DU MPhil PhD in Statistics

Topic:- STATS MPHIL

1) Consider the following BIBD with treatments A, B, C and D (columns denote incomplete blocks)

A	A	B	A
C	B	C	B
D	C	D	D

The incidence matrix associated with this BIBD is

[Question ID = 10744]

1.
$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

[Option ID = 42973]

2.
$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

[Option ID = 42974]

3.
$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

[Option ID = 42975]

4.
$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

[Option ID = 42976]

2) Consider a 3^3 confounded design in 3 blocks. The total number of effects that get confounded in this design are[Question ID = 10745]

1. 0 [Option ID = 42977]
2. 1 [Option ID = 42978]
3. 2 [Option ID = 42979]
4. 3 [Option ID = 42980]

3) Consider a 2^5 factorial experiment in 4 blocks of size 8 each. Some elements of the key block are {00000, 01100, 11001,11110, 10101, 10010}. Which of the following effects is not confounded?[Question ID = 10746]

1. ABCD [Option ID = 42981]
2. BCE [Option ID = 42982]
3. ACD [Option ID = 42983]
4. ADE [Option ID = 42984]

4) The alias table of a 2^{n-k} fractional factorial design contains $2^n - 1$ elements including the top most row. Out of these, the total number of estimable elements is[Question ID = 10747]

1. $2^{n-k} -1$ [Option ID = 42985]
2. $2^k - 1$ [Option ID = 42986]
3. 2^{n-k-1} [Option ID = 42987]
4. 2^{k-1} [Option ID = 42988]

5) A design has ‘s’ blocks, each of which is divided into ‘a’ whole plots and each of these is subdivided into ‘b’ split plots, giving a total of ‘sab’ observations. The whole plots within each block are assigned at random to the levels of one factor (A) and the split plots within each whole plot are assigned at random to levels of another factor B. The split [illegible]ror degrees

of freedom is[Question ID = 10748]

1. (b-1)(s-1) [Option ID = 42989]
2. (a-1)(b-1) [Option ID = 42990]
3. abs-1 [Option ID = 42991]
4. a(b-1)(s-1) [Option ID = 42992]

6) If the sample size is large, the ratio estimator is more efficient than the simple mean $\overline{y}$ if the following relation holds.

C_x and C_y are the co-efficient of variation of x and y respectively, and ρ is the population correlation coefficient between x and y.

[Question ID = 10749]

1. $\rho > \frac{2Cx}{Cy}$

   [Option ID = 42993]
2. $\rho < \frac{C_x}{2C_y}$

   [Option ID = 42994]
3. $\rho > \frac{C_x}{2C_y}$

   [Option ID = 42995]
4. $\rho > \frac{C_y}{2C_x}$

   [Option ID = 42996]

7) A random sample of size n is drawn without replacement from a finite population of size N with variance σ^2. The covariance between any two units of the sample is[Question ID = 10750]

1. $\frac{\sigma}{(N-1)}$

   [Option ID = 42997]
2. $-\frac{\sigma}{(N-1)}$

   [Option ID = 42998]
3. $\frac{\sigma^2}{(N-1)}$

   [Option ID = 42999]
4. $-\frac{\sigma^2}{(N-1)}$

   [Option ID = 43000]

8) Consider a population of NM elements grouped into N =50 first stage units and M=20 second stage units. A sample of n first stage units is selected and then m second stage units are drawn from each of these first stage units. If m = M this corresponds to[Question ID = 10751]

1. Cluster Sampling [Option ID = 43001]
2. SRSWOR [Option ID = 43002]
3. Stratified Sampling [Option ID = 43003]
4. None of the above [Option ID = 43004]

9) Two stage sampling is generally better than single stage sampling when

[Question ID = 10752]

1. Elements in the same stage are positively correlated

   [Option ID = 43005]
2. Elements in the same stage are negatively correlated

   [Option ID = 43006]

3. Elements in the same stage are uncorrelated

[Option ID = 43007]

4. Two stage sampling is always better than single stage sampling

[Option ID = 43008]

10) A sample of size n is drawn from a finite population of N units with probabilities proportional to size. Size here refers to

[Question ID = 10753]

1. Size of the sample

[Option ID = 43009]

2. Size of the population units

[Option ID = 43010]

3. Size as decided by the experimenter

[Option ID = 43011]

4. None of the above

[Option ID = 43012]

11) In a population of size N=250, suppose P denotes the proportion of units possessing a characteristic that is rare.

A sample of size n= 10 is drawn using inverse sampling procedure and an estimate of P is calculated as 0.09.

An estimator of variance of $\hat{p}$ is given by:

[Question ID = 10754]

1. 0.101

[Option ID = 43013]

2. 0.201

[Option ID = 43014]

3. 0.020

[Option ID = 43015]

4. 0.010

[Option ID = 43016]

12) Consider a random sample of size n from a distribution with the following p.d.f.

$f(x;\theta) = \frac{1}{\pi\{1+(x-\mu)^2\}}$; $-\infty < x < \infty$, **where μ is unknown.**

Which of the following is a consistent estimator of μ?

[Question ID = 10755]

1. Sample mean, $\bar{X}$.

[Option ID = 43017]

2. Sample median, $\tilde{X}$.

[Option ID = 43018]

3. A consistent estimator does not exist for μ.

[Option ID = 43019]

4. n-th order statistic, $X_{(n)}$.

[Option ID = 43020]

13) Let $X_1, X_2, X_3, \ldots, X_n$ be independently and identically distributed random variables following Normal distribution with mean θ and variance $a\theta^2$; where a is a known constant and θ > 0. Which of the following statements is TRUE about the two-dimensional statistic, $T = (\bar{X}, S^2)$**?**

[Question ID = 10756]

1. *T* is a complete-sufficient statistic for θ.

[Option ID = 43021]

2. *T* is a sufficient statistic for θ, but its family of distribution is not complete.

[Option ID = 43022]

3. *T*, being a two-dimensional statistic, cannot be considered as a sufficient statistic for a single parameter θ.

[Option ID = 43023]

4. *T* is a complete statistic but is not sufficient for θ.

[Option ID = 43024]

14)

Let $X_1, X_2, X_3, \ldots, X_n$ be independent random variables following Uniform$\left(\theta-\frac{1}{2}, \theta+\frac{1}{2}\right)$ distribution.

Maximum likelihood estimate of θ is:

[Note: $X_{(i)}$ is the i-th order statistic]

[Question ID = 10757]

1. $\left(X_{(n)} - \frac{1}{2}\right) + \alpha\left(X_{(1)} - X_{(n)} + 1\right); 0 \le \alpha \le 1$

   [Option ID = 43025]
2. $Max\left(X_{(n)} - \frac{1}{2}, X_{(1)} + \frac{1}{2}\right)$

   [Option ID = 43026]
3. $\left(X_{(1)} + \frac{1}{2}\right) + \alpha\left(X_{(n)} - X_{(1)} - 1\right); 0 \le \alpha \le 1$

   [Option ID = 43027]
4. $Max\left(X_{(1)}, X_{(n)}\right)$

   [Option ID = 43028]

15) Pivotal Quantity method is popularly used for constructing confidence intervals of unknown population parameters. Which of the following statements correctly describes a Pivotal Quantity?[Question ID = 10758]

1. It is a statistic whose distribution depends upon the unknown parameter. [Option ID = 43029]
2. It is a statistic whose distribution is independent of the unknown parameter. [Option ID = 43030]
3. It is a function of a statistic and the unknown parameter, and its distribution is independent of the unknown parameter. [Option ID = 43031]
4. It is a function of a statistic and the unknown parameter, and its distribution should depend upon the unknown parameter. [Option ID = 43032]

16)

Let $\underline{x} = (x_1, x_2, ..., x_n)$ be a random sample from a distribution with the following p.d.f.,

$f(x;\theta) = \theta x^{\theta-1};\ 0 < x < 1;\ \theta > 0$

The Most Powerful Critical Region (MPCR), W, of size α, for testing the null hypothesis, $H_0: \theta = \theta_0$

against the alternative, $H_1: \theta = \theta_1 (> \theta_0)$ is given as:

{Note: $P[\chi^2_{2n} > \chi^2_{2n,\alpha}] = \alpha$, where 2n is the degree of freedom of the χ^2 distribution followed by the variable χ^2_{2n}}

[Question ID = 10759]

1. $W = \left\{\underline{x}: \prod_{i=1}^{n} x_i > exp\left(-\chi^2_{2n,1-\alpha}/2\theta_0\right)\right\}$

   [Option ID = 43033]
2. $W = \left\{\underline{x}: \prod_{i=1}^{n} x_i > exp\left(-\chi^2_{2n,\alpha}/2\theta_0\right)\right\}$

   [Option ID = 43034]
3. $W = \left\{\underline{x}: \prod_{i=1}^{n} x_i > exp\left(\chi^2_{2n,1-\alpha}/2\theta_0\right)\right\}$

   [Option ID = 43035]
4. $W = \left\{\underline{x}: \prod_{i=1}^{n} x_i > exp\left(\chi^2_{2n,\alpha}/2\theta_0\right)\right\}$

   [Option ID = 43036]

17) Let $X_1, X_2, ..., X_n$ be independently and identically distributed random variables following $N(\mu, \sigma^2)$ distribution, where both μ and σ^2 are unknown. Which of the following statements is NOT TRUE? [Note: $S^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$]

[Question ID = 10760]

1. $(\bar{X}, S^2)$ and $(\sum X_i, \sum X_i^2)$ are both sufficient for (μ, σ^2).

   [Option ID = 43037]

2. $(\sum X_i, \sum X_i^2)$ is minimal sufficient for (μ, σ^2) but $(\bar{X}, S^2)$, although sufficient, is not minimal sufficient for (μ, σ^2).

[Option ID = 43038]

3. Minimal sufficient statistic of (μ, σ^2) is not unique.

[Option ID = 43039]

4. Sufficient statistic of (μ, σ^2) is not unique.

[Option ID = 43040]

18) Let $X_1, X_2, \ldots, X_n$ be observations on independently and identically distributed random variables following Uniform(θ, θ+1), $-\infty < \theta < \infty$. Also, let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ be the order-statistics from the sample.

Then, which of the following is an ancillary statistic?

[Question ID = 10761]

1. $X_{(n)}$

[Option ID = 43041]

2. $X_{(1)}$

[Option ID = 43042]

3. $X_{(n)} - X_{(1)}$

[Option ID = 43043]

4. $[X_{(n)} + X_{(1)}]/ 2$

[Option ID = 43044]

19) Match the stochastic processes defined in List I with their type in List II.

List I	List II
(Stochastic Process)	(Type)
A. Let X(t) represents the maximum temperature at a place in (0, t). Then, the stochastic process {X(t), t ∊ T} has:	I. Discrete time and continuous state-space.
B. Let X(t) represents the number of incoming calls at a switchboard in (0, t). Then, the stochastic process {X(t), t ∊ T} has:	II. Continuous time and continuous state-space.
C. Let Xn represents the weight measured at the n-th visit of a patient in a hospital. Then, the stochastic process {Xn} has:	III. Continuous time and discrete state-space.
D. Let Xn be the total number of sixes appearing in the first n throws of a die. Then, the stochastic process {Xn} has:	IV. Discrete time and discrete state-space.

Choose the correct answer from the options given below:

[Question ID = 10762]

1. A - II, B - III , C - I , D - IV

[Option ID = 43045]

2. A - III , B - II , C - I , D - IV

[Option ID = 43046]

3. A - II , B - III , C - IV , D - I

[Option ID = 43047]

4. A - III , B - II, C - IV , D - I

[Option ID = 43048]

20) Let $\{X_n, n \geq 0\}$ be a Markov chain having state-space S = {1, 2, 3, 4} and transition probability matrix,

	1	2	3	4
1	1/3	2/3	0	0
2	1	0	0	0
3	1/2	0	1/2	0
4	0	0	1/2	1/2

A. State 3 is transient.Which of the following statements is/ are TRUE about the type of the states?

B. State 1 is transient.

C. State 1 is ergodic.

D. State 4 is persistent (recurrent).

Choose the *correct* answer from the options given below:

[Question ID = 10763]

1. A, B and D only

[Option ID = 43049]

2. A and C only

[Option ID = 43050]

3. A only

[Option ID = 43051]

4. C and D only

[Option ID = 43052]

21) A machine goes out of order whenever a component part fails. The failure of this part is in accordance with a Poisson process with mean rate of 1 per week. Suppose that there are 5 spare parts of the component in an inventory and that the next supply is not due in 10 weeks. What is the probability that the machine will not be out of order in the next 10 weeks? [Question ID = 10764]

1. $$\sum_{k=0}^{5} \frac{e^{-10}10^k}{k!}$$

[Option ID = 43053]

2. $$\sum_{k=0}^{10} \frac{e^{-5}5^k}{k!}$$

[Option ID = 43054]

3. $$\sum_{k=10}^{\infty} \frac{e^{-5}5^k}{k!}$$

[Option ID = 43055]

4. $$\sum_{k=5}^{\infty} \frac{e^{-10}10^k}{k!}$$

[Option ID = 43056]

22) Let a stochastic process X(t) be defined as,

$$X(t) = W(1) - \sigma W(1-t);\ \ 0 \le t \le 1, \sigma > 0$$

where, W(t) is a Weiner process. What are the mean and variance of X(t)?

[Question ID = 10765]

1. Mean = 0, Var = $\sigma^2(1-t)^2$

[Option ID = 43057]

2. Mean = $1-\sigma$, Var = $\sigma^2(1-t)^2$

[Option ID = 43058]

3. Mean = 0, Var = $t + (1-t)(1-\sigma)^2$

[Option ID = 43059]

4. Mean = $1-\sigma$, Var = $1+\sigma^2(1-t)^2$

[Option ID = 43060]

23) Let Z_1, Z_2, Z_3, … be independent random variables uniformly distributed on (0, a). Let X_0 = 1 and $X_t = C_t.Z_1.Z_2.....Z_t$

For what value of the constant C_t is X_t a martingale?

[Question ID = 10766]

1. $C_t = \left(\frac{a}{2}\right)^t$

[Option ID = 43061]

2. $C_t = \frac{2}{a}$

[Option ID = 43062]

3. $C_t = \left(\frac{2}{a}\right)^t$

[Option ID = 43063]

4. $C_t = \frac{2t}{a}$

[Option ID = 43064]

24) During an eight-hour work day, starting from 9 a.m., customers of a company arrive at a Poisson rate of 2 per hour. Assuming that during the period from 10 a.m. to 11 a.m., no customer arrived, what is the probability that during the next half an hour there will be no customer either? (Take e = 2.718)[Question ID = 10767]

1. 0.135 [Option ID = 43065]
2. 0.368 [Option ID = 43066]
3. 0.049 [Option ID = 43067]
4. 0.007 [Option ID = 43068]

25) The average number of samples required to declare a process out of statistical control, when the probability of a sample point falling between the control limits is half, is [Question ID = 10768]

1. One [Option ID = 43069]
2. Two [Option ID = 43070]
3. Three [Option ID = 43071]
4. Four [Option ID = 43072]

26) The OC function of a sequential sampling plan with best incoming lot quality is-

[Question ID = 10769]

1. (1-α)

   [Option ID = 43073]
2. β

   [Option ID = 43074]
3. One

   [Option ID = 43075]
4. Zero

   [Option ID = 43076]

27) The L_x column of a life table represents :

[Question ID = 10770]

1. the number of persons alive at age x.

   [Option ID = 43077]
2. the number of deaths in the age interval (x, x+1).

   [Option ID = 43078]
3. the complete expectation of life at age x.

   [Option ID = 43079]
4. person-years lived by cohort between x and (x+1) years.

   [Option ID = 43080]

28) The expectation of life decreases as age increases with a single exception of-[Question ID = 10771]

1. first year of life. [Option ID = 43081]
2. young age. [Option ID = 43082]
3. old age. [Option ID = 43083]
4. none of the above. [Option ID = 43084]

29) The reliability of a parallel system can be improved by improving the reliability of its-[Question ID = 10772]

1. weakest component. [Option ID = 43085]
2. best component. [Option ID = 43086]
3. best as well as weakest component. [Option ID = 43087]
4. none of the above. [Option ID = 43088]

30) The reliability of a series system decreases with-[Question ID = 10773]

1. a decrease in the number of its components. [Option ID = 43089]
2. an increase in the number of its components. [Option ID = 43090]
3. no change in the number of its components. [Option ID = 43091]
4. none of the above. [Option ID = 43092]

31) If Σ_1 and Σ_2 are the two covariance matrices given by

$\Sigma_1 = \begin{pmatrix} 4 & 5 \\ 5 & 8 \end{pmatrix}$ and $\Sigma_2 = \begin{pmatrix} 6 & 4 \\ 4 & 7 \end{pmatrix}$,

which of the below is correct?

[Question ID = 10774]

1. $|\Sigma_1| > |\Sigma_2|$ and trace (Σ_1) > trace (Σ_2)

[Option ID = 43093]

2. $|\Sigma_1| > |\Sigma_2|$ and trace (Σ_1) < trace (Σ_2)

[Option ID = 43094]

3. $|\Sigma_1| < |\Sigma_2|$ and trace (Σ_1) > trace (Σ_2)

[Option ID = 43095]

4. $|\Sigma_1| < |\Sigma_2|$ and trace (Σ_1) < trace (Σ_2)

[Option ID = 43096]

32) If $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$, and $\underset{\sim qx1}{Z} = D\underset{\sim px1}{X}$ where $rank(D_{qxp}) = q \leq p$ then the distribution of $\underset{\sim}{Z}$ is given by

[Question ID = 10775]

1. $N_p\left(D\underset{\sim}{\mu}, D\Sigma D^T\right)$

[Option ID = 43097]

2. $N_q\left(D\underset{\sim}{\mu}, D\Sigma D^T\right)$

[Option ID = 43098]

3. $N_p\left(D\underset{\sim}{\mu}, D^T\Sigma D\right)$

[Option ID = 43099]

4. $N_q\left(D\underset{\sim}{\mu}, D^T\Sigma D\right)$

[Option ID = 43100]

33) Let $\underset{\sim}{X}_\alpha,\ \alpha = 1(1)N$ be a random sample of size N from $N_p\left(\underset{\sim}{\mu}, \Sigma\right)$ and let $\overline{\underset{\sim}{X}} = \frac{1}{N}\sum_{\alpha=1}^{N} \underset{\sim}{X}\alpha$,

$A = \sum_{\alpha=1}^{N}\left(\underset{\sim}{X}\alpha - \overline{\underset{\sim}{X}}\right)\left(\underset{\sim}{X}\alpha - \overline{\underset{\sim}{X}}\right)^T$. Also let us define partition A and Σ as

$A_{pxp} = \begin{pmatrix} A_{11\ qxq} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ and $\Sigma_{pxp} = \begin{pmatrix} \Sigma_{11\ qxq} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$, then the matrix

$A_{22.1} = A_{22} - A_{21}A_{11}^{-1}A_{12}$ is distributed as

[Question ID = 10776]

1. $\sum_{\alpha=1}^{N+q-1} \underset{\sim}{U}\alpha\underset{\sim}{U}\alpha^T$, where $\underset{\sim}{U}\alpha \sim N_{p-q}(\underset{\sim}{0}, \Sigma_{22.1})$

[Option ID = 43101]

2. $\sum_{\alpha=1}^{N-q} \underset{\sim}{U}\alpha\underset{\sim}{U}\alpha^T$, where $\underset{\sim}{U}\alpha \sim N_{p-q}(\underset{\sim}{0}, \Sigma_{22.1})$

[Option ID = 43102]

3. $\sum_{\alpha=1}^{N-1} \underset{\sim}{U}\alpha\underset{\sim}{U}\alpha^T$, where $\underset{\sim}{U}\alpha \sim N_{p-q}(\underset{\sim}{0}, \Sigma_{22.1})$

[Option ID = 43103]

4. $\sum_{\alpha=1}^{N-q-1} \underset{\sim}{U}\alpha\underset{\sim}{U}\alpha^T$, where $\underset{\sim}{U}\alpha \sim N_{p-q}(\underset{\sim}{0}, \Sigma_{22.1})$

[Option ID = 43104]

34)

If $\underset{\sim}{X}\alpha,\ \alpha = 1(1)N$ be N independent observations from $N_p\left(\underset{\sim}{\mu}, \Sigma\right)$.

Then the maximum value of likelihood function at the maximum likelihood estimators of the parameters is given by

[Question ID = 10777]

1.

$$\frac{e^{-N/2}}{(2\pi)^{N/2}\left|\frac{1}{N}\sum_{\alpha=1}^{N}\left(\underset{\sim}{X}\alpha-\overline{\underset{\sim}{X}}\right)\left(\underset{\sim}{X}\alpha-\overline{\underset{\sim}{X}}\right)^{T}\right|^{Np/2}}$$

[Option ID = 43105]

2.

$$\frac{e^{-Np/2}}{(2\pi)^{N/2}\left|\frac{1}{N}\sum_{\alpha=1}^{N}\left(\underset{\sim}{X}\alpha-\overline{\underset{\sim}{X}}\right)\left(\underset{\sim}{X}\alpha-\overline{\underset{\sim}{X}}\right)^{T}\right|^{Np/2}}$$

[Option ID = 43106]

3.

$$\frac{e^{-Np/2}}{(2\pi)^{Np/2}\left|\frac{1}{N}\sum_{\alpha=1}^{N}\left(\underset{\sim}{X}\alpha-\overline{\underset{\sim}{X}}\right)\left(\underset{\sim}{X}\alpha-\overline{\underset{\sim}{X}}\right)^{T}\right|^{N/2}}$$

[Option ID = 43107]

4.

$$\frac{e^{Np/2}}{(2\pi)^{N/2}\left|\frac{1}{N}\sum_{\alpha=1}^{N}\left(\underset{\sim}{X}\alpha-\overline{\underset{\sim}{X}}\right)\left(\underset{\sim}{X}\alpha-\overline{\underset{\sim}{X}}\right)^{T}\right|^{N/2}}$$

[Option ID = 43108]

35) Consider the following two statements

A. Principal components are invariant under a non-singular linear transformation

B. Principal components are uncorrelated

then, which of the above statement(s) is/are correct?

[Question ID = 10778]

1. Both A and B

   [Option ID = 43109]
2. Only B

   [Option ID = 43110]
3. Only A

   [Option ID = 43111]
4. None of the above

   [Option ID = 43112]

36) Using Cholesky decomposition, a positive definite matrix A can be factored into

[Question ID = 10779]

1.

$A = T'T$, where, T is a nonsingular upper triangular matrix

[Option ID = 43113]

2.

$A = T'T$, where, T is a nonsingular lower triangular matrix

[Option ID = 43114]

3.

$A = T^{1/2}T^{1/2}$, where, T is a nonsingular upper triangular matrix

[Option ID = 43115]

4.

$A = T^{1/2}T^{1/2}$, where, T is nonsingular lower triangular matrix

[Option ID = 43116]

37)

Consider the demand and supply model given below

$Q_t^d = \alpha_o + \alpha_1 P_t + \alpha_2 I_t + u_{1t}$, where $\alpha_1 < 0,\ \alpha_2 > 0$

$Q_t^s = \beta_o + \beta_1 P_t + \beta_2 P_{t-1} + u_{2t}$, where $\beta_1 > 0,\ \beta_2 > 0$

where, P_t and Q_t are endogenous variables and; I_t and P_{t-1} are predetermined variables.

Then which of the below is correct?

[Question ID = 10780]

1. Demand function is unidentified and Supply function is identified [Option ID = 43117]
2. Demand function is identified and Supply function is unidentified [Option ID = 43118]
3. Demand function is identified and Supply function is identified [Option ID = 43119]
4. Demand function is unidentified and Supply function is unidentified [Option ID = 43120]

38)

Consider the following structural model

$Q_t^d = \alpha_o + \alpha_1 P_t + \alpha_2 I_t + u_{1t}$, where $\alpha_1 < 0,\ \alpha_2 > 0$

$Q_t^s = \beta_o + \beta_1 P_t + u_{2t}$, where $\beta_1 > 0$

where, P_t and Q_t are endogenous variables and; I_t is predetermined variable.

Then, the number of reduced form coefficients corresponding to the above structural model is given by

[Question ID = 10781]

1. 4 [Option ID = 43121]
2. 8 [Option ID = 43122]
3. 5 [Option ID = 43123]
4. 6 [Option ID = 43124]

39)

If we want to impose the restriction $\gamma_{11} = \gamma_{12}$ in the following equation:

$y_1 + \beta_{12} y_2 + \gamma_{11} z_1 + \gamma_{12} z_2 = u_1$.

Then the column vector of restriction is given by

[Question ID = 10782]

1. $\emptyset = (0,0,-1,-1)^T$

   [Option ID = 43125]
2. $\emptyset = (0,0,1,1)^T$

   [Option ID = 43126]
3. $\emptyset = (0,0,1,-1)^T$

   [Option ID = 43127]
4. None of the above

   [Option ID = 43128]

40) Consider the process, $y_1 = 0.2y_{t-1} + 0.6y_{t-2} + \varepsilon_t$ and the three statements.

A. It is a second-order autoregressive process

B. Process is stationary

C. Process is non-stationary

Which of the above statements is/are correct?

[Question ID = 10783]

1. Both A and B

   [Option ID = 43129]
2. Both A and C

   [Option ID = 43130]
3. Both B and C

   [Option ID = 43131]
4. Only A

   [Option ID = 43132]

41) Consider the process, $y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$, then the first and second autocorrelation are given by

[Question ID = 10784]

1. $\rho(1) = -\theta_2/(1+\theta_1^2+\theta_2^2)$ and $\rho(2) = (-\theta_1+\theta_1\theta_2)/(1+\theta_1^2+\theta_2^2)$

[Option ID = 43133]

2. $\rho(1) = (-\theta_1+\theta_1\theta_2)/(1+\theta_1^2+\theta_2^2)$ and $\rho(2) = -\theta_2/(1+\theta_1^2+\theta_2^2)$

[Option ID = 43134]

3. $\rho(1) = -\theta_2/(1+\theta_1^2+\theta_2^2)$ and $\rho(2) = 0$

[Option ID = 43135]

4. $\rho(1) = 0$ and $\rho(2) = (-\theta_1+\theta_1\theta_2)/(1+\theta_1^2+\theta_2^2)$

[Option ID = 43136]

42) The simple exponential smoother is represented by[Question ID = 10785]

1. $\tilde{y}_T = \lambda\, y_T + (1-\lambda)\, \tilde{y}_{T-1}$

[Option ID = 43137]

2. $\tilde{y}_T = \lambda\, y_{T-1} + (1-\lambda)\, \tilde{y}_{T+1}$

[Option ID = 43138]

3. $\tilde{y}_T = \lambda\, y_{T+1} + (1-\lambda)\, \tilde{y}_T$

[Option ID = 43139]

4. $\tilde{y}_T = \lambda\, y_{T+1} + (1-\lambda)\, \tilde{y}_{T-1}$

[Option ID = 43140]

43) If a random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{1}{3}, & \text{if } x = 0 \\ \frac{1+x}{3}, & \text{if } 0 < x < 1 \\ 1, & \text{if } x \geq 1, \end{cases}$$

then, E[X] equals to

[Question ID = 10786]

1. $\frac{1}{3}$

[Option ID = 43141]

2. 1

[Option ID = 43142]

3. $\frac{1}{6}$

[Option ID = 43143]

4. $\frac{1}{2}$

[Option ID = 43144]

44) If the characteristic function of i.i.d. random variables X_1 and X_2 is $\phi(t)$ where $\phi: \mathfrak{R} \to \mathfrak{R}$, then the characteristic function of $X_1 - X_2$ is

[Question ID = 10787]

1. $\phi(t)$

[Option ID = 43145]

2. $\phi^2(t)$

[Option ID = 43146]

3. $-\phi(t)$

[Option ID = 43147]

4. $\phi\left(\frac{1}{t}\right)$

[Option ID = 43148]

45)

Let $X_1, X_2, \ldots$ is a sequence of i.i.d. uniform U(0,1) random variables. If $Y_n = \sum_{i=1}^{n} X_i, n = 1,2,\ldots$, then

$\lim_{n\to\infty} P\left(Y_n \leq \frac{n}{2} + \sqrt{\frac{n}{12}}\right) =$

[Question ID = 10788]
1. 0.8413 [Option ID = 43149]
2. 0.9413 [Option ID = 43150]
3. 0.7413 [Option ID = 43151]
4. 0.6413 [Option ID = 43152]

46)

If $X_n \xrightarrow{d} X$, $Z_n \xrightarrow{d} Z$ and $Y_n \xrightarrow{p} a$, where '*d*' stands for distribution and '*p*' stands for probability then,

which of the following is/are TRUE according to the Slutsky's theorem?

A. $X_n Y_n \xrightarrow{d} aX$

B. $X_n Y_n \xrightarrow{p} aX$

C. $X_n + Y_n \xrightarrow{d} X + a$

D. $X_n + Z_n \xrightarrow{d} X + Z$

Choose the ***correct*** answer from the options given below:

[Question ID = 10789]
1. Both A and C [Option ID = 43153]
2. Both B and D [Option ID = 43154]
3. B only [Option ID = 43155]
4. D only [Option ID = 43156]

47) Let $X_1, X_2, \dots, X_n$ be independent random variables with sum S_n and average A_n. Let $\epsilon > 0$. Which of the following can you approximate using the central limit theorem?

[Question ID = 10790]
1. P(A_n ≥ ∈)

   [Option ID = 43157]
2. P(S_n ≤ ∈)

   [Option ID = 43158]
3. P(-∈ < A_n ≤ 2∈)

   [Option ID = 43159]
4. All of the above

   [Option ID = 43160]

48) The cumulative distribution function F(x) of a discrete random variable X is given by F(0) = 0.30, F(1) = 0.70, F(2) = 0.90, and F(3) = 1.0, then the value of the probability mass function p(x) at x = 1 is[Question ID = 10791]
1. 0.30 [Option ID = 43161]
2. 0.40 [Option ID = 43162]
3. 0.20 [Option ID = 43163]
4. 0.80 [Option ID = 43164]

49)

Let X be a random variable which is symmetric about 0. Let F be the cumulative distribution function of X.

Which of the following statement is always true?

[Question ID = 10792]
1. $F(x) + F(-x) = 1$ for all $x \epsilon \Re$.

   [Option ID = 43165]
2. $F(x) - F(-x) = 0$ for all $x \epsilon \Re$.

   [Option ID = 43166]
3. $F(x) + F(-x) = 1 + P(X = x)$ for all $x \epsilon \Re$.

   [Option ID = 43167]
4. $F(x) + F(-x) = 1 - P(X = -x)$ for all $x \epsilon \Re$.

   [Option ID = 43168]

50) Which of the following statements is/are true of a BIBD with usual notations?

A. The necessary conditions for the existence of a BIBD are vr = bk, r(k-1) = λ (v-1), b ≤ v

B. BIBD is binary

C. BIBD exists for all choices of v, b, r and k.

Choose the correct option.

[Question ID = 11367]

1. A, B and C

   [Option ID = 45465]
2. A and B only

   [Option ID = 45466]
3. B only

   [Option ID = 45467]
4. B and C only

   [Option ID = 45468]