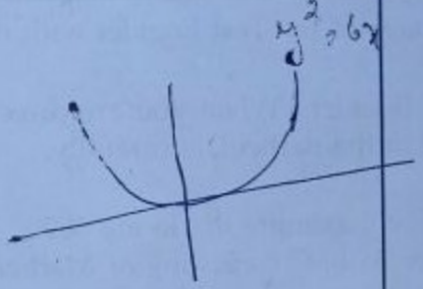
PART A – MATHEMATICS

1. If the curves $y^2=6x$, $9x^2+by^2=16$ intersect each other at right angles, then the value of b is :



(1) $\frac{9}{2}$

(2) 6

(3) $\frac{7}{2}$

(4) 4

2. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b}=24$, then $|\vec{u}|^2$ is equal to :

(1) 84

(2) 336

(3) 315

(4) 256

3. For each $t \in \mathbf{R}$, let [t] be the greatest integer less than or equal to t. Then

$$\lim_{x\to 0+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{15}{x}\right]\right)$$

(1) does not exist (in **R**).

(2) is equal to 0.

(3) is equal to 15.

(4) is equal to 120.

4. If L_1 is the line of intersection of the planes $2x-2y+3z-2=0$, $x-y+z+1=0$ and L_2 is the line of intersection of the planes $x+2y-z-3=0$, $3x-y+2z-1=0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is :

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{4\sqrt{2}}$

(3) $\frac{1}{3\sqrt{2}}$

(4) $\frac{1}{2\sqrt{2}}$

5. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}\,dx$ is :

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{8}$

(3) $\frac{\pi}{2}$

(4) 4π

6. Let $g(x)=\cos x^2$, $f(x)=\sqrt{x}$, and α, β ($\alpha<\beta$) be the roots of the quadratic equation $18x^2-9\pi x+\pi^2=0$. Then the area (in sq. units) bounded by the curve $y=(gof)(x)$ and the lines $x=\alpha$, $x=\beta$ and $y=0$, is :

(1) $\frac{1}{2}(\sqrt{2}-1)$

(2) $\frac{1}{2}(\sqrt{3}-1)$

(3) $\frac{1}{2}(\sqrt{3}+1)$

(4) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

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7. If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0, \pi]$ is $k\pi$, then k is equal to :

(1) $\frac{20}{9}$

(2) $\frac{2}{3}$

(3) $\frac{13}{9}$

(4) $\frac{8}{9}$

8. Let $f(x)=x^2+\frac{1}{x^2}$ and $g(x)=x-\frac{1}{x}$, $x \in R-\{-1, 0, 1\}$. If $h(x)=\frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is :

(1) $2\sqrt{2}$

(2) 3

(3) -3

(4) $-2\sqrt{2}$

9. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$$

is equal to :

(1) $\frac{-1}{1+\cot^3 x}+C$

(2) $\frac{1}{3(1+\tan^3 x)}+C$

(3) $\frac{-1}{3(1+\tan^3 x)}+C$

(4) $\frac{1}{1+\cot^3 x}+C$

(where C is a constant of integration)

10. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

(1) $\frac{3}{4}$

(2) $\frac{3}{10}$

(3) $\frac{2}{5}$

(4) $\frac{1}{5}$

11. Let the orthocentre and centroid of a triangle be A(−3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is :

(1) $\frac{3\sqrt{5}}{2}$

(2) $\sqrt{10}$

(3) $2\sqrt{10}$

(4) $3\sqrt{\frac{5}{2}}$

12. If the tangent at (1, 7) to the curve $x^2=y-6$ touches the circle $x^2+y^2+16x+12y+c=0$ then the value of c is :

(1) 95

(2) 195

(3) 185

(4) 85

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13. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to :

(1) 2

(2) -1

(3) 0

(4) 1

14. PQR is a triangular park with $PQ = PR = 200$ m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is :

(1) $50\sqrt{2}$

(2) 100

(3) 50

(4) $100\sqrt{3}$

15. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is :

(1) 3

(2) 9

(3) 4

(4) 2

16. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$, $(x > 1)$ is :

(1) 2

(2) -1

(3) 0

(4) 1

17. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is :

(1) $36\sqrt{5}$

(2) $45\sqrt{5}$

(3) $54\sqrt{3}$

(4) $60\sqrt{3}$

18. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) at least 750 but less than 1000

(2) at least 1000

(3) less than 500

(4) at least 500 but less than 750

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19. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) 30

(2) -10

(3) 10

(4) -30

20. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to :

(1) (4, 5)

(2) $(-4, -5)$

(3) $(-4, 3)$

(4) $(-4, 5)$

21. Two sets A and B are as under :

$A = \{(a, b) \in R \times R : |a-5| < 1$ and $|b-5| < 1\}$;

$B = \{(a, b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \leq 36\}$. Then :

(1) neither $A \subset B$ nor $B \subset A$

(2) $B \subset A$

(3) $A \subset B$

(4) $A \cap B = \phi$ (an empty set)

22. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is :

(1) $\frac{4}{3}$

(2) $\frac{1}{2}$

(3) 2

(4) 3

23. Let $S = \{t \in R : f(x) = |x-\pi| \cdot (e^{|x|} - 1) \sin|x|$ is not differentiable at t$\}$. Then the set S is equal to :

(1) $\{0, \pi\}$

(2) ϕ (an empty set)

(3) $\{0\}$

(4) $\{\pi\}$

24. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim q$

(2) $\sim p$

(3) p

(4) q

25. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

(1) $3x + 2y = 6xy$

(2) $3x + 2y = 6$

(3) $2x + 3y = xy$

(4) $3x + 2y = xy$

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26. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2+2\cdot2^2+3^2+2\cdot4^2+5^2+2\cdot6^2+.....$

If $B-2A=100\lambda$, then λ is equal to :

(1) 496

(2) 232

(3) 248

(4) 464

27. Let $y=y(x)$ be the solution of the differential equation

$\sin x\frac{dy}{dx}+y\cos x=4x,\ x\in(0,\pi)$. If $y\left(\frac{\pi}{2}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $-\frac{4}{9}\pi^2$

(2) $\frac{4}{9\sqrt{3}}\pi^2$

(3) $\frac{-8}{9\sqrt{3}}\pi^2$

(4) $-\frac{8}{9}\pi^2$

28. The length of the projection of the line segment joining the points $(5, -1, 4)$ and $(4, -1, 3)$ on the plane, $x+y+z=7$ is :

(1) $\sqrt{\frac{2}{3}}$

(2) $\frac{2}{\sqrt{3}}$

(3) $\frac{2}{3}$

(4) $\frac{1}{3}$

29. Let $S=\{x\in R : x\geq 0$ and $2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0\}$. Then S :

(1) contains exactly four elements.

(2) is an empty set.

(3) contains exactly one element.

(4) contains exactly two elements.

30. Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1}=416$ and $a_9+a_{43}=66$. If $a_1^2+a_2^2+.....+a_{17}^2=140\,m$, then m is equal to :

(1) 33

(2) 66

(3) 68

(4) 34

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