JEE-Main-26-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: How many 5 digit number can be formed such that product of digits is 30.
Answer: 80.00
Solution:

$x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 = 30 = 5 \times 2 \times 3$

$1\ 1\ 2\ 3\ 5 \rightarrow \frac{5!}{2!} = 60$

$1\ 1\ 1\ 6\ 5 \rightarrow \frac{5!}{3!} = 20$

$60 + 20 = 80$

Question: $f(3x) - f(x) = x, f(8) = 7,$ find $f(14)$.
Answer: 10.00
Solution:

$f(3x) - f(x) = x$

$f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3}$

$f\left(\frac{x}{3}\right) - f\left(\frac{x}{3^2}\right) = \frac{x}{3^2}$

$\vdots$

On adding, we get

$f(x) - \lim_{n \to \infty} f\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2} + \ldots + \infty\right)$

$\Rightarrow f(x) - f(0) = \frac{x}{2}$

$\because f(8) = 7,$ so $f(0) = 3$

$\therefore f(x) = \frac{x}{2} + 3$

$\therefore f(14) = 10$

Question: Coefficient of x & x^2 in $(1+x)^p \times (1-x)^q$ are -3 & -5 respectively. Find coefficient of x^3.
Answer: 23.00
Solution:

Given $(1+x)^p \times (1-x)^q$

$$\left({}^pC_0 + {}^pC_1x + {}^pC_2x^2 + {}^pC_3x^3 \ldots\right)\left({}^qC_0 - {}^qC_1x + {}^qC_2x^2 - {}^qC_3x^3 \ldots\right)$$

$$p - q = -3$$

$$-pq + \frac{q(q-1)}{2} + \frac{p(p-1)}{2} = -5$$

$$-2pq + q^2 - q + p^2 - p = -10$$

$$(p-q)^2 - p - q = -10$$

$$9 - p - q = -10$$

$$p + q = 19$$

$$\Rightarrow p = 8,\ q = 11$$

Coefficient of $x^3 = -{}^qC_3 + {}^pC_3 + p\,{}^qC_2 - q\,{}^pC_2$

$$= -{}^{11}C_3 + {}^8C_3 + 8\,{}^{11}C_2 - 11\,{}^8C_2$$

$$= \frac{-11\cdot10\cdot9}{6} + \frac{8\cdot7\cdot6}{6} + \frac{8\cdot11\cdot10}{2} - \frac{11\cdot8\cdot7}{2}$$

$$= 23$$

Question: $\frac{dy}{dx} + (2\tan x)y = \sin x,\ y\left(\frac{\pi}{3}\right) = 0,$ find $f(x)\big|_{\max}$

Answer: $\frac{1}{8}$

Solution:

$$\frac{dy}{dx} + (2\tan x)y = \sin x$$

$$\text{IF } = e^{\int 2\tan x\,dx} = e^{-2\ln\cos x} = \frac{1}{\cos^2 x}$$

$$\frac{y}{\cos^2 x} = \int \frac{\sin x}{\cos^2 x}dx$$

$$\frac{y}{\cos^2 x} = \frac{1}{\cos x} + C$$

$$0 = \frac{1}{\cos\frac{x}{3}} + C$$

$$C = -2$$

$$y = \cos x - 2\cos^2 x$$

$$= -2\left[\cos^2 x - \frac{1}{2}\cos x\right]$$

$$= -2\left[\cos^2 x - \frac{1}{2}\cos x + \frac{1}{16} - \frac{1}{16}\right]$$

$$= -2\left[\cos x - \frac{1}{4}\right]^2 + \frac{1}{8}$$

$$y_{\max} = \frac{1}{8}$$

Question: Find sum of elements in 11^{th} term: $(3);(6,9,12);(15,18,21,24,27);.....$

Answer: 6993.00

Solution:

$$(3);(6,9,12);(15,18,21,24,27);.....$$

$$1+3+5+....10 \text{ terms} = \frac{10}{2}\left[2\times 1+(10-1)2\right]$$

$$=100$$

$$a_{146} = 3+(99)3 = 300$$

$$11^{th} \text{ term} = (303+........21 \text{ terms})$$

$$= \frac{21}{2}\left[2\times 303+(20)3\right]$$

$$=6993$$

Question: $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{\sqrt{5}}{2}\right)+2\tan^{-1}\left(\frac{1}{8}\right)\right] = ?$

Answer: 2.00

Solution:

$$2\left(\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{1}{5}\right)\right) = 2\tan^{-1}\left(\frac{\frac{1}{8}+\frac{1}{5}}{1-\frac{1}{40}}\right)$$

$$= 2\tan^{-1}\left(\frac{1}{3}\right)$$

$$= \tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right)$$

$\therefore$ The given terms reduces to

$$\tan\left(\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$= \tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{8}}\right)$$

$$= 2$$

Question: $x+y-z=0=x-2y+3z-5$. There is a line parallel to this & passing through $(1,-2,3)$. Find distance of this line from $(1,4,5)$.

Answer: ()

Solution:

$x+y-z=0,\ x-2y+3z-5=0$

$$n=\begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix}=i(3-2)-j(3+1)+k(-2-1)$$

$$= i-4j-3k$$

Line $\Rightarrow \dfrac{x-1}{1}=\dfrac{y+2}{-4}=\dfrac{z-3}{-3}=\lambda$

$P \equiv x=\lambda+1, y=-4\lambda+2, z=-3\lambda+3$

$PQ=(\lambda,-4\lambda-2,-3\lambda-2)$

Now, $\lambda-4(-4\lambda-2)-3(-3\lambda-2)=0$

$\lambda+16\lambda+8+9\lambda+6=0$

$26\lambda+14=0$

$\lambda=\dfrac{-14}{26}=\dfrac{-7}{13}$

$$PQ=\sqrt{\lambda^2+(-4\lambda-2)^2+(-3\lambda-2)^2}$$

$$=\sqrt{\lambda^2+16\lambda^2+16\lambda+4+9\lambda^2+12\lambda+4}$$

$$=\sqrt{26\lambda^2+18\lambda+4}$$

$$=\sqrt{26\left(\frac{-7}{13}\right)^2+18\left(\frac{-7}{13}\right)+4}$$

$$=\sqrt{\frac{2\times 49}{13}-\frac{7\times 18}{13}+4}$$

$$=\sqrt{\frac{24}{13}}$$

Question: Area under $y=f(x)$ from 3 to x (where $x>3$) is $\left(\frac{y}{x}\right)^3\cdot f(3)=3$ then for $y=6\sqrt{10}$, what will be x?

Answer: 6.00

Solution:

$$\int_3^x f(t)\,dt = \left(\frac{y}{x}\right)^3$$

$$f(x) = 3\left(\frac{y}{x}\right)^2 \left(\frac{y'x - y}{x^2}\right)$$

$$y = \frac{3y^2}{x^2}\left(\frac{y'x - y}{x^2}\right)$$

$$\Rightarrow x^4 = 3y(y'x - y)$$

$$\Rightarrow x^2 dx = 3y\left(\frac{xdy - ydx}{x^2}\right)$$

$$\Rightarrow x^2 dx = 3y\, d\left(\frac{y}{x}\right)$$

$$\Rightarrow xdx = 3\frac{y}{x} d\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{x^2}{2} = \frac{3}{2}\left(\frac{y}{x}\right)^2 + C$$

$$\Rightarrow x^2 = \frac{3y^2}{x^2} + C$$

$$\Rightarrow 3^2 = 3 + C$$

$$\Rightarrow C = 6$$

$$\Rightarrow x^2 = \frac{3y^2}{x^2} + 6$$

$$y = 6\sqrt{10}$$

$$\Rightarrow x^2 = \frac{3\cdot 36\times 10}{x^2} + 6$$

$$\Rightarrow x = 6$$

Question: From a group of 10 boys $B_1, B_2, \ldots, B_{10}$ and 5 girls $G_1, G_2, \ldots G_5$, the number of ways of selection of group of 3 boys and 3 girls, such that B_1 & B_2 are not together in group is ____.

Answer: 1120.00

Solution:

Number of ways to select 3 boys = Total ways – No. of ways when both B_1 & B_2 are selected

$= {}^{10}C_3 - {}^{8}C_1 = 112$

Number of ways to select 3 girls $= {}^{5}C_3 = 10$

Required number of ways $= 112 \times 10 = 1120$

Question: $f(x) = \sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}}$, find relation between $f\left(\dfrac{a}{2}\right)$ & $f'\left(\dfrac{a}{2}\right)$

Options:

(a) $\sqrt{2f\left(\frac{a}{2}\right)}=f'\left(\frac{a}{2}\right)$

(b)

(c)

(d)

Answer: ()

Solution:

$$f(x)=\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$$

$$f(x)=\sqrt{\frac{\sin^2 x}{\cos^2 x}}=\tan x$$

$$f'(x)=\sec^2 x=1+\tan^2 x$$

$$f'(x)=1+f^2(x)$$

$$f'\left(\frac{a}{2}\right)=1+f^2\left(\frac{a}{2}\right)$$

Question: $f(x)=\begin{cases}\dfrac{\ln\left(1-x+x^2\right)+\ln\left(1+x+x^2\right)}{\sec x-\cos x} & ; \quad x<0 \\ k & ; \quad x\geq 0\end{cases}$ is continuous, find k.

Answer: 1.00

Solution:

$$\lim_{x\to 0}\frac{\ln\left(1-x+x^2\right)+\ln\left(1+x+x^2\right)}{\sec x-\cos x}$$

$$\lim_{x\to 0}\frac{\frac{1}{1-x+x^2}(-1+2x)+\frac{1}{1+x+x^2}(1+2x)}{\sec x\tan x+\sin x}$$

$$\lim_{x\to 0}\frac{(2x-1)\left(1+x+x^2\right)+(2x+1)\left(1-x+x^2\right)}{\left(1+x^2+x^4\right)(\sec x\tan x+\sin x)}$$

$$=\lim_{x\to 0}\frac{2x+2x^2+2x^3-1-x-x^2+2x-2x^2+2x^3+1-x+x^2}{\left(1+x^2+x^4\right)(\sec x\tan x+\sin x)}$$

$$=\lim_{x\to 0}\frac{4x^3+2x}{\left(1+x^2+x^4\right)\left(\sec^2 x\sin x+\sin x\right)}$$

$$=\lim_{x\to 0}\frac{4x^2+2}{\left(1+x^2+x^4\right)\left(\sec^2 x+1\right)}$$

$$=\frac{2}{1\times 2}=1$$

Question: Normal to $y^2 = 24x$ at (α, β) is perpendicular to $2x + 2t = 5$. Find equation of normal to $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at $(\alpha + 4, \beta + 10)$.

Answer: ()

Solution:

$$\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1 \quad (\alpha + 4, \beta + 10)$$

$$\beta^2 = 24\alpha$$

$$2yy' = 24$$

$$y' = \frac{12}{y} = \frac{12}{\beta}$$

$$\Rightarrow \frac{12}{\beta} = 1 \Rightarrow \beta = 12$$

$$12^2 = 24 \cdot \alpha$$

$$\Rightarrow \alpha = 6$$

$$\frac{x^2}{36} - \frac{y^2}{144} = 1$$

$$\frac{2x}{36} - \frac{2yy'}{144} = 0$$

$$\frac{x}{36} = \frac{yy'}{144}$$

$$\Rightarrow y' = \frac{144}{136} \cdot \frac{x}{y} = \frac{4x}{y} = \frac{4 \times 10}{22}$$

$$= \frac{20}{11}$$

$$y - 22 = \frac{20}{11}(x - 10)$$

$$11y - 242 = 20x - 200$$

$$\Rightarrow 20x - 11y + 42 = 0$$