MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}\right)$ equals

(1) $\ln 2$ (2) $\ln\frac{3}{2}$

(3) $\ln\frac{2}{3}$ (4) 0

Answer (1)

Sol. $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{n+r}\right)$

$\Rightarrow \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$

$\Rightarrow \int_0^1 \frac{dx}{1+x}$

$\Rightarrow \ln(1+x)\Big|_0^1 = \ln 2$

2. For solution of $\frac{dy}{dx}+y\tan x=\sec x,\ y(0)=1$, then $y\left(\frac{\pi}{6}\right)=$

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1+\sqrt{3}}{2}$

(3) $\frac{1}{2}$ (4) $-\frac{\sqrt{3}}{2}$

Answer (2)

Sol. $\text{IF}=e^{\int \tan x\,dx}=|\sec x|$

Solution of equation is

$y\cdot|\sec x|=\int \sec x\,|\sec x|\,dx$

$\Rightarrow y\sec x=\tan x+C$

at $x = 0$, $y = 1$

$C = 1$

at $x=\frac{\pi}{6}$

$y\cdot\frac{2}{\sqrt{3}}=\frac{1}{\sqrt{3}}+1$

$y=\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)\frac{\sqrt{3}}{2}$

$=\frac{1+\sqrt{3}}{2}$

3. The sum $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\ldots\infty$ terms equals

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{1}{4}$ (4) $\frac{1}{5}$

Answer (1)

Sol. $\sum_{r=1}^{\infty}\frac{r}{1+r^2+r^4}$

$\Rightarrow \frac{1}{2}\sum_{r=1}^{\infty}\frac{(r^2+r+1)-(r^2-r+1)}{(r^2+r+1)\times(r^2-r+1)}$

$\Rightarrow \frac{1}{2}\sum_{r=1}^{\infty}\left(\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right)$

$\Rightarrow \frac{1}{2}\begin{bmatrix}1-\frac{1}{3}\\+\frac{1}{3}-\frac{1}{7}\\+\frac{1}{7}-\frac{1}{13}\\\vdots\end{bmatrix}$

$\Rightarrow \frac{1}{2}\times 1=\frac{1}{2}$

4. The number of ways by which letter of word ASSASSINATION can be arranged such that all vowels come together is

(1) $\frac{8!3!}{6!}$ (2) $\frac{8!}{4!3!}$

(3) $\frac{8!\cdot 6!}{4!(2!)^2\cdot 3!}$ (4) $\frac{8!\cdot 6!}{4!3!2!}$

Answer (3)

Sol. $A \to 3$

$S \to 4$

$I \to 2$

$N \to 2$

$T \to 1$

$O \to 1$

$\therefore$ $A \to 3, I \to 2$ & $O \to 1$ are vowels

$\therefore$ Number of ways = $\frac{8!}{4!2!}\cdot\frac{6!}{3!2!}$

5. $f(x)+f'(x)=\int_0^2 f(t)dt$ and $f(0) = e^{-2}$, then the value of $f(2) - 2f(0)$ is

(1) 0 (2) –1

(3) 1 (4) 2

Answer (2)

Sol. $f(x)+f'(x)=\int_0^2 f(t)dt = k$ (let)

$\Rightarrow e^x f(x)+e^x\cdot f'(x)=k\cdot e^x$

$\Rightarrow \int d\left(f(x)\cdot e^x\right)=\int k\cdot e^x dx$

$\Rightarrow f(x)\cdot e^x = ke^x + c$

$f(0) = e^{-2} \Rightarrow x = 0, y = f(x) = e^{-2}$

$e^{-2} = k + c \Rightarrow c = e^{-2} - k$

$y\cdot e^x = k\cdot e^x + (e^{-2} - k)$

$\Rightarrow y = k + (e^{-2} - k)e^{-x}$

Now, $\int_0^2 f(x) = k$

$\Rightarrow \int_0^2\left(k+\left(e^{-2}-k\right)e^{-x}\right)dx = k$

$\Rightarrow kx\Big|_0^2 - e^{-x}\left(e^{-2}-k\right)\Big|_0^2 = k$

$\Rightarrow 2k - (e^{-2} - k)(e^{-2} - 1) = k$

$\Rightarrow 2k - (e^{-4} - ke^{-2} - e^{-2} + k) = k$

$\Rightarrow 2k - e^{-4} + ke^{-2} + e^{-2} - k = k$

$\Rightarrow k\cdot e^{-2} = e^{-4} - e^{-2}$

$\Rightarrow k = e^{-2} - 1$

$\therefore f(x) = e^{-2} - 1 + e^{-x}$

Now, $f(2) - 2f(0)$

$= (e^{-2} - 1 + e^{-2}) - 2(e^{-2} - 1 + 1)$

$\Rightarrow 2e^{-2} - 1 - 2e^{-2}$

$= -1$

6. If set $S=\left\{\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10\right\}$ then $n(S)$ equals

(1) 2 (2) 3

(3) 4 (4) 6

Answer (1)

Sol. Let $\left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=t$

$\therefore t+\frac{1}{t}=10$

$\Rightarrow t^2-10t+1=0$

$\Rightarrow (t-5)^2=24$

$\Rightarrow \left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=5\pm 2\sqrt{6}$

$\therefore$ if $\left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=5+2\sqrt{6}$

then $x^2-4=2 \Rightarrow x=\pm\sqrt{6}$

if $\left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=5-2\sqrt{6}$

then $x^2-4=-2 \Rightarrow x^2=-2$ not possible

$\therefore$ 2 solutions

7. 1, 3, 5, x, y are 5 observations. Mean of the observations is 5 and variance is 8. The sum of the cubes of the two missing number equals

(1) 1072 (2) 513

(3) 1079 (4) 516

Answer (1)

Sol. $\bar{x} = 5$

$\Rightarrow$ $1 + 3 + 5 + x + y = 25$

$\Rightarrow$ $x + y = 16$

$$\sigma^2 = 8 = \frac{\sum x_i^2}{5} - \left(\bar{x}\right)^2$$

$$\Rightarrow \quad 8 = \frac{1^2 + 3^2 + 5^2 + x^2 + y^2}{5} - 25$$

$\Rightarrow$ $165 = 35 + x^2 + y^2$

$\Rightarrow$ $x^2 + y^2 = 130$

$\Rightarrow$ $(x + y)^2 - 2xy = 130$

$\Rightarrow$ $xy = 63$

$\Rightarrow$ $x = 7, y = 9$

Now, $x^3 + y^3$

$\Rightarrow$ $7^3 + 9^3$

$\Rightarrow$ $343 + 729$

$\Rightarrow$ 1072

8. Sum of series

$$\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \ldots + \frac{1}{51!0!} \text{ equals}$$

(1) $\frac{2^{51}}{50!}$ (2) 2^{51}

(3) $51 \cdot 2^{50}$ (4) $\frac{2^{50}}{51!}$

Answer (4)

Sol. $\frac{1}{51!}\left(\frac{51!}{1!50!} + \frac{51!}{3!48!} + \frac{51!}{5!46!} + \ldots. + \frac{51!}{51!0!}\right)$

$$= \frac{1}{51!}\left({}^{51}C_1 + {}^{51}C_3 + \ldots + {}^{51}C_{51}\right)$$

$$= \frac{1}{51!}\left(\frac{2^{51}}{2}\right) = \frac{2^{50}}{51!}$$

9. Let $R = \{(a, b) : 3a - 3b + \sqrt{7}$ is irrational$\}$

(1) R is an equivalence relation

(2) R is symmetric but not reflexive

(3) R is reflexive but not symmetric

(4) R is reflexive and symmetric but not transitive

Answer (3)

Sol. For reflexive

$3a - 3b + \sqrt{7} = \sqrt{7}$ is irrational

$\therefore$ $(a, a) \in R$

$\therefore$ reflexive

For symmetric

$(a, b) \in R$

$\Rightarrow$ $3a - 3b + \sqrt{7}$ is irrational

$\Rightarrow$ $3b - 3a + \sqrt{7}$ is irrational

$\Rightarrow$ $(b, a) \in R$

$\therefore$ Not symmetric

For transitive

$(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $3a - 3b + \sqrt{7}$ is irrational

$3b - 3c + \sqrt{7}$ is irrational

$3a - 3c + \sqrt{7}$ is irrational

$\therefore$ R is not transitive

10. Negation of the statement $p \vee (p \wedge \sim q)$ is

(1) p

(2) $\sim p$

(3) q

(4) $\sim q$

Answer (2)

Sol. $p \vee (p \wedge \sim q)$

$\equiv p$

$\sim (p \vee (p \wedge \sim q)) \equiv \sim p$

11. Let S be solution set for values of x satisfying $\cos^{-1}(2x)+\cos^{-1}\sqrt{1-x^2}=\pi$ then $\sum_{x\in S} 2\sin^{-1}(x^2-1)$ is equal to

(1) 0

(2) $-\sin^{-1}\left(\frac{24}{25}\right)$

(3) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(4) $\pi-\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Answer (2)

Sol. $\frac{\pi}{2}-\sin^{-1}(2x)+\frac{\pi}{2}-\sin^{-1}\sqrt{1-x^2}=\pi$

$\Rightarrow \sin^{-1}(2x)+\sin^{-1}\sqrt{1-x^2}=0$

$\Rightarrow \sin^{-1}(-2x)=\sin^{-1}\sqrt{1-x^2}$

$\Rightarrow -2x=\sqrt{1-x^2}$

$4x^2=1-x^2$

$\Rightarrow x=\pm\sqrt{\frac{1}{5}}$

$\boxed{x=-\frac{1}{\sqrt{5}}}$ is only possible solution

$\sum_{x\in S} 2\sin^{-1}(x^2-1)=2\sin^{-1}\left(-\frac{4}{5}\right)$

$=-2\sin^{-1}\frac{4}{5}$

$=-\sin^{-1}\left(\frac{24}{25}\right)$

12. A triangle be such that cos2A + cos2B + cos2C is minimum. If inradius of the triangle is 3 then which of the following is CORRECT?

(1) Area of Δ is $\frac{6\sqrt{3}}{2}$

(2) Perimeter of Δ is $18\sqrt{3}$

(3) sin2A + sin2B + sin2C = sin$A$ + sin$B$ – sin$C$

(4) Perimeter of triangle is $9\sqrt{3}$

Answer (2)

Sol. If k = cos2A + cos2B + cos2C is minimum then $k=\frac{-3}{2}$ and $A=B=C=\frac{\pi}{3}$

$\therefore \quad r=\frac{\Delta}{s}=3=\frac{\sqrt{3}a^2\cdot 2}{4\cdot 3a}$

$\Rightarrow \quad a=6\sqrt{3}$

$\therefore \quad \text{Area}=\frac{\sqrt{3}}{4}\cdot 36.3=27\sqrt{3}$

Perimeter is $18\sqrt{3}$

13. ?

14. ?

15. ?

16. ?

17. ?

18. ?

19. ?

20. ?

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Area bounded by $y = x|x - 3|$ and x-axis between $x = -1$ and $x = 2$ is A then $12A$ equals

Answer (18.00)

Sol. $y=\begin{cases} x(x-3) & x>3 \\ -x(x-3) & x\le 3 \end{cases}$

$A=\int_{-1}^{2}-x(x-3)dx=\int_{-1}^{2}\left(-x^2+3x\right)dx$

$=\left.\frac{-x^3}{3}+\frac{3x^2}{2}\right|_{-1}^{2}$

$=\left(-\frac{8}{3}+\frac{3}{2}\cdot 4\right)-\left(\frac{1}{3}+\frac{3}{2}\right)$

$=\frac{-8}{3}+6-\frac{11}{6}$

$=6-\frac{27}{6}=\frac{9}{6}$

$12A = 18$

22. Remainder when $23^{200} + 19^{200}$ divided by 49 equals

Answer (02.00)

Sol. $23^{200} + 19^{200} = (21+2)^{200} + (21-2)^{200}$

$= 2\left[{}^{200}C_0 21^{200} + {}^{200}C_2 21^{198} + {}^{200}C_4 21^{196} + ... + {}^{200}C_{198} 21^2 + {}^{200}C_{200} 21^0\right]$

= 2(49K + 1)

Remainder = 2

23. 8, a_1, a_2 … a_n are terms m A.P. Sum of first 4 terms of series is 50 and sum of last 4 terms of series is 170. Find product of middle terms of series

Answer (754)

Sol. $\frac{4}{2}[16 + 3d] = 50$

$\Rightarrow \quad d = 3$

$\frac{4}{2}[2a_n + 3(-d)] = 170$

$2a_n - 3d = 85$

$2a_n = 94$

$a_n = 47$

$8 + (n-1)d = 47$

$\boxed{n = 14}$

So 7th and 8th are middle terms

T_7 = 8 + 6.3 = 26

T_8 = 8 + 7.3 = 29

$T_7 . T_8$ = 754

24. A circle is represented by $\frac{|z-2|}{|z-3|} = 2$. Its radius is γ units and centre is (α, β) then find 3($\alpha + \beta + \gamma$)

Answer (12.00)

Sol. Let $z = x + iy$

$(x-2)^2 + y^2 = 4(x-3)^2 + 4y^2$

$x^2 + y^2 - 4x + 4 = 4x^2 - 24x + 36 + 4y^2$

$\Rightarrow \quad 3x^2 + 3y^2 - 20x + 32 = 0$

or $x^2 + y^2 - \frac{20x}{3} + \frac{32}{3} = 0$

centre $= \left(\frac{10}{3}, 0\right)$

$\gamma = \sqrt{\left(\frac{10}{3}\right)^2 + 0^2 - \frac{32}{3}} = \frac{2}{3}$

$3(\alpha + \beta + \gamma) = 12$

25. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = 2x + f'(1)$, then $f(4) - g(4)$ equals

Answer (12.00)

Sol. $g(x) = 2x + f'(1)$

$\Rightarrow \quad g'(x) = 2 \Rightarrow g'(1) = 2$ and $g''(x) = 0$

Now,

$f(x) = x^2 + g'(1)x + g''(1)$

$f(x) = x^2 + 2x$

$f'(x) = 2x + 2 \quad \Rightarrow f'(1) = 4$

$\therefore \quad g(x) = 2x + 4$

$f(4) - g(4) = (16+8) - (8+4)$

$= 24 - 12 = 12$

26. For some values of λ, system of equations

$\lambda x + y + z = 1$

$x + \lambda y + z = 1$

$x + y + \lambda z = 1$

has no solution, then $\sum\left(|\lambda|^2 + |\lambda|\right)$ equal

Answer (06.00)

Sol. $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$\Rightarrow \quad \lambda(\lambda^2 - 1) - 1(\lambda - 1) + 1(1 - \lambda) = 0$

$\Rightarrow \quad (\lambda - 1)(\lambda^2 + \lambda - 1 - 1) = 0$

$\Rightarrow \quad \lambda = 1, -2$

For λ = 1 there are infinite solution

for λ = –2, system has no solution

27. If solution of $\frac{dy}{dx}+\frac{(x+a)}{y-2}=0$ is a circle and $y(0) = 1$, area of circle is = 2π. P and Q are point of intersection of circle with y-axis. Normals at P and Q intersect x-axis at R and S. The length of RS is

Answer (0, 4)

Sol. $(y - 2)dy + (x + a)dx = 0$

$\Rightarrow \frac{y^2}{2}-2y+\frac{x^2}{2}+ax=c$

$\Rightarrow x^2 + y^2 + 2ax - 4y + 3 = 0$

$\Rightarrow (x + a)^2 + (y - 2)^2 = a^2 + 1$

Also $\sqrt{a^2+1}=\sqrt{2}$ (as area is 2π sq. units)

$\therefore C \equiv (x\pm 1)^2+(y-2)^2=2$

For P and Q put $x = 0$

$\Rightarrow y - 2 = 1$ or $-1 \Rightarrow P \equiv (0, 3)$ and $Q \equiv (0, 1)$

If $C \equiv (x + 1)^2 + (y - 2)^2 = 2$ then normal at P

$y = x + 3 \Rightarrow R \equiv (-3, 0)$

and normal at Q

$x + y = 1 \Rightarrow S \equiv (1, 0)$

$\therefore RS = 4$

If $C \equiv (x - 1)^2 + (y - 2)^2 = 2$

Normal at P $y = x + 1 \Rightarrow R \equiv (-1, 0)$

Normal at Q $x + y = 3 \Rightarrow S \equiv (3, 0)$

$\therefore RS = 4$

28. Find number of 3-digit number which are divisible by 2 or 3 but not divisible by 7.

Answer (536.00)

Sol. Numbers divisible by 2 = 450 = $n(A)$

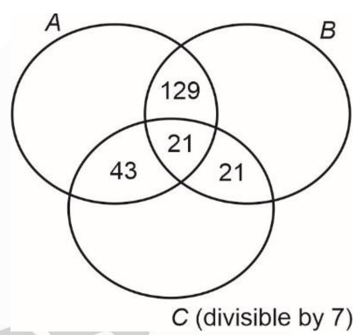
Numbers divisible by 3 = 300 = $n(B)$

Numbers divisible by 6 = 150

Numbers divisible by 2 and 7 = 64

Numbers divisible by 3 and 7 = 42

Numbers divisible by 2, 3 and 7 = 21



Total numbers = 450 + 300 – 150 – 43 – 21

= 600 – 64 = 536

29. ?

30. ?

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