QUESTION PAPER CODE 65/3/A

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Order 2 or degree = 1 $\frac{1}{2}$ m

 $\frac{1}{2}$ m sum = 3

Writing $\int \frac{y}{\sqrt{1+y^2}} dy = -\int \frac{x dx}{\sqrt{1+x^2}}$ $\frac{1}{2}$ m

Getting $\sqrt{1+y^2} + \sqrt{1+x^2} = c$ $\frac{1}{2}$ m

3.

Vector Perpendicular to \vec{a} and $\vec{b} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$ [Finding or using]

Required Vector = $\hat{i} - 11\hat{j} - 7\hat{k}$ $\frac{1}{2}$ m

Writing standard form

$$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$$
 and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

Finding
$$\theta = \frac{\pi}{2}$$

6.
$$\overrightarrow{OB} = \frac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$$

$$\overrightarrow{OC} = \overrightarrow{2b} - \overrightarrow{a}$$
 1/2 m

Marks

SECTION - B

7.
$$I = \int_{0}^{\frac{\pi}{2}} \frac{dx}{1 + 4 \tan^{2} x} = \int_{0}^{\frac{\pi}{2}} \frac{\sec^{2} x}{(1 + \tan^{2} x) (1 + 4 \tan^{2} x)} dx$$

Put tan x = t

$$I = \int_{0}^{\infty} \frac{dt}{(1+t^{2})(1+4t^{2})} = -\frac{1}{3} \int_{0}^{\infty} \frac{dt}{1+t^{2}} + \frac{4}{3} \int_{0}^{\infty} \frac{dt}{1+(2t)^{2}}$$
1 m

$$I = -\frac{1}{3} \tan^{-1}t \Big]_{0}^{\infty} + \frac{4}{3 \times 2} \tan^{-1}(2t) \Big]_{0}^{\infty}$$

$$= -\frac{1}{3} \tan^{-1}t \Big]_{0}^{\infty} + \frac{4}{3 \times 2} \tan^{-1}(2t) \Big]_{0}^{\infty}$$

$$= -\frac{1}{3} \left(\frac{\pi}{2}\right) + \frac{2}{3} \left(\frac{\pi}{2}\right) = \frac{\pi}{6}$$

$$I = -\frac{\pi}{4} \frac{\sin x + \cos x}{(\sin x - \cos x)^{2} - 2^{2}} dx$$

$$Put \sin x - \cos x = t \implies t = -1 \text{ to } 0$$

$$I = -\frac{\pi}{4} \frac{\sin x + \cos x}{(\sin x - \cos x)^{2} - 2^{2}} dx$$

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$$I = -\frac{\pi}{4} \frac{\sin x$$

8.
$$I = -\int_{0}^{\frac{\pi}{4}} \frac{\sin x + \cos x}{(\sin x - \cos x)^{2} - 2^{2}} dx$$

Put
$$\sin x - \cos x = t \implies t = -1$$
 to 0

 $(\cos x + \sin x) dx = dt$

$$I = -\int_{-1}^{0} \frac{dt}{t^2 - 2^2}$$

$$= -\frac{1}{4} \log \left| \frac{t-2}{t+2} \right|^0$$

$$= -\frac{1}{4} \left\{ 0 - \log 3 \right\}$$
¹/₂ m

$$= \frac{1}{4} \log 3$$



9. Writing
$$\vec{d} = \lambda \left(\vec{a} \times \vec{b} \right)$$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$$1 \text{ m}$$

$$= \lambda \left(32 \,\hat{i} - \hat{j} - 14 \,\hat{k}\right) \dots (1)$$

$$\overrightarrow{c} \cdot \overrightarrow{d} = 27$$

$$\left(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}\right) \cdot \lambda \left(32\hat{\mathbf{i}} - \hat{\mathbf{j}} - 14\hat{\mathbf{k}}\right) = 27$$

$$9 \lambda = 27$$

$$\lambda = 3$$
1 m

$$\therefore \vec{d} = 96\hat{i} - 3\hat{j} - 42\hat{k}$$

$$\therefore S.D = \left| \frac{\begin{pmatrix} \overrightarrow{a}_2 - \overrightarrow{a}_1 \end{pmatrix} \times \overrightarrow{b}}{\begin{vmatrix} \overrightarrow{b} \end{vmatrix}} \right|$$
1 m

$$\overrightarrow{a}_2 - \overrightarrow{a}_1 = \overrightarrow{i} + 2 \overrightarrow{j} + 2 \overrightarrow{k}$$
 and $\rightarrow b = 2 \overrightarrow{i} + 3 \overrightarrow{j} + 4 \overrightarrow{k}$

$$\therefore \text{ S. D} = \left| \frac{2 \hat{i} - \hat{k}}{\sqrt{29}} \right| = \frac{\sqrt{5}}{\sqrt{29}} \text{ or } \frac{\sqrt{145}}{29}$$

OR

Required equation of plane is

$$2x + y - z - 3 + \lambda (5x - 3y + 4z + 9) = 0 \rightarrow (1)$$

$$x(2+5\lambda) + y(1-3\lambda) + z(-1+4\lambda) + 9\lambda - 3 = 0$$

(1) is parallel to
$$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$$

$$\therefore 2(2+5\lambda)+4(1-3\lambda)+5(-1+4\lambda)=0$$

(1) is parallel to
$$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$$

$$\therefore 2(2+5\lambda)+4(1-3\lambda)+5(-1+4\lambda)=0$$

$$\Rightarrow \lambda = -\frac{1}{6}$$
1 m
(1) $\Rightarrow 7x+9y-10z-27=0$
1 m
1. P (step forward) = $\frac{2}{5}$, P (step backword) = $\frac{3}{5}$

$$(1) \Rightarrow 7x + 9y - 10z - 27 = 0$$
1 m

11. P (step forward) =
$$\frac{2}{5}$$
, P (step backword) = $\frac{3}{5}$

He can remain a step away in either of the

or 2 steps forward & 3 backwards

$$\therefore \text{ required possibility } = {}^{5}C_{3} \left(\frac{2}{5}\right)^{3} \left(\frac{3}{5}\right)^{2} + {}^{5}C_{2} \left(\frac{2}{5}\right)^{2} \left(\frac{3}{5}\right)^{3}$$
 2 m

$$=\frac{72}{125}$$
 1/2 m

OR



A die is thrown

Let E₁ be the event of getting 1 or 2

Let E, be the event of getting 3, 4, 5 or 6

Let A be the event of getting a tail

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3}$$

$$\Rightarrow P\left(A_{E_1}\right) = \frac{3}{8}, \& P\left(A_{E_2}\right) = \frac{1}{2}$$

$$P(E_{2}) \times P(A_{E_{1}}) = \frac{P(E_{2}) \times P(A_{E_{2}})}{P(E_{1}) \times P(A_{E_{1}}) + P(E_{2}) \times P(A_{E_{2}})}$$
1 m

$$P(E_{2}/A) = \frac{P(E_{2}) \times P(A/E_{2})}{P(E_{1}) \times P(A/E_{1}) + P(E_{2}) \times P(A/E_{2})}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}}$$

$$= \frac{8}{11}$$
India's largest Student Review 1 m

12.
$$A = IA$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Using elementary row trans formations to get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} A$$
2 m

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$$
1 m



$$AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$$
1 m

$$BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$$
1 m

$$AC + BC = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$
¹/₂ m

$$(A+B) C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

Yes,
$$(A + B) C = AC + BC$$

13.
$$f(x) = \begin{cases} -2x+1 & \text{if } x < 0 \\ 1 & \text{if } 0 \le x < 1 \\ 2x-1 & \text{if } x \ge 1 \end{cases}$$

Only possible discontinuties are at x = 0, x = 1

at
$$x = 0$$
: at $x = 1$

L. H. limit
$$= 1$$
: L. H. limit $= 1$

$$f(0) = R. H. limit = 1 : f(1) = R. H. limit = 1$$

f(x) is continuous in the interval (-1, 2)

At x = 0

L. H. D =
$$-2 \neq R$$
. H. D = 1

 \therefore f(x) is not differentiable in the interval (-1, 2)

14.
$$x = a (\cos 2t + 2t \sin 2t)$$

$$y = a (\sin 2t - 2t \cos 2t)$$

$$\Rightarrow \frac{dx}{dt} = 4 at \cos 2 t$$

$$\Rightarrow \frac{dy}{dt} = 4 \text{ at } \sin 2 t$$

$$\Rightarrow \frac{dy}{dx} = \tan 2 t$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\sec^2 2t \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2 \operatorname{at} \cos^3 2 t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2 \operatorname{at} \cos^3 2 t}$$

15.
$$\frac{y}{x} = \log x - \log (ax + b)$$

$$= \frac{x \frac{dy}{dx} - y}{x^{2}} = \frac{1}{x} - \frac{a}{ax + b} = \frac{b}{x (ax + b)}$$

$$= x \cdot \frac{dy}{dx} - y = \frac{bx}{(ax+b)}$$
(1)

differentiating w.r.t. x again

$$x \frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(ax+b)b - abx}{(ax+b)^2}$$



$$x \frac{d^2y}{dx^2} = \frac{b^2}{(ax+b)^2}$$

Writing
$$\Rightarrow x^3 \frac{d^2y}{dx^2} = \left(\frac{bx}{ax+b}\right)^2$$
(2)

From (1) and (2) \Rightarrow

$$x^3 \frac{d^2y}{dx^2} = \left(x \cdot \frac{dy}{dx} - y\right)^2$$

16.
$$I = \int \frac{x + \sin x - x (1 + \cos x)}{x (x + \sin x)} dx$$

$$= \int \frac{1}{x} dx - \int \frac{1 + \cos x}{x + \sin x} dx$$

$$= \log |x| - \log |x + \sin x| + c$$
1 m

1 m

$$= \int \frac{1}{x} dx - \int \frac{1 + \cos x}{x + \sin x} dx$$

$$\Rightarrow (1 + \cos x) dx = dt$$

$$2 \text{ m}$$

$$= \log |x| - \log |x + \sin x| + c$$
OR

$$I = \int \frac{(x-1)(x^2+x+1)+1}{(x-1)(x^2+1)} dx$$
¹/₂ m

$$= \int \frac{x^2 + x + 1}{x^2 + 1} dx + \int \frac{dx}{(x - 1)(x^2 + 1)}$$

$$= \int \left(1 + \frac{x}{x^2 + 1} + \frac{1}{2} \frac{1}{x - 1} - \frac{1}{2} \frac{x}{x^2 + 1} - \frac{1}{2} \frac{1}{x^2 + 1}\right) dx$$

$$= x + \frac{1}{4} \log |x^2 + 1| + \frac{1}{2} \log |x - 1| - \frac{1}{2} \tan^{-1} x + c$$



17. Family A
$$\Rightarrow$$
 $\begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$ 2 m

Writing Matrix Multiplication as
$$\begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix}$$
 1 m

Writing about awareness of balanced diet

Alt: Method

Taking the given data for all Men, all Women, all Children for each family, the solution must be given marks accordingly

18.
$$\tan \left\{ \tan^{-1} \left(\frac{1}{5} \right) + \frac{\pi}{4} \right\} = \tan \left\{ \tan^{-1} \left(\frac{\frac{2}{5}}{1 - \frac{1}{25}} \right) + \frac{\pi}{4} \right\}$$

$$= \tan \left\{ \tan^{-1} \left(\frac{5}{12} \right) + \frac{\pi}{4} \right\}$$

$$= \frac{\frac{5}{12} + 1}{1 - \frac{5}{12}} = \frac{17}{7}$$
 1+1 m

19. Writing $C_1 \leftrightarrow C_2$

$$A = -2 \begin{vmatrix} 1 & a^{3} & a \\ 1 & b^{3} & b \\ 1 & c^{3} & c \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_2 \& R_2 \rightarrow R_2 - R_3$$



$$A = -2 \begin{vmatrix} 0 & a^{3} - b^{3} & a - b \\ 0 & b^{3} - c^{3} & b - c \\ 1 & c^{3} & c \end{vmatrix}$$
 1+1 m

$$A = -2 (a-b) (b-c) \begin{vmatrix} 0 & a^{2} + ab + b^{2} & 1 \\ 0 & b^{2} + c^{2} + bc & 1 \\ 1 & c^{3} & c \end{vmatrix}$$
1 m

$$= -2 (a-b) (b-c) \{a^2 + ab + b^2 - b^2 - bc - c^2\}$$
 1/2 m

$$= 2 (a-b) (b-c) (c-a) (a+b+c)$$
 ½ m

SECTION - C

$$0 \qquad \frac{{}^{2}C_{0} \times {}^{5}C_{2}}{{}^{7}C_{2}} = \frac{20}{42} \qquad 0 \qquad \text{For P (x)} \qquad 1\frac{1}{2} \text{ m}$$

$$1 \qquad \frac{{}^{2}C_{1} \times {}^{5}C_{1}}{{}^{7}C_{2}} = \frac{20}{42} \qquad \frac{20}{42} \qquad \frac{20}{42} \qquad \text{For x P (x)}$$

$$\frac{{}^{2}C_{2} \times {}^{5}C_{0}}{{}^{7}C} = \frac{2}{42} \qquad \frac{4}{42} \qquad \frac{8}{42} \qquad \text{For } x^{2} P(x) \qquad \frac{1}{2} m$$

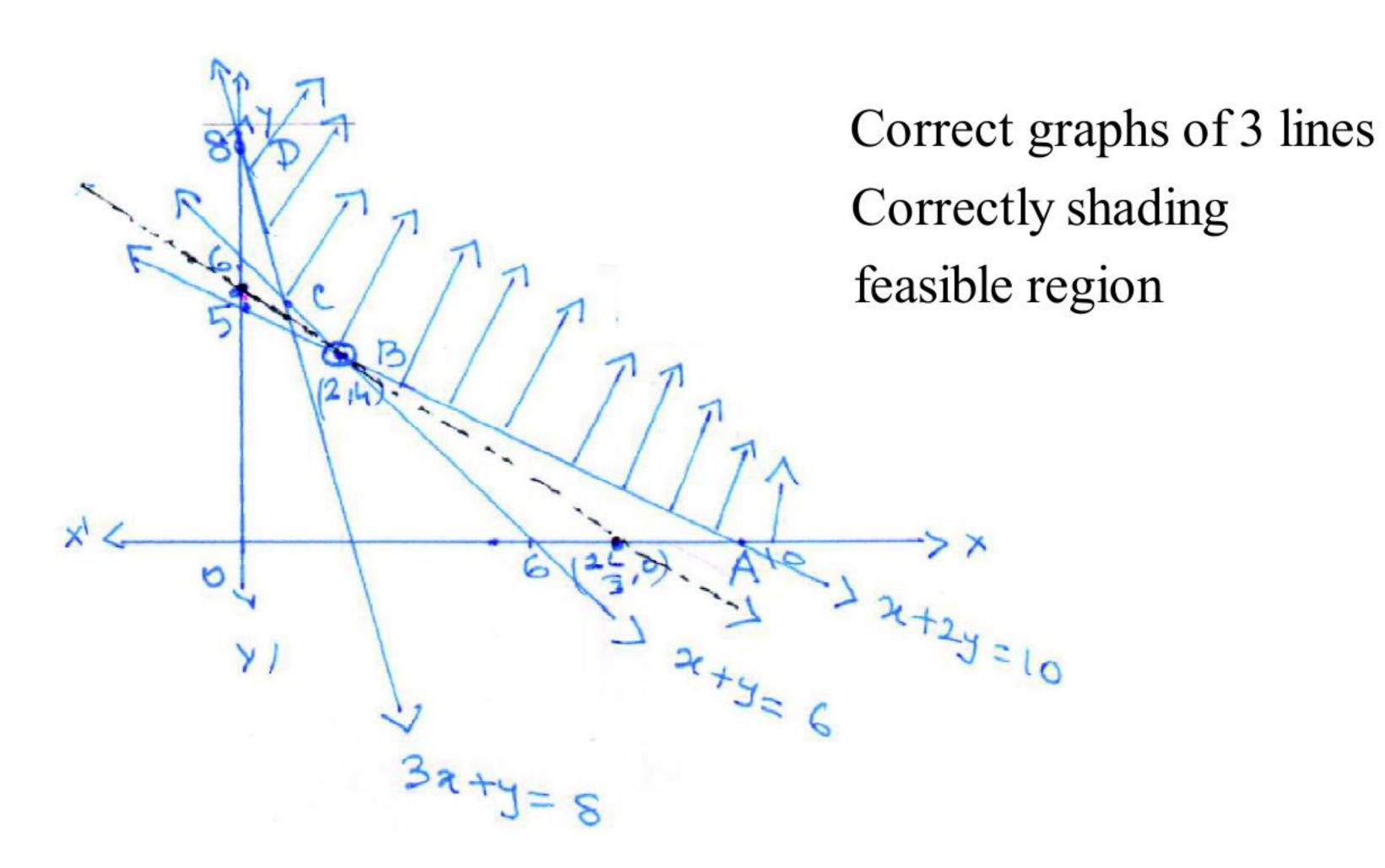
$$\sum x P(x) = \frac{24}{42}; \sum x^2 P(x) = \frac{28}{42}$$
 1 m

Mean =
$$\sum x P(x) = \frac{4}{7}$$
; var iance = $\sum x^2 P(x) - \left[\sum x P(x)\right]^2$

Variance =
$$\frac{50}{147}$$
 = $\frac{2}{3} - \frac{16}{49} = \frac{50}{147}$



21.



Vertices are A (10, 0), B (2, 4), C (1, 5) & D (0, 8)

1 m

3 m

 $\frac{1}{2}$

Z = 3x + 5y is minimum

region, Hence, x = 2, y = 4 gives minimum Z Here $R = \{(a, b) : a, b \in \Re \text{ and } a - b + \sqrt{3} \in S, \text{ where } a \in \mathbb{R} \}$

S is the set of all irrational numbers.}

(i) $\forall a \in \Re$, $(a, a) \in R$ as $a - a + \sqrt{3}$ is irrational

: R is reflexive

 $1\frac{1}{2}$ m

(ii) Let for a, $b \in \Re$, $(a, b) \in R$ i. e. $a - b + \sqrt{3}$ is irrational

$$a-b+\sqrt{3}$$
 is irrational $\Rightarrow b-a+\sqrt{3} \in S$: $(b,a) \in R$

Hence R is symmetric

 $2 \, \mathrm{m}$

(iii) Let $(a, b) \in R$ and $(b, c) \in R$, for $a, b, c \in \Re$

$$\therefore$$
 a-b+ $\sqrt{3} \in S$ and b-c+ $\sqrt{3} \in S$

adding to get $a-c+2\sqrt{3} \in S$ Hence $(a, c) \in R$

 $2\frac{1}{2}$ m

R is Transitive

OR

 \forall a, b, c, d, e, f \in \Re

$$((a,b)*(c,d))*(e,f) = (a+c,b+d)*(e,f)$$

$$= (a+c+e,b+d+f) \rightarrow (3)$$

$$(a, b)*((c, d)*(e, f)) = (a, b)*(c + e, d + f)$$

$$= (a + c + e, b + d + f) \rightarrow (4)$$

$$\therefore * \text{ is Associative}$$
Let (x, y) be on identity element in $\Re \times \Re$

$$\Rightarrow (a, b)*(x, y) = (a, b) = (x, y)*(a, b)$$

$$\Rightarrow a + x = a, b + y = b$$

$$x = 0, y = 0$$

$$2 \text{ m}$$

$$\Rightarrow a + x = a, b + y = b$$

$$x = 0$$
, $y = 0$

 \therefore (0, 0) is identity element

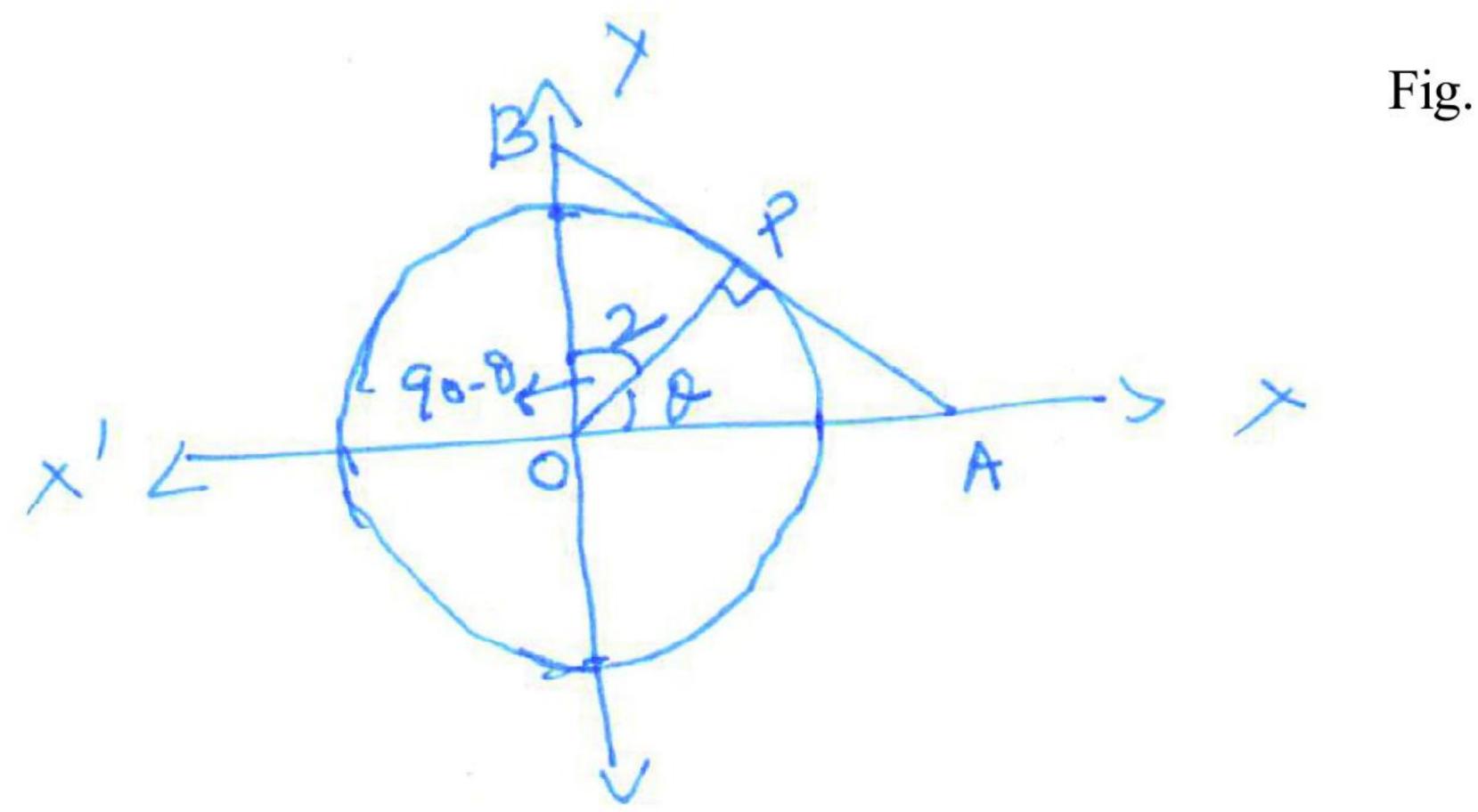
Let the inverse element of (3, -5) be $(x_1, y_1,)$

$$\Rightarrow (3,-5) * (x_1, y_1) = (0,0) = (x_1, y_1) * (3,-5)$$
$$3 + x_1 = 0, -5 + y_1 = 0$$
$$x_1 = -3, y_1 = 5$$

$$\Rightarrow$$
 (-3, 5) is an inverse of (3, -5)

 $2 \, \mathrm{m}$

23.



$$x^2 + y^2 = 4$$
. OP is \perp to AB

$$\cos \theta = \frac{2}{OA}$$
; $OA = 2 \sec \theta$

 $\frac{1}{2}$ m

 $\frac{1}{2}$ m

$$\cos(90^{\circ} - \theta) = \frac{2}{OB}$$

$$OB = 2 \csc \theta$$

Let
$$S = OA + OB = 2 (\sec \theta + \csc \theta) \dots (1)$$

$$\cos(90^{\circ} - \theta) = \frac{2}{\text{OB}}$$

$$OB = 2 \csc \theta$$

$$\text{Let } S = OA + OB = 2 (\sec \theta + \csc \theta) \dots (1)$$

$$\frac{dS}{d\theta} = 2 (\sec \theta \tan \theta - \csc \theta \cot \theta)$$

$$1 \text{ m}$$

$$\cos(90^{\circ} - \theta) = \frac{2}{OB}$$

$$1 \text{ m}$$

$$1 \text{ m}$$

$$= 2 \left(\frac{\sin^3 \theta - \cos^3 \theta}{\sin^2 \theta \cdot \cos^2 \theta} \right) \tag{2}$$

for maxima or minima $\frac{dS}{d\theta} = 0$

$$\Rightarrow \theta = \frac{\pi}{4},$$

(2)
$$\Rightarrow \frac{d^2S}{d\theta^2} > 0 \text{ when } \theta = \frac{\pi}{4}$$

 \therefore OA + OB is minimum

$$\Rightarrow$$
 OA + OB = $4\sqrt{2}$ unit

24.
$$(x-y) \frac{dy}{dx} = x + 2y$$

$$\frac{dy}{dx} = \frac{x + 2y}{x - y}$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1+2\frac{y}{x}}{1-\frac{y}{x}} = f\left(\frac{y}{x}\right)...(1)$$

: differential equation is homogeneous Eqn.

1 m

y = vx to give

$$v + x \cdot \frac{dv}{dx} = \frac{1 + 2v}{1 - v}$$

$$\Rightarrow \int \frac{1-v}{1+v+v^2} dv = \int \frac{dx}{x}$$

$$y = vx \text{ to give}$$

$$v + x \cdot \frac{dv}{dx} = \frac{1+2v}{1-v}$$

$$\Rightarrow \int \frac{1-v}{1+v+v^2} dv = \int \frac{dx}{x}$$

$$1 \text{ m}$$

$$\Rightarrow \frac{1}{2} \int \frac{2v+1}{1+v+v^2} dv + \frac{3}{2} \int \frac{dv}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \int \frac{dx}{x}$$

$$1\frac{1}{2} \log \left|1+v+v^2\right| + \sqrt{3} \tan^{-1} \left(\frac{2v+1}{\sqrt{3}}\right) = \log |x| + c$$

$$1 \text{ m}$$

$$-\frac{1}{2}\log \left|1+v+v^{2}\right| + \sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = \log \left|x\right| + c$$

1 m

$$-\frac{1}{2} \log \left| \frac{x^2 + xy + y^2}{x^2} \right| + \sqrt{3} \tan^{-1} \left(\frac{2y + x}{x\sqrt{3}} \right) = \log |x| + c$$

1 m

OR

$$(x-h)+(y-k)\frac{dy}{dx} = 0$$

1 m

and
$$1 + (y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

1 m

$$\Rightarrow (y-k) = \frac{-\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$
1 m

(1)
$$\Rightarrow$$
 $(x-h) = \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \frac{dy}{dx}$ 1 m

Putting in the given eqn.

$$\frac{\left(1 + \left(\frac{dy}{dx}\right)^{2}\right)^{2}}{\left(\frac{d^{2}y}{dx^{2}}\right)^{2}} \cdot \left(\frac{dy}{dx}\right)^{2} + \frac{\left(1 + \left(\frac{dy}{dx}\right)^{2}\right)^{2}}{\left(\frac{d^{2}y}{dx^{2}}\right)^{2}} = r^{2}$$
or
$$\left[1 + \left(\frac{dy}{dx}\right)^{2}\right]^{3} = r^{2} \left(\frac{d^{2}y}{dx^{2}}\right)^{2}$$
1 m

25. Eqn. of a plane through

and Points A (6, 5, 9), B (5, 2, 4) & C (-1, -1, 6) is

$$\Rightarrow \begin{vmatrix} x-6 & y-5 & z-9 \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 0$$
2½ m

$$\Rightarrow 3x - 4y + 3z - 25 = 0 \rightarrow (2)$$

distance from (3, -1, 2) to (2)

$$d = \left| \frac{9 + 4 + 6 - 25}{\sqrt{9 + 16 + 9}} \right| = \frac{6}{\sqrt{34}} \text{ units}$$



26.

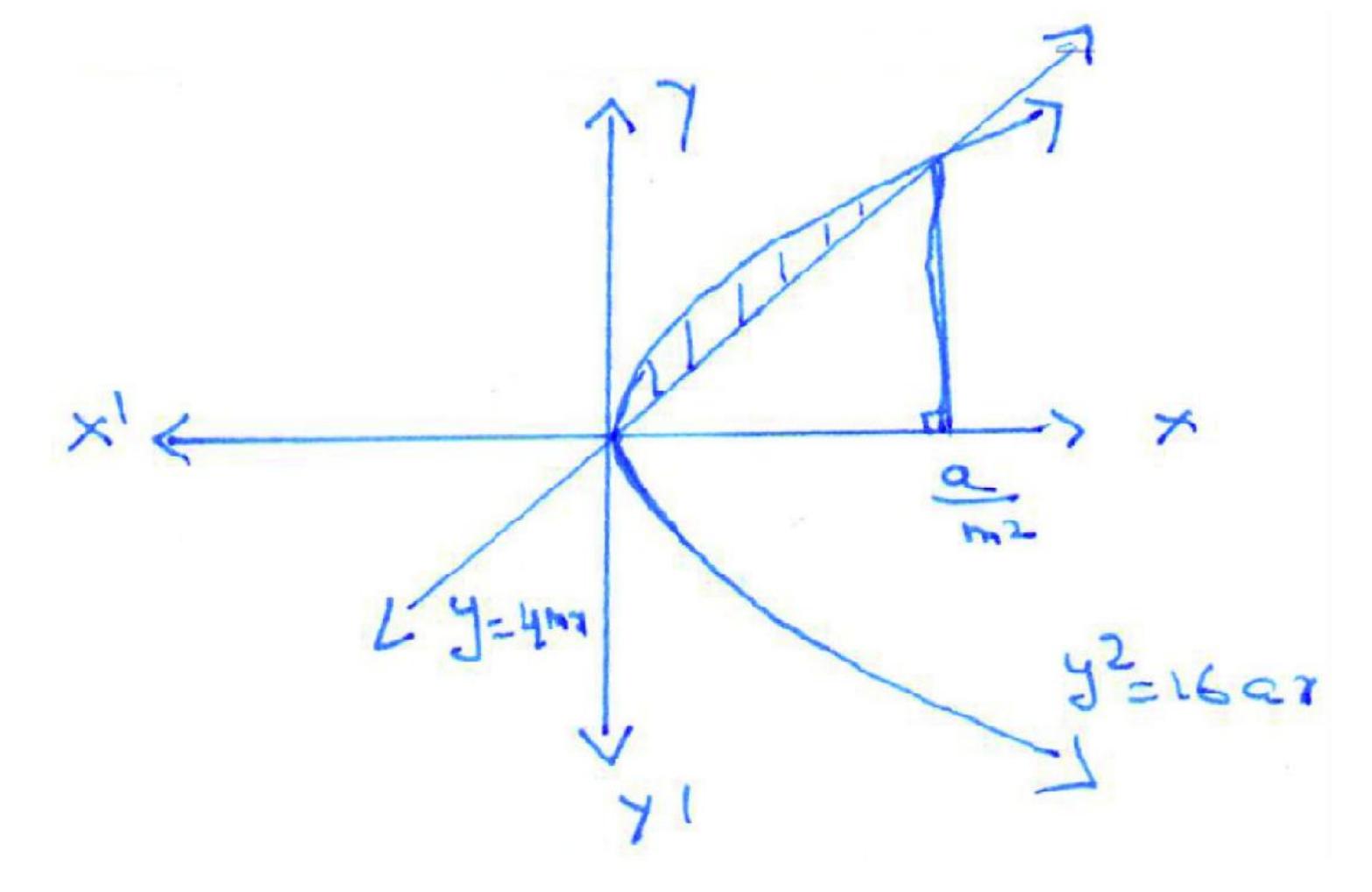


Figure $\frac{1}{2}$ m

 $y = 4mx \rightarrow (1)$ and $y^2 = 16$ ax \rightarrow (2)

$$\Rightarrow x = \frac{a}{m^2}$$

Required area $= 4\sqrt{a} \int_{0}^{\frac{a}{m^{2}}} \sqrt{x} \, dx - 4m \int_{0}^{\frac{a}{m^{2}}} x \, dx$ $= \frac{8}{3} \sqrt{a} x^{\frac{3}{2}} \Big]_{0}^{\frac{a}{m^{2}}} - 2m x^{2} \Big]_{0}^{\frac{a}{m^{2}}}$ 2 m

$$= \frac{8}{3} \sqrt{a} x^{\frac{3}{2}} \Big]_{0}^{\frac{a}{m^{2}}} - 2m x^{2} \Big]_{0}^{\frac{a}{m^{2}}}$$

$$= \frac{8}{3} \frac{a^2}{m^3} - \frac{2a^2}{m^3} = \frac{2}{3} \frac{a^2}{m^3}$$

2 m

$$\Rightarrow \frac{2}{3} \cdot \frac{a^2}{m^3} = \frac{a^2}{12} \text{ given}$$

$$m^3 = 8$$

$$m = 2$$

 $\frac{1}{2}$ m