regular graph

134. For $n \geq 4$, let G be a graph with n vertices and n edges. Then

(A) G is a star
(B) G should contain a cycle
(C) G is acyclic
(D) G is a complete graph

135. If Z is the optimal value of the objective function of a LPP and Z' is the optimal value of the objective function of its dual, then

(A) $Z < Z'$
(B) $Z > Z'$
(C) $Z \neq Z'$
(D) $Z = Z'$

136. Solving by variation of parameter $y'' - 2y' + y = e^x \log x$, the value of Wronskion W is

(A) e^{2x}
(B) 2
(C) e^{-2x}
(D) None of the above

137. The value of Wronskion $W\left(x, x^2, x^3\right)$ is

(A) $2x^4$
(B) $2x^2$
(C) $2x^3$
(D) None of the above

138. The complementary function of $\left(D^4 - a^4\right)y = 0$ is

(A) $y = C_1e^{ax} + C_2e^{-ax}$
(B) $y = C_1e^{ax} + C_2e^{-ax} + C_3 \cos ax + C_4 \sin ax$
(C) $y = C_1e^{-ax} + C_2e^{ax} + C_3 \sin ax + C_4 \cos ax$
(D) None of the above

139. The differential equation $f_{xx} + 2f_{xy} + 4f_{yy} = 0$, is classified as

(A) elliptic
(B) hyperbolic
(C) parabolic
(D) None of the above

140. Using Binomial theorem the 7^{th} power of 11 is

(A) 1,94,87,171
(B) 1,94,87,121
(C) 1,94,77,171
(D) 1,94,77,121

141. Find the coefficient of x^7 in $\left(1 - x - x^2 + x^3\right)^6$

(A) 124
(B) 144
(C) –144
(D) –124

142. The matrix $\begin{pmatrix} i & 1+i \\ -1+i & i \end{pmatrix}$ is a

(A) symmetric matrix
(B) skew symmetric matrix
(C) Hermitian matrix
(D) skew Hermitian matrix

143. If A is a matrix of order 4×5, its rank is

(A) 4
(B) ≤5
(C) ≤4
(D) 5

144. If $A^T = A^{-1}$, then A is

(A) Hermitian matrix
(B) orthogonal matrix
(C) unitary matrix
(D) skew symmetric matrix

145. The basis of R_3 from the set $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$, where $\alpha_1 = (1,-3,2)$, $\alpha_2 = (2,4,1)$, $\alpha_3 = (3,1,3)$ and $\alpha_4 = (1,1,1)$ is

(A) $(\alpha_1 \alpha_2 \alpha_3)$
(B) $(\alpha_1 \alpha_2 \alpha_4)$
(C) both $(\alpha_1 \alpha_2 \alpha_3)$ and $(\alpha_1 \alpha_2 \alpha_4)$
(D) None of the above

146. If $A=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}$, then

(A) first three rows are linearly independent
(B) first and third rows are linearly independent
(C) first and fourth rows are linearly independent
(D) all columns are linearly independent

147. For which value of x will the matrix given below become singular?

$$\begin{pmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{pmatrix}$$

(A) 4
(B) 6
(C) 8
(D) 12

148. The sum of coefficients in the binomial expansion of $(5p-4q)^n$, where 'n' is a positive integer is

(A) 0
(B) 2
(C) 1
(D) 4

149. The range of $f(x) = x^2 + |x| + 1$ defined on R is

(A) $(0,\infty)$
(B) $[0, \infty)$
(C) R
(D) $[1, \infty)$

150. The curvature of a circle is

(A) zero
(B) always < 1
(C) constant
(D) None of the above

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